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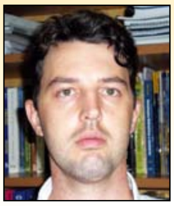
The minimum steel consumption in beams according to the prescriptions of the NBR 6118-2003 code – case study

Consumo mínimo de armadura em vigas segundo as prescrições da norma NBR 6118-2003 – estudo de caso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Abstract

The Brazilian code, NBR 6118:2003, recommends the usage of truss models with strut angle variation (θ) and tie angle (α) for the concrete beams design subjected to bending and shear force. This code allows the designer to choose between two design models of the shear steel: The model I, where the strut angle has the value of 45º and the portion of shear force resisted by the concrete V_c has a constant value, and the model II, where the strut angle varies from 30º to 45º, with V_c varying in function of the shear force calculation (V_{sd}) and the shear force resisted by the strut (V_{Rd2}). The stirrups angle varies from 45° to 90°. With the aim of helping the designers in the choice of optimum models and angles, the mentioned models were evaluated, in order to determine which of these models and (θ) and (α) angles lead to a minimum consumption of longitudinal and transverse steel in simply supported beams, subjected to bending combined with shear, with distributed loading.

Keywords: truss model, strut angle, shear strength, minimum steeel consumption

Resumo

A NBR 6118:2003 [1] recomenda o uso de modelos de treliça com variação do ângulo da escora (θ) e do tirante transversal (α) para o dimensionamento de vigas de concreto submetidas à flexão e força cortante. Esta norma permite ao projetista optar entre dois modelos de dimensionamento da armadura transversal: O modelo I, onde o ângulo da escora tem valor de 45º e a parcela de força cortante resistida pelo concreto V_c tem valor constante, e o modelo II, onde o ângulo da escora varia de 30º a 45º, com V_c variando em função da força cortante de cálculo (V_{sd}) e da força cortante resistida pela escora (V_{Rd2}). O ângulo α dos tirantes transversais varia entre 45º e 90º. Com o objetivo de auxiliar os projetistas na escolha do modelo e ângulos ótimos, foram avaliados os modelos mencionados, determinando-se qual destes modelos e os ângulos (θ) e (α) levam ao consumo mínimo de armaduras longitudinal e transversal em vigas simplesmente apoiadas, submetidas à flexão combinada com cortante, com carregamento distribuído.

Palavras-chave: Modelo de treliça, ângulo da escora, resistência a cortante, consumo mínimo de armadura

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1. Introduction

The design of the beams reinforced subjected to combined actions of bending and shear can be done based only on the plane truss model, both to the transverse steel and to the longitudinal steel.
The NBR 6118:2003 [1] code prescribes the calculation conditions for beams based on the truss model of parallel flange. Two calculation models are admitted for the transversal steel design, being the designer in charge of choosing one of them.
In general, the designer does not have criteria to choose one or other model, usually choosing the most simplified calculation model, without taking into account which model leads to smaller amounts of longitudinal and transverse steel and what values of the strut and the tie angles lead to a more economical design.
Differently from other papers done before about this theme [2, 3, 4, 5, 6], where the "corrective term V_c" was not regarded, this one considers when analyses the models described by the code, evaluating the influence of the strut angles "θ" and the transverse tie "α", in the volume of transverse steel and the total steel of the beam, considering the resistant contribution of the concrete (V_c). The angles α e θ that lead to a lower cost for a certain value of shear force are obtained. It is considered the case of simply supported beams, with constant height and width with uniformly distributed loading. The found values for each model are compared, searching thus, to help the designer in the choice of the more economical model.

