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Comparative analysis among standards of the area calculation of transversal reinforcement on reinforced concrete beams of high resistance subjected by shear force

Análise comparativa entre normas do cálculo da área da armadura transversal em vigas de concreto armado de resistência elevada submetidas à ação de força cortante

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Abstract

High strength concretes (HSC) correspond to a characteristic compression strength between 55 e 90 MPa. With the growing use of HSC, studies about the regular design standards of elements made of it, specifically standards about design on shear, become necessary. Hence, the main aspects of the NBR, Model Code 1990 e 2010, Portuguese Standard and German Standard related to the design on shear are presented. From the numerical simulations, with the addition of Cladera and Marí's experimental contributions, it is confirmed that the Brazilian design standard procedure produces lower transverse reinforcement areas in comparison to the ones predicted by the international codes; these, excepted by LoA III, do not consider the concrete contribution, in spite of being experimentally verified, leading to very conservative results.

Keywords: design, shear, high strength.

Resumo

Concretos de alta resistência (CAR) correspondem a uma resistência à compressão característica compreendida entre 55 e 90 MPa. Com a possibilidade crescente da utilização de CAR, faz-se necessária a realização de estudos que abordem os tratamentos normativos usuais acerca do dimensionamento de elementos por ele constituídos, especificamente, à ação de força cortante. Portanto, são apresentadas os principais aspectos da NBR, Model Code 1990 e 2010, Norma portuguesa e alemã acerca dos dimensionamento à cortante. Das simulações numéricas, acrescidas das contribuições experimentais de Cladera e Marí, constata-se que o procedimento de cálculo da NBR produz áreas de estribos inferiores às previstas pelos códigos internacionais; estes, com exceção do LoA III, não adotam a contribuição do concreto, apesar de esta ser verificada experimentalmente, levando a resultados muito conservadores.

Palavras-chave: dimensionamento, cortante, alta resistência.

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1. Introduction

High strength concretes (HSC) correspond to a characteristic compression strength (f_{ck}) between 55 e 90 MPa, according to ABNT NBR 6118:2014 [5]. Its use has been disseminated due to the demand for structures for which weight reduction is important and/or for when the architecture imposes the use of more slender pieces (Silva [1]). HSC are obtained by improving the compaction of the concrete mixtures, which improves the resistance of the paste and its interface with coarse aggregates (Cladera [2]).

From the analysis of the standard's procedures, the importance of the concrete class for the design of the elements is prominent. Thus, with the increased possibility of using concretes from group II, it is necessary to conduct studies that approach the usual codes' treatments regarding the design of high strength concrete elements, specifically the design on shear. According to Arslan [3], the

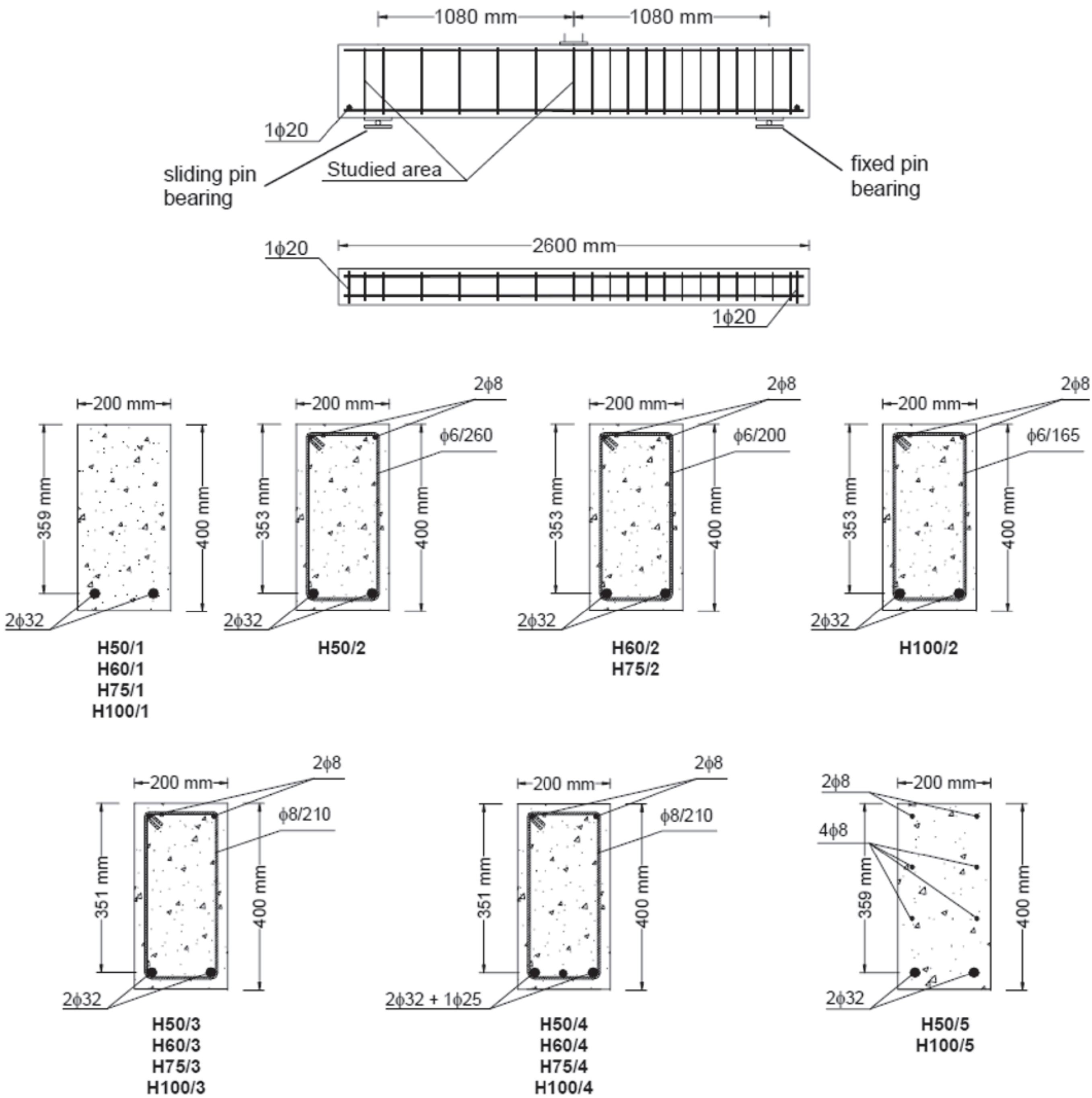

Figure 1
Test set-up and cross-section of the beam specimens. (Cladera [2])

Table 1
ANN for beams with web reinforcement.
Ranges of parameters in the database [4]