2. General procedures to determine the longitudinal steel volume according to the generalized truss model

The resultant of tension in the longitudinal steel can be determined through the plane truss model, making possible, thus, to express the longitudinal steel area in terms of the angles α e θ .
From the Figure 1, and applying moment in relation to the O point it´s obtained:

$$R_{st}.z + R_t.\frac{z}{2}.(\cot\theta + \cot\alpha).\sin\alpha = M + \Delta M \quad (1)$$

Where:
R_{cc} is the resultant of compression in the concrete;
R_{st} is the resultant of tension in the longitudinal steel;
R_t is the resultant of tension in the transverse steel;
V_{sd} is the shear force;
s é o spacing between the stirrups;
z is the lever arm.
Reorganizing and replacing the values of ΔM and R_t, it´s obtained the resultant of tension in the longitudinal steel.

$$R_{st} = \frac{M_{sd}}{z} + \frac{V_{sd}}{2}(\cot\theta - \cot\alpha) \quad (2)$$

The longitudinal steel area is given by the Equation 3, where f_{yld} is the design strength to the longitudinal steel tension [4, 5].

$$A_{sL} = \frac{1}{f_{yld}}.\left(\frac{|M_{sd}|}{z} + \frac{|V_{sd}|}{2}.(\cot\theta - \cot\alpha)\right) \quad (3)$$

Therefore, the longitudinal steel volume ($V_{ol,L}$) between the sections of the maximum and minimum moment (Figure 2) can be obtained through the integration along of the length L_o, that is:

$$V_{ol,L} = \int_0^{Lo} \frac{1}{f_{yld}}\left(\frac{|M_{sd}|}{z} + \frac{|V_{sd}|}{2}(\cot\theta - \cot\alpha)\right)dx \quad (4)$$

The Figure 2 shows the diagrams moment and shear for uniformly distributed loading. Replacing the equations of shear force and bending moment in the Equation 4 and integrating, it´s obtained the Equation 5.

$$V_{ol,L} = \frac{1}{f_{yld}}\left(\frac{V_{sd}.L_0^2}{3.z} + \frac{1}{2}\frac{V_{sd}.L_o}{2}(\cot\theta - \cot\alpha)\right) \quad (5)$$

Figure 1 – Forces in a beam section

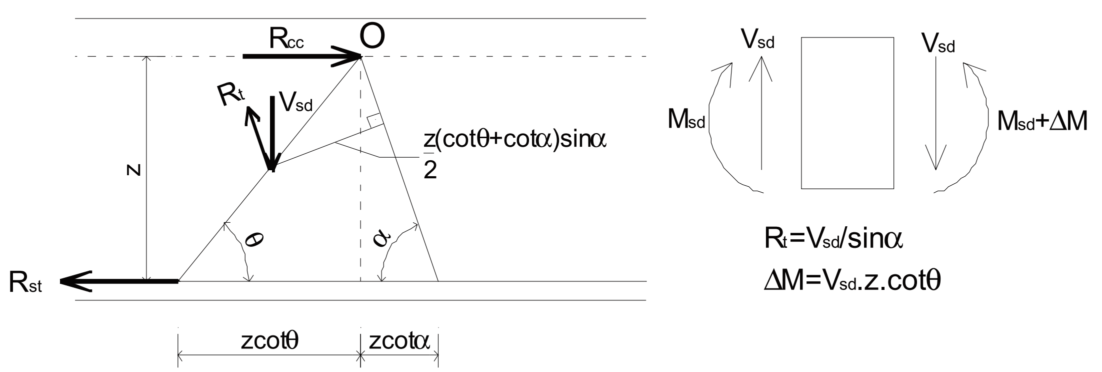
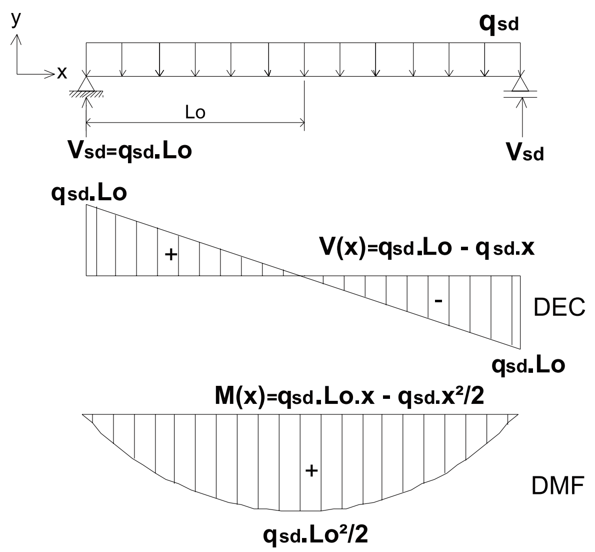


Figure 2 – Forces diagram in simply supported beams with distributed loading



3. Transverse steel volume

In the chapter 17, item 17.4 of the NBR 6118:2003 [1] code, it is described how must be done the calculations of the transverse steel and the compression strut verification for the linear elements that are subject to shear force in the Ultimate Limit State.