Parameter	Minimum	Maximum
d (mm)	198	925
d/b	0.792	4.5
ρ_l (%)	0.50	5.80
ρ_w (MPa)	0.33	3.57
f_c (MPa)	21	125.2
a/d	2.49	5.00
V (kN)	63.28	1172.19

concrete´s contribution is important in the design of beams where the factored shear force is close to the value of the shear force required to produce diagonal tension cracking, and also necessary for the economic design of beams and slabs with little or no shear reinforcement.
Among the experimental studies regarding the high strength concrete beams subjected to shear force, those of Cladera [2] and Cladera & Marí [4] are highlighted. In the first, 18 beams of reinforced concrete beams – which characteristics are illustrated in Figure 1 – with compressive strength between 50 and 87 MPa, were tested at the Structural Technology Laboratory of the Department of Construction Engineering at the School of Civil Engineering of Barcelona. The main objectives of the experimental program were to study the influence of concrete compressive strength in beams with and without shear reinforcement; to propose and verify a more adequate minimum amount of web reinforcement then the proposed by the Spanish code EHE Instrución de Hormigón Estructural of 1998; to evaluate the efficiency of the amount of shear and longitudinal reinforcement as a function of f_{ck}; and to study the influence of the longitudinally-distributed web reinforcement in beams without stirrups.
In the second [4], Eurocode 2, AASHTO LRFD and ACI 318-02 were evaluated with an Artificial Neural Network (ANN) based on 123 test-beams of high strength concrete. From the results of the ANN, the authors analyzed the influences of the amount of transverse web reinforcement, the effect of the beams' size and effective depth, of the concrete compressive strength, of the amount of longitudinal reinforcement and the ratio between shear span and the effective depth on the shear strength. Hence, an alternate design method was proposed. The ANN contemplated the test-beams with the characteristics indicated in Table 1.

1.1 Justification

The expansion of the use of high strength concretes indicates the need for better understanding the structural behavior of the elements made of it. This understanding comprehends the usual design standard's procedures. From the numerical analyses and simulations, comparisons are made to explain how each code approaches the issue, specifically about shear design. In addition, the experimental contributions of Cladera [2] and Cladera and Marí [4] will base the comparative analyses between the standard's predictions of shear reinforcement area and those demanded according to experimental results. The ultimate shear forces, the experimentally predicted and the obtained through normative calculation, are also contemplated in the analysis.

2. Analysed design standards

2.1 ABNT NBR 6118:2014 [5]

The Brazilian code presents two design models for linear elements subjected to shear force. For both, the minimum ratio of web reinforcement is given by:

$$\rho_{sw} = \frac{A_{sw}}{b_w \cdot s \cdot \text{sen}\alpha} \geq 0{,}2\frac{f_{ct,m}}{f_{ywk}} \quad (1)$$

where:
A_{sw} is the shear reinforcement area;
s is the longitudinal space between stirrups, along the longitudinal axis of the structural element;
α is the inclination of the transverse reinforcement related to the longitudinal axis of the structural element; it is contained in the interval 45° ≤ α ≤ 90°;
b_w is the effective web width;
f_{ywk} is the characteristic value of yield strength of the reinforcing steel;
$f_{ct,m}$ = 2,12 ln (1 + 0,11 f_{ck}) for concretes C55 até C90.
The resistance is considered satisfactory when simultaneously observes the following conditions:

$$V_{Sd} \leq V_{Rd2} \quad (2)$$

$$V_{Sd} \leq V_{Rd3} = V_c + V_{sw} \quad (3)$$

where:
V_{Sd} is the design value of applied shear force;
V_{Rd2} is the design shear strength related to the concrete failure by diagonal compression;
V_{Rd3} is the design shear force related to the diagonal tension failure, where V_c is the amount of shear force absorbed by the complementary mechanisms of the truss and V_{sw} the amount of shear force resisted by the transverse reinforcement.

2.1.1 Calculation Model I

This Model, conducted by the following expressions, admits struts' inclination of θ = 45° in relation to the longitudinal axis of the structural element and complementary portion V_c constant and independent from V_{Sd}.
a) Verification of the concrete failure by diagonal compression:

$$V_{Rd2} = 0{,}27 \propto_{v2} f_{cd}\, b_w\, d \quad (4)$$

where:
$\propto_{v2} = (1 - f_{ck}/250)$ and f_{ck} in MPa;
b) Calculation of the transverse reinforcement:

$$V_{Rd3} = V_c + V_{sw} \quad (5)$$

where:
$V_{sw} = (A_{sw} / s)\ 0{,}9 d f_{ywd}\ (\text{sen}\alpha + \cos\alpha)$
$V_c = V_{c0} = 0{,}6\ f_{ctd}\ b_w\ d$
$f_{ctd} = f_{ctk,inf} / \gamma_c = 0{,}7\ f_{ct,m} / \gamma_c$

where:
d is the distance between the compressed edge and the center of gravity of the reinforcement;
f_{ywd} is the stress on the passive transverse reinforcement not superior to 435 MPa.

2.1.2 Calculation Model II

This Model admits θ inclinations between 30° and 45° and the decrease of V_c with the increase of V_{Sd}.

a) Verification of the concrete failure by diagonal compression:

$$V_{Rd2} = 0{,}54 \propto_{v2} f_{cd}\, b_w\, d\, \mathrm{sen}^2\theta\, (\cot g\, \alpha + \cot g\, \theta) \quad (6)$$

b) Calculation of the transverse reinforcement, as shown on Equation 5, where:
$V_{sw} = (A_{sw} / s)\ 0{,}9 d f_{ywd}\ (\cot g\ \alpha + \cot g\ \theta)\ \mathrm{sen}\alpha$;
$V_{c1} = V_{c0}$ when $V_{Sd} \leq V_{c0}$;
$V_{c1} = 0$ when $V_{Sd} = V_{Rd2}$;
$V_{c1} = \left(\frac{V_{Rd2} - V_{Sd}}{V_{Rd2} - V_{c0}}\right) V_{c0}$ for intermediate values.

Table 2
Results of Model I of ABNT NBR 6118:2014

f_{ck} (MPa)	$f_{ct,m}$ (MPa)	f_{ctd} (MPa)	b_w (cm)	d (cm)	V_{Sd} (kN)	V_{Rd2} (kN)	V_{c0} (kN)	V_{sw} (kN)	A_{sw} (cm²/m)
55	4.14	2.07	12	40	100.00	397.13	59.62	40.38	2.58
55	4.14	2.07	12	40	125.00	397.13	59.62	65.38	4.17
55	4.14	2.07	12	40	150.00	397.13	59.62	90.38	5.77
55	4.14	2.07	12	40	175.00	397.13	59.62	115.38	7.37
60	4.30	2.15	12	40	100.00	422.13	61.92	38.08	2.43
60	4.30	2.15	12	40	125.00	422.13	61.92	63.08	4.03
60	4.30	2.15	12	40	150.00	422.13	61.92	88.08	5.62
60	4.30	2.15	12	40	175.00	422.13	61.92	113.08	7.22
70	4.59	2.29	12	40	100.00	466.56	66.04	33.96	2.20
70	4.59	2.29	12	40	125.00	466.56	66.04	58.96	3.76
70	4.59	2.29	12	40	150.00	466.56	66.04	83.96	5.36
70	4.59	2.29	12	40	175.00	466.56	66.04	108.96	6.96
80	4.84	2.42	12	40	100.00	503.59	69.68	30.32	2.32
80	4.84	2.42	12	40	125.00	503.59	69.68	55.32	3.53
80	4.84	2.42	12	40	150.00	503.59	69.68	80.32	5.13
80	4.84	2.42	12	40	175.00	503.59	69.68	105.32	6.73
90	5.06	2.53	12	40	100.00	533.21	72.92	27.08	2.43
90	5.06	2.53	12	40	125.00	533.21	72.92	52.08	3.33
90	5.06	2.53	12	40	150.00	533.21	72.92	77.08	4.92
90	5.06	2.53	12	40	175.00	533.21	72.92	102.08	6.52

Table 3
Results of Model II of ABNT NBR 6118:2014

f_{ck} (MPa)	$f_{ct,m}$ (MPa)	f_{ctd} (MPa)	θ°	b_w (cm)	d (cm)	V_{Sd} (kN)	V_{Rd2} (kN)	V_{c1} (kN)	V_{sw} (kN)	A_{sw} (cm²/m)
55	4.14	2.07	45	12	40	100.00	397.13	52.49	47.51	3.03
55	4.14	2.07	45	12	40	125.00	397.13	48.07	76.93	4.91
55	4.14	2.07	45	12	40	150.00	397.13	43.66	106.34	6.79
55	4.14	2.07	45	12	40	175.00	397.13	39.24	135.76	8.67
60	4.30	2.15	45	12	40	100.00	422.13	55.37	44.63	2.85
60	4.30	2.15	45	12	40	125.00	422.13	51.07	73.93	4.72
60	4.30	2.15	45	12	40	150.00	422.13	46.77	103.23	6.59
60	4.30	2.15	45	12	40	175.00	422.13	42.48	132.52	8.46
70	4.59	2.29	45	12	40	100.00	466.56	60.44	39.56	2.53
70	4.59	2.29	45	12	40	125.00	466.56	56.32	68.68	4.39
70	4.59	2.29	45	12	40	150.00	466.56	52.20	97.80	6.25
70	4.59	2.29	45	12	40	175.00	466.56	48.08	126.92	8.11
80	4.84	2.42	45	12	40	100.00	503.59	64.81	35.19	2.32
80	4.84	2.42	45	12	40	125.00	503.59	60.79	64.21	4.10
80	4.84	2.42	45	12	40	150.00	503.59	56.78	93.22	5.95
80	4.84	2.42	45	12	40	175.00	503.59	52.76	122.24	7.81
90	5.06	2.53	45	12	40	100.00	533.21	68.63	31.37	2.43
90	5.06	2.53	45	12	40	125.00	533.21	64.67	60.33	3.85
90	5.06	2.53	45	12	40	150.00	533.21	60.71	89.29	5.70
90	5.06	2.53	45	12	40	175.00	533.21	56.75	118.25	7.55

2.2 CEB-FIP Model Code 1990 [6]

The model code of 1990 brings in its section 6.3.3 *Shear and axial action effects* the considerations presented below. This standard establishes the θ inclination of the struts between 18.4º and 45º.

a) Minimum transverse reinforcement ratio:

$$\omega_{sw} = \frac{A_{sw} \cdot f_{yk}}{b_w \cdot s \cdot f_{ctm} \cdot \text{sen}\alpha} \geq 0{,}2 \quad (7)$$

where:

$$f_{ctm} = f_{ctko,m}\left(\frac{f_{ck}}{f_{cko}}\right)^{2/3}$$

f_{cko} = 10 MPa

$f_{ctko,m}$ = 1,40 MPa

b) Maximum shear resistance, with θ = 45°:

$$V_{Rd,max} = \frac{f_{cd2}}{2}\, b_w\, z(1 + \text{cotg}\ \alpha) \quad (8)$$

where:

$$f_{cd2} = 0{,}60\left[1 - \frac{f_{ck}}{250}\right] f_{cd}$$

Table 4
Results of MC 1990

f_{ck} (MPa)	$f_{ct,m}$ (MPa)	f_{cd2} (MPa)	b_w (cm)	d (cm)	θ°	$V_{Rd,max}$ (kN)	V_{Sd} (kN)	F_{Stw} (kN)	A_{sw} (cm²/m)
55	4.36	17.16	12	40	45	370.66	100.00	100.00	6.39
55	4.36	17.16	12	40	45	370.66	125.00	125.00	7.99
55	4.36	17.16	12	40	45	370.66	150.00	150.00	9.58
55	4.36	17.16	12	40	45	370.66	175.00	175.00	11.18
60	4.62	18.24	12	40	45	393.98	100.00	100.00	6.39
60	4.62	18.24	12	40	45	393.98	125.00	125.00	7.99
60	4.62	18.24	12	40	45	393.98	150.00	150.00	9.58
60	4.62	18.24	12	40	45	393.98	175.00	175.00	11.18
70	5.12	20.16	12	40	45	435.46	100.00	100.00	6.39
70	5.12	20.16	12	40	45	435.46	125.00	125.00	7.99
70	5.12	20.16	12	40	45	435.46	150.00	150.00	9.58
70	5.12	20.16	12	40	45	435.46	175.00	175.00	11.18
80	5.60	21.76	12	40	45	470.02	100.00	100.00	6.39
80	5.60	21.76	12	40	45	470.02	125.00	125.00	7.99
80	5.60	21.76	12	40	45	470.02	150.00	150.00	9.58
80	5.60	21.76	12	40	45	470.02	175.00	175.00	11.18
90	6.06	23.04	12	40	45	497.66	100.00	100.00	6.39
90	6.06	23.04	12	40	45	497.66	125.00	125.00	7.99
90	6.06	23.04	12	40	45	497.66	150.00	150.00	9.58
90	6.06	23.04	12	40	45	497.66	175.00	175.00	11.18

Table 5
Results of MC 2010 for LoA I

f_{ck} (MPa)	$f_{ct,m}$ (MPa)	b_w (cm)	d (cm)	θ°	k_ε	k_c	V_{Ed} (kN)	$V_{Rd,max}$ (kN)	$V_{Rd,s}$ (kN)	A_{sw} (cm²/m)
55	4.21	12	40	45	0.55	0.45	100.00	355.91	100.00	6.39
55	4.21	12	40	45			125.00	355.91	125.00	7.99
55	4.21	12	40	45			150.00	355.91	150.00	9.58
55	4.21	12	40	45			175.00	355.91	175.00	11.18
60	4.35	12	40	45	0.55	0.44	100.00	377.17	100.00	6.39
60	4.35	12	40	45			125.00	377.17	125.00	7.99
60	4.35	12	40	45			150.00	377.17	150.00	9.58
60	4.35	12	40	45			175.00	377.17	175.00	11.18
70	4.61	12	40	45	0.55	0.41	100.00	417.99	100.00	6.39
70	4.61	12	40	45			125.00	417.99	125.00	7.99
70	4.61	12	40	45			150.00	417.99	150.00	9.58
70	4.61	12	40	45			175.00	417.99	175.00	11.18
80	4.84	12	40	45	0.55	0.40	100.00	456.90	100.00	6.39
80	4.84	12	40	45			125.00	456.90	125.00	7.99
80	4.84	12	40	45			150.00	456.90	150.00	9.58
80	4.84	12	40	45			175.00	456.90	175.00	11.18
90	5.04	12	40	45	0.55	0.38	100.00	494.23	100.00	6.39
90	5.04	12	40	45			125.00	494.23	125.00	7.99
90	5.04	12	40	45			150.00	494.23	150.00	9.58
90	5.04	12	40	45			175.00	494.23	175.00	11.18

c) Tension of web steel:

$$F_{Stw} = \frac{V_{Sd}}{\text{sen } \propto} \quad (9)$$

$$F_{Rtw} = \left[\frac{A_{sw} \cdot f_{yd}}{s}\right] z(\text{cotg } \theta + \text{cotg } \alpha) \quad (10)$$

where:

F_{Stw} is the applied shear force on the transverse reinforcement;

F_{Rtw} is the resistant shear force of the reinforcement.