The calculation conditions set for the beams are based on the analogy with the truss model, of parallel flange, associated to the parcel of shear force resisted by the concrete, V_c, absorbed by complementary mechanisms to the truss: aggregate interlock in the face of the inclined fissure and dowel action of the longitudinal steel.

The methodology for the design establishes that the strength to the shear force, in a certain cross-section, must be considered satisfactory when the following conditions are done simultaneously:

- **Verification of the compressed strut of the concrete**

$$V_{sd} \le V_{Rd2} \quad (6)$$

- **Verification of the rupture for diagonal tension**

$$V_{sd} \le V_{Rd3} = V_c + V_{sw} \quad (7)$$

Where:

- V_{Sd} is the shear force of calculation;
- V_{Rd2} is the resistant shear force of calculation, pertinent to the rupture of the compressed diagonals of concrete;
- V_{Rd3} is the strength shear force of calculation pertinent to the rupture for diagonal tension.

To evaluate which of the alternative models due to the NBR 6118:2003 [1] code leads to a more economical solution it was realized a procedure based in the obtained equations of the truss model with the addition of the "corrective term V_c".

The volume of transverse steel ($V_{ol,w}$) along of the length L_0, between the sections of maximum and minimum shear force can be expressed by the following equation:

$$V_{ol,w} = \int_0^{Lo} K.\frac{A_{sw}}{s}.dx \quad (8)$$

The K value of this equation is related to the dimensions, shape and angle of the stirrups. If all the stirrups be rectangular with the same dimension, where b' and h' be the width and the height, respectively, this value will be:

$$K = \frac{h'}{\sin\alpha} + b' \quad (9)$$

Considering the plane truss model shown in the Figure 3, the resultant of tension stress in the steel can be written through the equation 10.

$$\underbrace{\frac{z(\cot\theta + \cot\alpha)}{s}}_{n^\circ \text{ the stirrups}} A_{sw}.f_{ywd} = R_t = \frac{V_{sd}}{\sin\alpha} \quad (10)$$

The transverse steel area per unit of length (A_{sw}/s) can be defined through the truss model, considering the lever arm "z" equals to 0.9.d, as:

$$\frac{A_{sw}}{s} = \frac{|V|}{0{,}9.f_{ywd}.d.(\cot\theta + \cot\alpha)\sin\alpha} \quad (11)$$

The parcel of shear force resisted by the concrete V_c must be considered in the calculation of the transverse steel and being "θ" and "α" the struts and stirrups angles, respectively, and considering the other constant factors in the length L_0, the transverse steel volume can be defined according to the Equation 12:

$$V_{ol,w} = \frac{K}{0{,}9.f_{ywd}.d.(\cot\theta + \cot\alpha)\sin\alpha}.\int_0^{Lo}(V_{sd} - V_c)dx \quad (12)$$

For the distributed loading, the V_{sd} shear force varies along of the length L_0 (Figure 2). In such case, in a certain length (a), the shear force V_{sd} reaches a certain value of $V_{sd,min}$, that leads to the minimal transverse steel. It is considered that from this length, the beam will have minimum transverse steel. In the Figure 4, it can be noticed that the value of (a) can be determined through similarity of triangles, according to the Equation 13:

$$a = \frac{(V_{sd} - V_{sd,min}).L_o}{V_{sd}} \quad (13)$$

Where the $V_{sd,min}$ can be calculated through the Equation 14, taking the value of V_c and the value of the angles α and θ according to each model recommended by the code:

$$V_{sd,min} = \left(\frac{A_{sw,min}}{s}\right).0{,}9.d.f_{ywd}.(\cot\alpha + \cot\theta)\sin\alpha + V_c \quad (14)$$

$$V_{sd}(x) = V_{sd} - \frac{V_{sd}.x}{L_o} \quad (15)$$

$$V_{ol,w} = \int_0^{a} K.\frac{A_{sw}}{s}.dx + \int_a^{Lo} K.\frac{A_{sw,min}}{s}.dx \quad (16)$$

Equation 15 defines the V_{sd} variation along of the length L_0 (Figure 2).
The transverse steel volume is calculated by adding the necessary volume for region 1 to the minimum volume of transverse steel for the region 2, calculated as the item 17.4.1.1 of the NBR 6118:2003 [1], resulting in Equation 16.