2.3 CEB-FIP Model Code 2010 [7]

Ahead, the considerations of the 2010 version of the model code are presented.

a) Minimum shear reinforcement area:

$$A_{\text{sw,min}} = 0{,}08 \cdot \sqrt{f_{ck}} \cdot \frac{b_w \cdot s_w}{f_{yk}} \quad (11)$$

b) Design shear resistance:

$$V_{Rd} = V_{Rd,c} + V_{Rd,s} \geq V_{Ed} \quad (12)$$

Table 6
Results of MC 2010 for LoA II

f_{ck} (MPa)	b_w (cm)	d (cm)	k_ε	k_c	V_{Ed} (kN)	$V_{Rd,max}$ (kN)	$V_{Rd,s}$ (kN)	A_{sw} (cm²/m)
55	12	40	0.65	0.53	100.00	420.62	100.00	6.39
55	12	40			125.00	420.62	125.00	7.99
55	12	40			150.00	420.62	150.00	9.58
55	12	40			175.00	420.62	175.00	11.18
60	12	40	0.65	0.52	100.00	445.74	100.00	6.39
60	12	40			125.00	445.74	125.00	7.99
60	12	40			150.00	445.74	150.00	9.58
60	12	40			175.00	445.74	175.00	11.18
70	12	40	0.65	0.49	100.00	493.99	100.00	6.39
70	12	40			125.00	493.99	125.00	7.99
70	12	40			150.00	493.99	150.00	9.58
70	12	40			175.00	493.99	175.00	11.18
80	12	40	0.65	0.47	100.00	539.98	100.00	6.39
80	12	40			125.00	539.98	125.00	7.99
80	12	40			150.00	539.98	150.00	9.58
80	12	40			175.00	539.98	175.00	11.18
90	12	40	0.65	0.38	100.00	584.09	100.00	6.39
90	12	40			125.00	584.09	125.00	7.99
90	12	40			150.00	584.09	150.00	9.58
90	12	40			175.00	584.09	175.00	11.18

Table 7
Results of MC 2010 for LoA III

f_{ck} (MPa)	b_w (cm)	d (cm)	θ_{min}	ε_x	k_ε	k_c	k_v	V_{Ed} (kN)	$V_{Rd,min}$ (kN)	$V_{Rd,c}$ (kN)	$V_{Rd,s}$ (kN)	A_{sw} (cm²/m)
55	12	40	30	0.001	0.65	0.53	0.116	100.00	364.27	24.79	75.21	4.80
55	12	40	30				0.105	125.00	364.27	22.45	102.55	6.55
55	12	40	30				0.094	150.00	364.27	20.10	129.90	8.30
55	12	40	30				0.083	175.00	364.27	17.76	157.24	10.05
60	12	40	30	0.001	0.65	0.52	0.119	100.00	386.02	26.45	73.55	4.70
60	12	40	30				0.108	125.00	386.02	24.14	100.86	6.44
60	12	40	30				0.098	150.00	386.02	21.82	128.18	8.19
60	12	40	30				0.087	175.00	386.02	19.51	155.49	9.93
70	12	40	30	0.001	0.65	0.49	0.123	100.00	427.80	28.25	71.75	4.58
70	12	40	30				0.113	125.00	427.80	26.09	98.91	6.32
70	12	40	30				0.104	150.00	427.80	23.94	126.06	8.05
70	12	40	30				0.095	175.00	427.80	21.78	153.22	9.79
80	12	40	30	0.001	0.65	0.47	0.126	100.00	467.63	28.98	71.02	4.54
80	12	40	30				0.117	125.00	467.63	27.01	97.99	6.26
80	12	40	30				0.109	150.00	467.63	25.04	124.96	7.98
80	12	40	30				0.100	175.00	467.63	23.07	151.93	9.71
90	12	40	45	0.001	0.65	0.45	0.128	100.00	505.83	29.58	70.42	4.50
90	12	40	45				0.120	125.00	505.83	27.75	97.25	6.21
90	12	40	45				0.113	150.00	505.83	25.93	124.07	7.93
90	12	40	45				0.105	175.00	505.83	24.11	150.89	9.64

where:
V_{Rd} is the design shear resistance;
$V_{Rd,c}$ is the design shear resistance attributed to the concrete;
$V_{Rd,s}$ is the design shear resistance provided by shear reinforcement;
V_{Ed} is the design shear force.

c) Maximum shear resistance for stirrups at 90°:

$$V_{Rd,max} = k_c \cdot \frac{f_{ck}}{\gamma_c} \cdot b_w \cdot z \cdot \text{sen}\,\theta \cdot \cos\theta \tag{13}$$

where:
$k_c = k_\varepsilon \cdot \eta_{fc}$ is the strength reduction factor;

$$\eta_{f_c} = \left(\frac{30}{f_{ck}}\right)^{1/3} \leq 1{,}0.$$

d) Design shear resistance provided by stirrups at 90°:

$$V_{Rd,s} = \frac{A_{sw}}{s_w} \cdot f_{ywd} \cdot z \cdot \text{cotg}\,\theta \tag{14}$$

e) Design shear resistance attributed to the concrete:

$$V_{Rd,c} = k_v \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_w \cdot z \tag{15}$$

Table 8
Results of NP EN 1992-1-1, with υ

f_{ck} (MPa)	$f_{ct,m}$ (MPa)	υ	b_w (cm)	d (cm)	V_{Ed} (kN)	$V_{Rd,max}$ (kN)	$V_{Rd,s}$ (kN)	A_{sw} (cm²/m)
55	4.21	0.468	12	40	100.00	370.66	100.00	6.39
55	4.21	0.468	12	40	125.00	370.66	125.00	7.99
55	4.21	0.468	12	40	150.00	370.66	150.00	9.58
55	4.21	0.468	12	40	175.00	370.66	175.00	11.18
60	4.35	0.456	12	40	100.00	393.98	100.00	6.39
60	4.35	0.456	12	40	125.00	393.98	125.00	7.99
60	4.35	0.456	12	40	150.00	393.98	150.00	9.58
60	4.35	0.456	12	40	175.00	393.98	175.00	11.18
70	4.61	0.432	12	40	100.00	435.46	100.00	6.39
70	4.61	0.432	12	40	125.00	435.46	125.00	7.99
70	4.61	0.432	12	40	150.00	435.46	150.00	9.58
70	4.61	0.432	12	40	175.00	435.46	175.00	11.18
80	4.84	0.408	12	40	100.00	470.02	100.00	6.39
80	4.84	0.408	12	40	125.00	470.02	125.00	7.99
80	4.84	0.408	12	40	150.00	470.02	150.00	9.58
80	4.84	0.408	12	40	175.00	470.02	175.00	11.18
90	5.04	0.384	12	40	100.00	497.66	100.00	6.39
90	5.04	0.384	12	40	125.00	497.66	125.00	7.99
90	5.04	0.384	12	40	150.00	497.66	150.00	9.58
90	5.04	0.384	12	40	175.00	497.66	175.00	11.18

Table 9
Results of NP EN 1992-1-1, with υ

f_{ck} (MPa)	υ_1	b_w (cm)	d (cm)	V_{Ed} (kN)	$V_{Rd,max}$ (kN)	$V_{Rd,s}$ (kN)	A_{sw} (cm²/m)
55	0.60	12	40	100.00	475.20	100.00	6.94
55	0.60	12	40	125.00	475.20	125.00	8.68
55	0.60	12	40	150.00	475.20	150.00	10.42
55	0.60	12	40	175.00	475.20	175.00	12.15
60	0.60	12	40	100.00	518.40	100.00	6.94
60	0.60	12	40	125.00	518.40	125.00	8.68
60	0.60	12	40	150.00	518.40	150.00	10.42
60	0.60	12	40	175.00	518.40	175.00	12.15
70	0.55	12	40	100.00	554.40	100.00	6.94
70	0.55	12	40	125.00	554.40	125.00	8.68
70	0.55	12	40	150.00	554.40	150.00	10.42
70	0.55	12	40	175.00	554.40	175.00	12.15
80	0.50	12	40	100.00	576.00	100.00	6.94
80	0.50	12	40	125.00	576.00	125.00	8.68
80	0.50	12	40	150.00	576.00	150.00	10.42
80	0.50	12	40	175.00	576.00	175.00	12.15
90	0.50	12	40	100.00	648.00	100.00	6.94
90	0.50	12	40	125.00	648.00	125.00	8.68
90	0.50	12	40	150.00	648.00	150.00	10.42
90	0.50	12	40	175.00	648.00	175.00	12.15

where:

$\sqrt{f_{ck}} \leq 8\text{MPa}$.

f) Compressive stress field inclination:

$$\theta_{min} \leq \theta \leq 45° \quad (16)$$

The code also presents the levels-of-approximation approach. According to Muttoni and Ruiz [8], this approach is based on the use of theories based on physical parameters where the hypotheses for their applications can be refined according to the demand for accuracy. As Barros [9] highlights, the increase in the approximation level (I to IV) is followed by the increase of precision and of the time expended for the analyses.

2.3.1 Level I Approximation

In this level, the $V_{Rd,c}$ of Equation 12 is not considered. For reinforced concrete members, θ_{min} = 30°. In addition, to calculate $V_{Rd,max}$, $k_\varepsilon = 0,55$.

Table 10
Results of DIN 1045-1

f_{ck} (MPa)	$f_{ct,m}$ (MPa)	b_w (cm)	d (cm)	V_{Ed} (kN)	$V_{Rd,max}$ (kN)	$V_{Rd,s}$ (kN)	A_{sw} (cm^2/m)
55	4.21	12	40	100.00	891.00	100.00	6.39
55	4.21	12	40	125.00	891.00	125.00	7.99
55	4.21	12	40	150.00	891.00	150.00	9.58
55	4.21	12	40	175.00	891.00	175.00	11.18
60	4.35	12	40	100.00	972.00	100.00	6.39
60	4.35	12	40	125.00	972.00	125.00	7.99
60	4.35	12	40	150.00	972.00	150.00	9.58
60	4.35	12	40	175.00	972.00	175.00	11.18
70	4.61	12	40	100.00	1134.00	100.00	6.39
70	4.61	12	40	125.00	1134.00	125.00	7.99
70	4.61	12	40	150.00	1134.00	150.00	9.58
70	4.61	12	40	175.00	1134.00	175.00	11.18
80	4.84	12	40	100.00	1296.00	100.00	6.39
80	4.84	12	40	125.00	1296.00	125.00	7.99
80	4.84	12	40	150.00	1296.00	150.00	9.58
80	4.84	12	40	175.00	1296.00	175.00	11.18
90	5.04	12	40	100.00	1458.00	100.00	6.39
90	5.04	12	40	125.00	1458.00	125.00	7.99
90	5.04	12	40	150.00	1458.00	150.00	9.58
90	5.04	12	40	175.00	1458.00	175.00	11.18

Table 11
Transverse reinforcement areas (cm^2/m) for beams of 20 cm x 60 cm

f_{ck} (MPa)	V_{Sd} (kN)	NBR		MC 1990	MC 2010			NP EN 1992		DIN 1045
		M I	M II		LoA I	LoA II	LoA III	ν	ν_1	
55	200.00	3.31	3.31	8.52	8.52	8.52	5.68	8.52	9.26	8.52
55	250.00	4.30	5.06	10.65	10.65	10.65	8.01	10.65	11.57	10.65
55	300.00	6.43	7.56	12.78	12.78	12.78	10.34	12.78	13.89	12.78
55	375.00	9.62	11.32	15.97	15.97	15.97	13.83	15.97	17.36	15.97
60	200.00	3.44	3.44	8.52	8.52	8.52	5.51	8.52	9.26	8.52
60	250.00	4.05	4.75	10.65	10.65	10.65	7.83	10.65	11.57	10.65
60	300.00	6.18	7.24	12.78	12.78	12.78	10.16	12.78	13.89	12.78
60	375.00	9.37	10.99	15.97	15.97	15.97	13.65	15.97	17.36	15.97
70	200.00	3.67	3.67	8.52	8.52	8.52	5.33	8.52	9.26	8.52
70	250.00	3.67	4.21	10.65	10.65	10.65	7.64	10.65	11.57	10.65
70	300.00	5.74	6.69	12.78	12.78	12.78	9.95	12.78	13.89	12.78
70	375.00	8.94	10.41	15.97	15.97	15.97	13.42	15.97	17.36	15.97
80	200.00	3.87	3.87	8.52	8.52	8.52	5.26	8.52	9.26	8.52
80	250.00	3.87	3.87	10.65	10.65	10.65	7.56	10.65	11.57	10.65
80	300.00	5.36	6.22	12.78	12.78	12.78	9.86	12.78	13.89	12.78
80	375.00	8.55	9.92	15.97	15.97	15.97	13.31	15.97	17.36	15.97
90	200.00	4.05	4.05	8.52	8.52	8.52	5.21	8.52	9.26	8.52
90	250.00	4.05	4.05	10.65	10.65	10.65	7.50	10.65	11.57	10.65
90	300.00	5.01	5.80	12.78	12.78	12.78	9.78	12.78	13.89	12.78
90	375.00	8.20	9.50	15.97	15.97	15.97	13.21	15.97	17.36	15.97