Figure 3 – Forces in a beam section

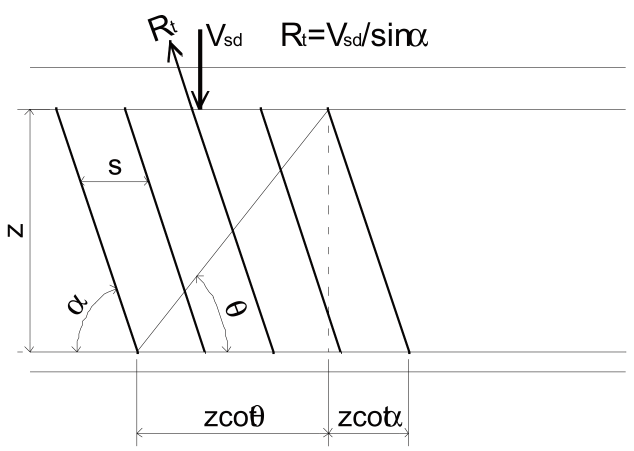
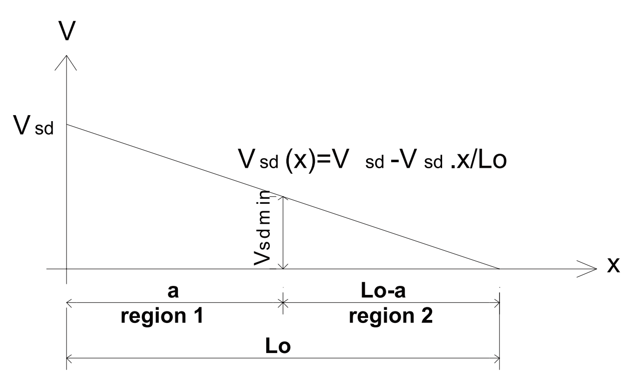


Figure 4 – Illustrative schema of the shear force variation along of L_o



$$\frac{A_{sw,\min}}{s}=\frac{0{,}06.\sqrt[3]{f_{ck}^{\ 2}}.b_w.\sin\alpha}{f_{ywk}} \tag{17}$$

Being the minimal transverse steel per unit of length defined as Equation 17:

The transverse steel volume, considering the generalized truss model, for simply supported beams with uniformly distributed loading is obtained through the Equation 18,

$$V_{ol,w}=\frac{K}{0{,}9.d.f_{ywd}.(\cot\theta+\cot\alpha)\sin\alpha}.\int_0^a V_{sd}-V_c.dx+\frac{K.0{,}06.\sqrt[3]{f_{ck}^{\ 2}}.b_w.\sin\alpha}{f_{ywd}}.(L_o-a) \tag{18}$$

3.1 Calculation of the transverse steel volume considering Model I

$$\mathrm{V_{co}}=\frac{0{,}126\sqrt[3]{\mathrm{f_{ck}}^{2}}\mathrm{b_w}.\mathrm{d}}{\gamma_c} \tag{19}$$

The Model I admits θ = 45° in relation to the longitudinal axis of the structural member and also admits the complementary parcel V_c has a constant value equals to V_{co}, according to the Equation 19, with f_{ck} in MPa.

Knowing that Equation 20.