2.3.2 Level II Approximation

As occurs in LoA I, the $V_{Rd,c}$ portion of Equation 12 is disregarded. The minimum inclination of the compressive stress field is given by Equation 17, but will be adopted as 30°. In addition, the parameter k_ε, given by Equation 18, will assume its maximum value of 0,65 on the further developed simulations.

$$\theta_{\min} = 20° + 10000\varepsilon_x \tag{17}$$

$$k_\varepsilon = \frac{1}{1{,}2 + 55\varepsilon_1} \leq 0{,}65 \tag{18}$$

2.3.3 Level III Approximation

In this level, the whole Equation 12 is valid. The maximum shear resistance is given by Equation 13 for $\theta = \theta_{min}$, given by Equation 17 and admitted as 30°. To determinate the $V_{Rd,c}$ attributed to the concrete (Equation 15), the parameter k_υ is calculated as shown on Equation 19. For the following examples, $\varepsilon_x = 0{,}001$.

$$k_v = \frac{0{,}4}{1 + 1500\varepsilon_x}\left(1 - \frac{V_{Ed}}{V_{Rd,max}(\theta_{\min})}\right) \geq 0 \tag{19}$$

2.3.4 Level IV approximation

The model code does not bring specific expressions for this level-of-approximation, but establishes that the resistance of members in shear or shear combined with torsion may be determined by satisfying the applicable conditions of equilibrium and compatibility of strains and by using appropriate stress-strain models for steel and for diagonally cracked concrete.

2.4 NP EN 1992-1-1:2010 [10]

Ahead, the Portuguese standard considerations on shear are shown.

a) Minimum shear reinforcement area: also given by Equation 11.

b) Design shear resistance:

$$V_{\mathrm{Rd}} = V_{\mathrm{Rd,s}} + V_{\mathrm{ccd}} + V_{\mathrm{td}} \tag{20}$$

where:

$V_{Rd,s}$ is the design value of the shear force which can be sustained by the yielding shear reinforcement;

V_{ccd} is the design value of the shear component of the force in the compression area (inclined compression chord);

V_{td} is the design value of the shear component of the force in the tensile reinforcement (inclined tensile chord).

c) Concrete compression strut inclination:

$$21{,}8° \leq \theta \leq 45° \tag{21}$$

d) Design value of the maximum shear force for stirrups at 90°:

$$V_{\mathrm{Rd,max}} = \propto_{\mathrm{cw}} \cdot b_{\mathrm{w}} \cdot z \cdot \nu_1 \cdot \frac{f_{\mathrm{ck}}}{\gamma_{\mathrm{c}}} / (\mathrm{cotg}\,\theta + \mathrm{tg}\,\theta) \tag{22}$$

where:

v_1 is the strength reduction factor for concrete cracked in shear;

$\propto_{cw}$ is the coefficient taking account of the state of the stress in the compression chord. For reinforced concrete members, $\propto_{cw} = 1{,}0$. The value of v_1 is given by Equation 23.

$$\nu = 0{,}6\left[1 - \frac{f_{\mathrm{ck}}}{250}\right] \tag{23}$$

If the design stress of the shear reinforcement is below 80% of the characteristic yield stress f_{yk}, v_1 may be taken as:

$v_1 = 0{,}6$ for $f_{ck} \leq 60$ MPa

$v_1 = 0{,}9 - f_{ck} / 200 > 0{,}5$ for $f_{ck} \geq 60$ MPa

e) Shear resistance of the stirrups at 90°:

$$V_{\mathrm{Rd},s} = \frac{A_{\mathrm{sw}}}{s} \cdot f_{\mathrm{ywd}} \cdot z \cdot \mathrm{cotg}\,\theta \tag{24}$$

f) Maximum effective cross-sectional area of the shear reinforcement for $\theta = 45°$:

$$\frac{A_{\mathrm{sw,max}} \cdot f_{\mathrm{ywd}}}{b_{\mathrm{w}} \cdot s} \leq \frac{0{,}5 \cdot \propto_{\mathrm{cw}} \cdot \nu_1 \cdot f_{\mathrm{cd}}}{sen\ \alpha} \tag{25}$$

Table 12
Transverse reinforcement areas (cm²/m) for beams of 60 cm x 165 cm

f_{ck} (MPa)	V_{Sd} (kN)	NBR		MC 1990	MC 2010			NP EN 1992		DIN 1045
		M I	M II		LoA I	LoA II	LoA III	ν	ν_1	
55	3000	27.41	32.25	46.46	46.46	46.46	39.91	46.46	50.51	46.46
55	3200	30.50	35.89	49.56	49.56	49.56	43.30	49.56	53.87	49.56
55	3400	33.60	39.53	52.66	52.66	52.66	46.68	52.66	57.24	52.66
55	3600	36.69	43.18	55.76	55.76	55.76	50.07	55.76	60.61	55.76
60	3000	26.67	31.26	46.46	46.46	46.46	39.36	46.46	50.51	46.46
60	3200	29.77	34.89	49.56	49.56	49.56	42.74	49.56	53.87	49.56
60	3400	32.86	38.51	52.66	52.66	52.66	46.13	52.66	57.24	52.66
60	3600	35.96	42.14	55.76	55.76	55.76	49.51	55.76	60.61	55.76
70	3000	25.36	29.54	46.46	46.46	46.46	38.69	46.46	50.51	46.46
70	3200	28.45	33.14	49.56	49.56	49.56	42.06	49.56	53.87	49.56
70	3400	31.55	36.75	52.66	52.66	52.66	45.42	52.66	57.24	52.66
70	3600	34.64	40.36	55.76	55.76	55.76	48.79	55.76	60.61	55.76
80	3000	24.19	28.08	46.46	46.46	46.46	38.35	46.46	50.51	46.46
80	3200	27.29	31.67	49.56	49.56	49.56	41.69	49.56	53.87	49.56
80	3400	30.39	35.27	52.66	52.66	52.66	45.04	52.66	57.24	52.66
80	3600	33.48	38.86	55.76	55.76	55.76	48.38	55.76	60.61	55.76
90	3000	23.16	26.83	46.46	46.46	46.46	38.07	46.46	50.51	46.46
90	3200	26.25	30.41	49.56	49.56	49.56	41.40	49.56	53.87	49.56
90	3400	29.35	34.00	52.66	52.66	52.66	44.72	52.66	57.24	52.66
90	3600	32.45	37.59	55.76	55.76	55.76	48.05	55.76	60.61	55.76

2.5 DIN 1045-1:2001-07 [11]

The German standard establishes the following considerations on shear design.

a) Inclination of struts:

$$18{,}43° \leq \theta \leq 59{,}88° \quad (26)$$

b) Design shear resistance, limited by the strength of the struts:

$$V_{\mathrm{Rd,max}} = \frac{b_{\mathrm{w}} \cdot z \cdot \alpha_{\mathrm{c}} \cdot f_{\mathrm{ck}}}{\mathrm{cotg}\,\theta + \mathrm{tg}\,\theta} \quad (27)$$

where:

α_c is a reduction factor equal to 0.75 η_1, that is, 0.75 for normal-weight concrete.

c) Design shear resistance, limited by the capacity of the shear reinforcement:

$$V_{\mathrm{Rd},sy} = \frac{A_{\mathrm{sw}}}{s_{\mathrm{w}}} \cdot f_{\mathrm{yd}} \cdot z \cdot \mathrm{cotg}\,\theta \quad (28)$$

3. Numerical simulations of the reinforcement areas according to the design standards

With the objective of evaluating each analyzed procedure, three situations are proposed, each one with four intensities of shear force and considering the high strength classes of concrete. For comparison purposes, the inclinations of all struts will be taken as θ = 45°. The areas were calculated using the expressions presented in section 2.