Doing the due replacements in Equation 18, the transverse steel volume for the model I can be obtained through the equation below (Equation 21).

$$\int_0^a \mathrm{V_{sd}}.\mathrm{dx}=\mathrm{a}\left(\mathrm{V_{sd}}-\frac{\mathrm{V_{sd}}.\mathrm{a}}{2.\mathrm{L_o}}\right) \tag{20}$$

$$V_{ol,w}=\frac{K.a}{0{,}9.d.f_{ywd}.(\sin\alpha+\cos\alpha)}\left(V_{sd}-V_{co}-\frac{V_{sd}.a}{2.L_o}\right)+\frac{0{,}06.K.\sqrt[3]{f_{ck}^{\ 2}}.b_w.\sin\alpha}{f_{ywk}}.(L_o-a) \tag{21}$$

3.2 Calculation of the transverse steel volume considering the Model II

$$V_c=\frac{(V_{Rd2}-V_{sd}).V_{co}}{(V_{Rd2}-V_{co})} \tag{22}$$

In this model it´s admitted that the complementary parcel V_c suffers reduction with the increase of V_{sd} (Figure 5), as the following Equation 22.

Replacing the Equation 22 in the Equation 18 and reorganizing the terms it´s obtained Equation 23.

$$\mathrm{V_{ol,w}}=\frac{\mathrm{K}}{0{,}9\mathrm{d}.\mathrm{f_{ywd}}(\cot\theta+\cot\alpha)\mathrm{sen}\,\alpha}\left[\left(1+\frac{\mathrm{V_{co}}}{(\mathrm{V_{Rd2}}-\mathrm{V_{co}})}\right)\int_0^a \mathrm{V_{sd}}.\mathrm{dx}-\frac{\mathrm{V_{Rd2}}\,\mathrm{V_{co}}\,\mathrm{a}}{(\mathrm{V_{Rd2}}-\mathrm{V_{co}})}\right]+\mathrm{K}\frac{\mathrm{A_{sw,min}}}{\mathrm{s}}\sin\alpha(\mathrm{L_o}-\mathrm{a}) \tag{23}$$

Integrating and replacing the value of $\frac{\mathrm{A_{sw,min}}}{\mathrm{s}}$ after mathematical transformations (Equation 24).

$$V_{ol,w}=\frac{K}{0{,}9.d.f_{ywd}.(\cot\theta+\cot\alpha)\sin\alpha}.\left(\frac{V_{Rd2}.a}{(V_{Rd2}-V_{co})}\left(V_{sd}-V_{co}-\frac{V_{sd}.a}{2.L_o}\right)\right)+\frac{0{,}06.K.\sqrt[3]{f_{ck}^{\ 2}}.b_w.\sin\alpha}{f_{yw}}.(L_o-a) \tag{24}$$

$$V_{Rd2} = 0{,}54\left(1 - \frac{f_{ck}}{250}\right)\frac{f_{ck}}{\gamma_c}.b_w.d\sin^2\theta\,(\cot\alpha + \cot\theta) \quad (25)$$

The value of V_{co} is obtained through the Equation 20 and the value of V_{Rd2}, pertinent to the verification of the concrete diagonal compression is obtained through the Equation 25 with f_{ck} in MPa.
The value of V_{co} is obtained through the Equation 20 and the value of V_{Rd2}, relative to the verification of the diagonal compression of the concrete is obtained through the Equation 25, with f_{ck} in MPa.

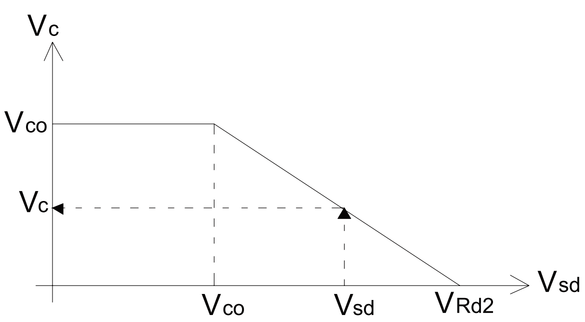
4. Parameterized case study

The necessary total steel volume is given by the sum of the transversal and longitudinal steel equations, for both models.
Considering that the cost of the volume per unit of the transverse steel is K_{ce} times the cost of volume per unit of the longitudinal steel, the most economical model will be determined by the difference of cost between the Model II and the Model I, considering for it the same angle value for both models and varying the strut angle of the Model II.