After the proposed situations, the ultimate shear strengths obtained through the codes' expressions will be evaluated.

3.1 Exemple 01

The first example consists of a beam of 12 cm width by 40 cm, subjected to four shear force intensities: 100 kN, 125 kN, 150 kN and 175 kN. The transverse reinforcement areas obtained are presented in Tables 2 to 10.

Table 13
Ultimate shear strengths

Design standard	V/bd (MPa)				
	C55	C60	C70	C80	C90
NBR – I	8.27	8.79	9.72	10.49	11.11
NBR – II	8.27	8.79	9.72	10.49	11.11
MC 1990	7.72	8.21	9.07	9.79	10.37
MC 2010 LoA I	7.41	7.86	8.71	9.52	10.30
MC 2010 LoA II	8.76	9.29	10.29	11.25	12.17
MC 2010 LoA III	7.59	8.04	8.91	9.74	10.54
NP EN 1992 ν	7.72	8.21	9.07	9.79	10.37
NP EN 1992 ν_1	9.90	10.80	11.55	12.00	13.50
DIN 1045	18.56	20.25	23.63	27.00	30.38

Table 14
Comparative percentages of the areas of example 01

f_{ck} (MPa)	V_{Sd} (kN)	A_{sw} / $A_{sw\,(MC\,1990)}$ %								
		NBR		MC 1990	MC 2010			NP EN 1992		DIN 1045
		M I	M II		LoA I	LoA II	LoA III	ν	ν_1	
55	100	40.36	47.49	100.00	100.00	100.00	75.21	100.00	108.70	100.00
55	125	52.28	61.51	100.00	100.00	100.00	82.04	100.00	108.70	100.00
55	150	60.22	70.86	100.00	100.00	100.00	86.60	100.00	108.70	100.00
55	175	65.90	77.54	100.00	100.00	100.00	89.85	100.00	108.70	100.00
60	100	38.07	44.61	100.00	100.00	100.00	73.55	100.00	108.70	100.00
60	125	50.44	59.11	100.00	100.00	100.00	80.69	100.00	108.70	100.00
60	150	58.69	68.78	100.00	100.00	100.00	85.45	100.00	108.70	100.00
60	175	64.59	75.69	100.00	100.00	100.00	88.85	100.00	108.70	100.00
70	100	34.46	39.54	100.00	100.00	100.00	71.75	100.00	108.70	100.00
70	125	47.14	54.92	100.00	100.00	100.00	79.13	100.00	108.70	100.00
70	150	55.94	65.17	100.00	100.00	100.00	84.04	100.00	108.70	100.00
70	175	62.23	72.49	100.00	100.00	100.00	87.55	100.00	108.70	100.00
80	100	36.35	36.35	100.00	100.00	100.00	71.02	100.00	108.70	100.00
80	125	44.24	51.34	100.00	100.00	100.00	78.39	100.00	108.70	100.00
80	150	53.52	62.12	100.00	100.00	100.00	83.31	100.00	108.70	100.00
80	175	60.15	69.81	100.00	100.00	100.00	86.82	100.00	108.70	100.00
90	100	38.05	38.05	100.00	100.00	100.00	70.42	100.00	108.70	100.00
90	125	41.64	48.24	100.00	100.00	100.00	77.80	100.00	108.70	100.00
90	150	51.36	59.49	100.00	100.00	100.00	82.71	100.00	108.70	100.00
90	175	58.30	67.54	100.00	100.00	100.00	86.22	100.00	108.70	100.00

3.2 Exemple 02

The second example consists of a beam of 20 cm width by 60 cm, subjected to four shear force intensities: 200 kN, 250 kN, 300 kN and 375 kN. Table 11 demonstrates the area values obtained from each normative treatment.

3.3 Exemple 03

The third example consists of a beam of 60 cm width by 165 cm, subjected to shear forces of 3000 kN, 3200 kN, 3400 kN and 3600 kN. The calculation areas obtained are presented in Table 12.

Table 15
Comparative percentages of the areas of example 02

f_{ck} (MPa)	V_{Sd} (kN)	A_{sw} / $A_{sw\ (MC\ 1990)}$ %								
		NBR		MC 1990	MC 2010			NP EN 1992		DIN 1045
		M I	M II		LoA I	LoA II	LoA III	ν	ν_1	
55	200	38.88	38.88	100.00	100.00	100.00	66.66	100.00	108.70	100.00
55	250	40.36	47.49	100.00	100.00	100.00	75.21	100.00	108.70	100.00
55	300	50.29	59.17	100.00	100.00	100.00	80.90	100.00	108.70	100.00
55	375	60.22	70.86	100.00	100.00	100.00	86.60	100.00	108.70	100.00
60	200	40.38	40.38	100.00	100.00	100.00	64.63	100.00	108.70	100.00
60	250	38.07	44.61	100.00	100.00	100.00	73.55	100.00	108.70	100.00
60	300	48.38	56.70	100.00	100.00	100.00	79.50	100.00	108.70	100.00
60	375	58.69	68.78	100.00	100.00	100.00	85.45	100.00	108.70	100.00
70	200	43.07	43.07	100.00	100.00	100.00	62.54	100.00	108.70	100.00
70	250	34.46	39.54	100.00	100.00	100.00	71.75	100.00	108.70	100.00
70	300	44.94	52.35	100.00	100.00	100.00	77.90	100.00	108.70	100.00
70	375	55.94	65.17	100.00	100.00	100.00	84.04	100.00	108.70	100.00
80	200	45.44	45.44	100.00	100.00	100.00	61.80	100.00	108.70	100.00
80	250	36.35	36.35	100.00	100.00	100.00	71.02	100.00	108.70	100.00
80	300	41.92	48.65	100.00	100.00	100.00	77.16	100.00	108.70	100.00
80	375	53.52	62.12	100.00	100.00	100.00	83.31	100.00	108.70	100.00
90	200	47.56	47.56	100.00	100.00	100.00	61.21	100.00	108.70	100.00
90	250	38.05	38.05	100.00	100.00	100.00	70.42	100.00	108.70	100.00
90	300	39.21	45.42	100.00	100.00	100.00	76.57	100.00	108.70	100.00
90	375	51.36	59.49	100.00	100.00	100.00	82.71	100.00	108.70	100.00