$$\Delta C = C_{MII} - C_{MI} \quad (26)$$

It can be noticed from the presented equations that on the contrary to the realized work by [3, 4], it becomes impossible to generalize the process in a dimensionless way, being necessary to accomplish a case study.
In this study it was considered the average steel cost in Brazil equals to R$4,00/kg, being the transverse steel cost considered bigger than the longitudinal steel. To evaluate the influence of this cost difference between the two steels, the results were compared using the factor K_{ce} equals to 1,1, as already evaluated by [2,3], and 1,3.
With the aim of not considering the distributed loading (q_{sd}) in the equations developing process, it was considered these loadings in function of V_{sd}. The value of the shear force of calculation (V_{sd}) was limited with the objective of the section to have minimal ductility to bending with simple steel. This value was obtained in accordance with the NBR 6118:2003 code, resulting in Vsd=103,5 kN for the uniformly distributed loading, for the realized model.
With the previously presented equations it was developed an algorithm and using the computational programming in MATLAB® version 7.0, it was calculated the longitudinal and transversal steel volume and the total cost, for all combinations of the strut and the tie angles defined by the code. In possession of this information, the differences of cost between the two models were calculated.
Description of the considered beam:
- Transverse section = 15x45 cm
- f_{ck} = 25 MPa, f_{ylk} =500 MPa, f_{ywk} = 600 MPa, d = 0,94.h, z = 0,9.d, L = 14.d
- Covering: 3,0cm

Figure 5 – Graphic of the V_c variation in relation to V_{sd}



5. Results and discussions

The figure 6 shows the volume variation of the longitudinal steel in relation to the transverse tie angle (α), for θ values equals to 30°, 35°, 40° and 45°. It can be noticed that the increase of the strut angle results in a volume decrease of the longitudinal steel. In this case, the models I and II with α=45° presented equal volumes and they correspond to the smaller volumes of longitudinal steel between the models.
The figure 7 shows the volume variation of the transverse steel in relation to the transverse tie angle (α), for θ values equals to 30°, 35°, 40° and 45° and also the minimum transverse steel volume recommended by the code. It can be observed that the increase of the strut angle results in a volume increase of the transverse steel. The smallest volume was obtained for the model II with θ=30° and α=60°. The volume variation for the vertical stirrup case, when compared in the model II the angles of θ=45° with θ=30° can achieve up to 19%. It´s also found that the model II due to the adoption of smaller values for V_c, even for the value of θ=45°, provides a transverse steel volume bigger than the model I whose the value is approximately equals to the model II, with θ=40°.
The figures 8 and 9 show the cost variation of the total steel in relation to the transverse tie angle (α) and the cost difference between the models II and I, for K_{ce}=1,1 e K_{ce}=1,3, respectively. It can be noticed that the model I provides the lowest cost when compared to the model II, being possible to reach approximately 14% of economy when compared the models I with α=45° and the model II with θ=30° and α=90°. For the model II the lowest cost was obtained when α=45° and θ=45°. For the vertical stirrups case it´s obtained a cost difference of 4,8%, 2,8% and 1,5% for K_{ce}=1,1 and 4,5%, 2,6% and 1,5% for K_{ce}=1,3 for θ of 30°, 35° and 40°, respectively.

6. Conclusions

From the realized study in this paper, can be deduced some conclusions in relation to what optimal model, inclined strut an-

Figure 6 – Volume variation of the longitudinal steel, with relation to the transversal tie angle (α) for θ values equal to 30°, 35°, 40° e 45°

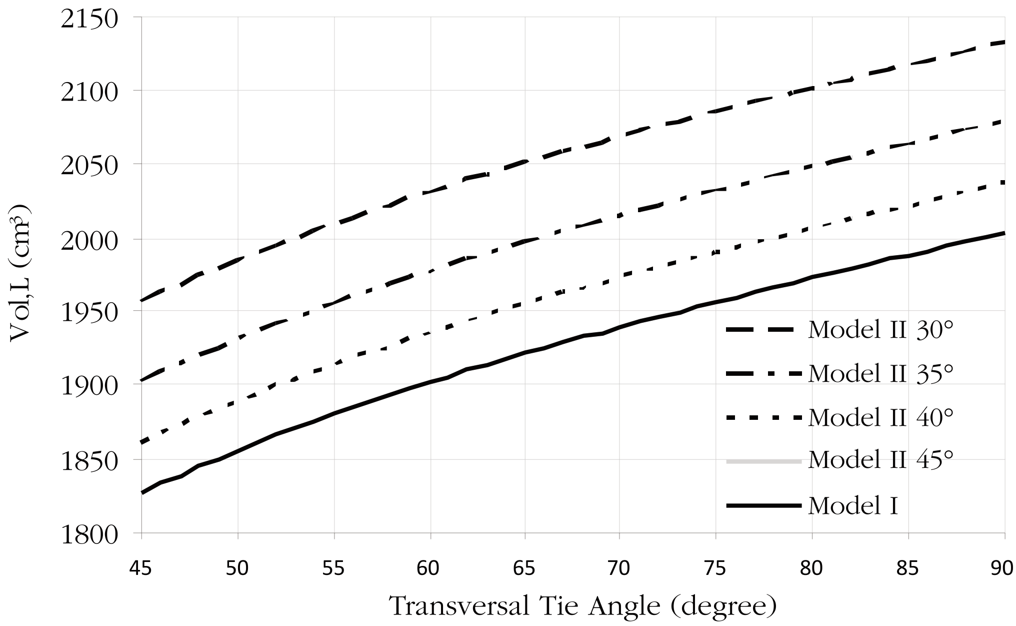
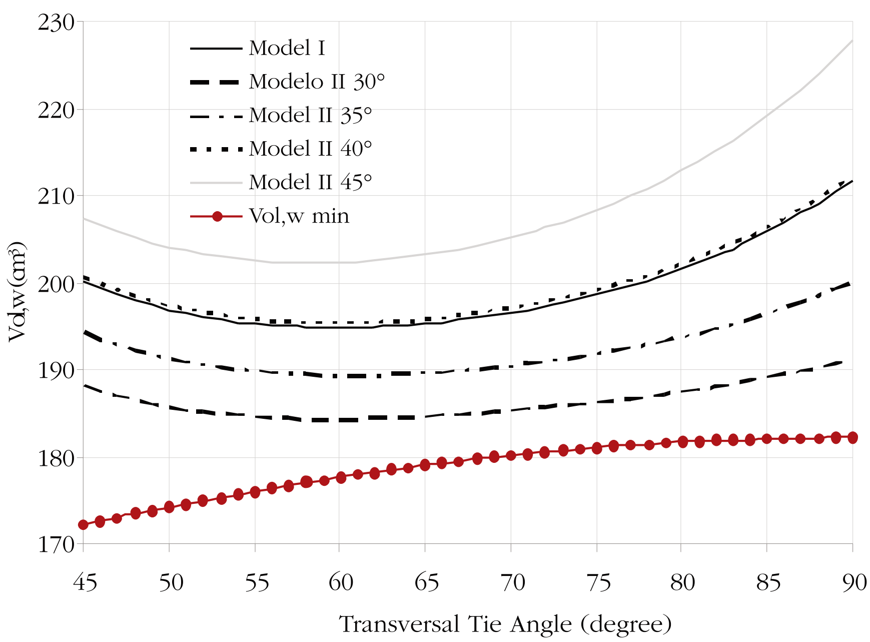


Figure 7 – Volume variation of the transverse steel, with relation to the transversal tie angle (α) for θ values equal to 30°, 35°, 40° e 45°



gle (θ) and transverse tie angle (α), would lead to the smallest consumption of longitudinal and transversal steel, according to the prescriptions of the NBR 6118:2003 code, these are:
For the transverse steel, the smaller is the inclined strut angle (θ), the smaller is the steel consumption, being the model II with θ=30° and α=60° the one that leads to the smallest steel consumption.
When a designer intends to achieve economy in the total steel consumption (transversal and longitudinal) in the beams design using the truss model, with vertical stirrups ($\alpha=90^0$), he must use the model I, where the strut angle is always 45^0. To have economy in the transverse steel, with vertical stirrups, it must used the model II, with $\theta=30^0$, using for the longitudinal steel design the traditional sectional method.
The longitudinal steel volume was the determinant factor of the lower cost, thus the Model I is the one that leads to lower costs of steel prevailing the transversal tie optimal angle equals to 45°. In the case of vertical stirrups, the economy can be approximately 5%, when compared the model I and the model II with θ=30°, for the analyzed values of K_{ce}.

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Figure 8 – Total costs of steel obtained using models I and II and difference of total costs of steel between the models II and I, for, K_{ce} = 1,1

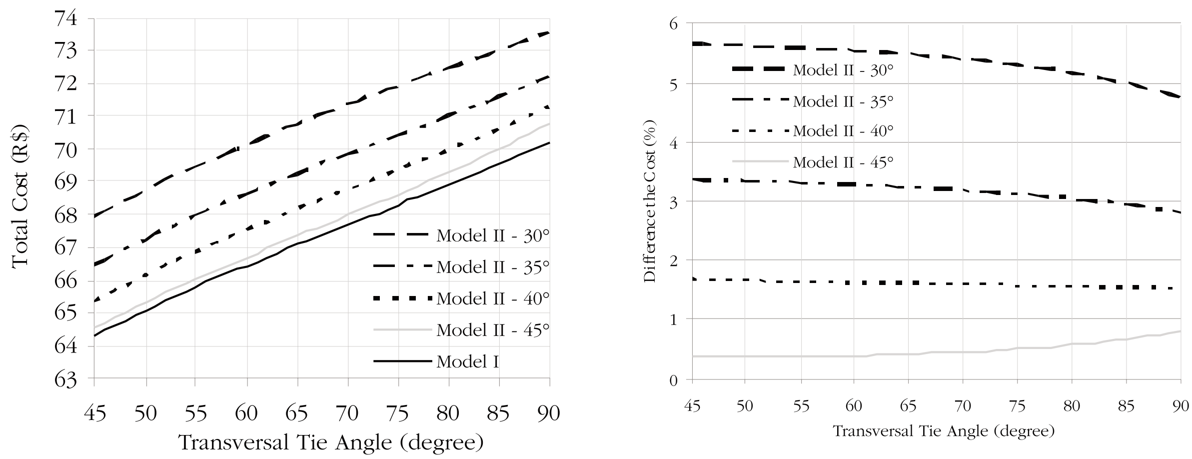
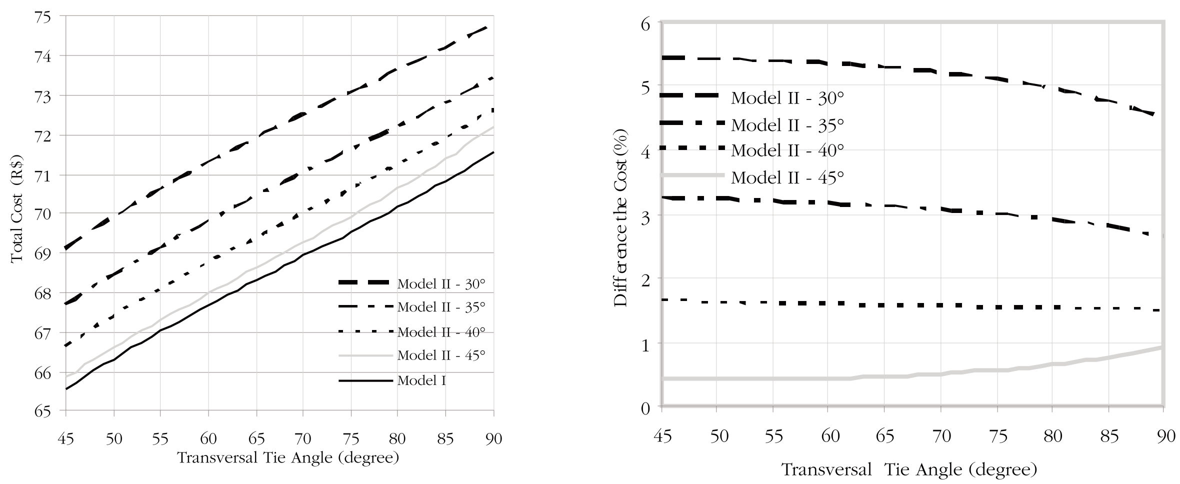


Figure 9 – Total costs of steel obtained using models I and II and difference of total costs of steel between the models II and I, for, K_{ce} = 1,3



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