Table 16
Comparative percentages of the areas of example 03

f_{ck} (MPa)	V_{Sd} (kN)	A_{sw} / $A_{sw\ (MC\ 1990)}$ %								
		NBR		MC 1990	MC 2010			NP EN 1992		DIN 1045
		M I	M II		LoA I	LoA II	LoA III	ν	ν_1	
55	3000	58.98	69.40	100.00	100.00	100.00	85.89	100.00	108.70	100.00
55	3200	61.54	72.41	100.00	100.00	100.00	87.36	100.00	108.70	100.00
55	3400	63.80	75.07	100.00	100.00	100.00	88.65	100.00	108.70	100.00
55	3600	65.81	77.43	100.00	100.00	100.00	89.80	100.00	108.70	100.00
60	3000	57.40	67.27	100.00	100.00	100.00	84.71	100.00	108.70	100.00
60	3200	60.06	70.39	100.00	100.00	100.00	86.24	100.00	108.70	100.00
60	3400	62.41	73.14	100.00	100.00	100.00	87.59	100.00	108.70	100.00
60	3600	64.50	75.58	100.00	100.00	100.00	88.80	100.00	108.70	100.00
70	3000	54.57	63.57	100.00	100.00	100.00	83.27	100.00	108.70	100.00
70	3200	57.41	66.87	100.00	100.00	100.00	84.86	100.00	108.70	100.00
70	3400	59.91	69.79	100.00	100.00	100.00	86.25	100.00	108.70	100.00
70	3600	62.13	72.38	100.00	100.00	100.00	87.50	100.00	108.70	100.00
80	3000	52.07	60.43	100.00	100.00	100.00	82.54	100.00	108.70	100.00
80	3200	55.06	63.91	100.00	100.00	100.00	84.12	100.00	108.70	100.00
80	3400	57.70	66.97	100.00	100.00	100.00	85.52	100.00	108.70	100.00
80	3600	60.05	69.69	100.00	100.00	100.00	86.76	100.00	108.70	100.00
90	3000	49.84	57.74	100.00	100.00	100.00	81.94	100.00	108.70	100.00
90	3200	52.97	61.36	100.00	100.00	100.00	83.53	100.00	108.70	100.00
90	3400	55.74	64.57	100.00	100.00	100.00	84.93	100.00	108.70	100.00
90	3600	58.19	67.41	100.00	100.00	100.00	86.17	100.00	108.70	100.00

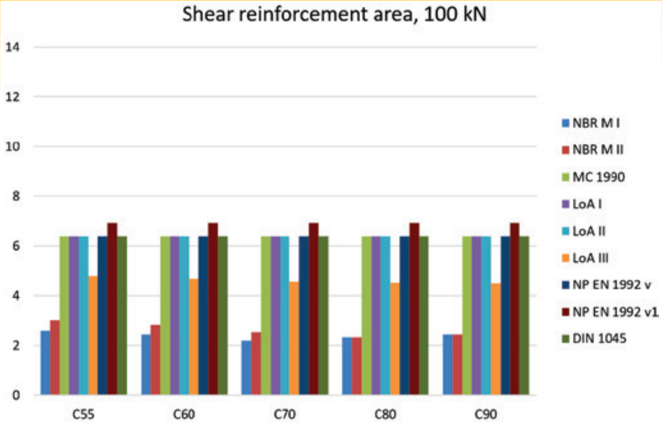

Figure 2
Comparative graph of the transverse reinforcement areas (cm²/m) of example 01 (beams of 12 cm by 40 cm), for applied shear force of 100 kN

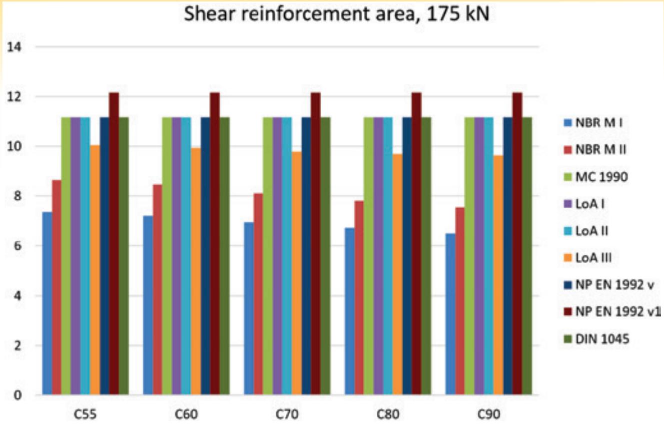

Figure 5
Comparative graph of the transverse reinforcement areas (cm²/m) of example 01 (beams of 12 cm by 40 cm), for applied shear force of 175 kN

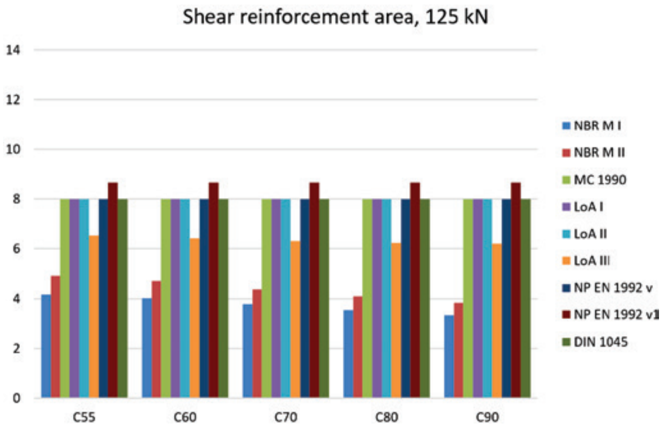

Figure 3
Comparative graph of the transverse reinforcement areas (cm²/m) of example 01 (beams of 12 cm by 40 cm), for applied shear force of 125 kN

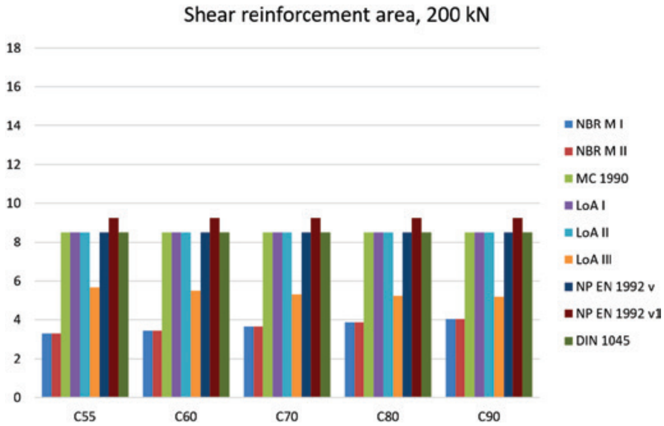

Figure 6
Comparative graph of the transverse reinforcement areas (cm²/m) of example 02 (beams of 20 cm by 60 cm), for applied shear force of 200 kN

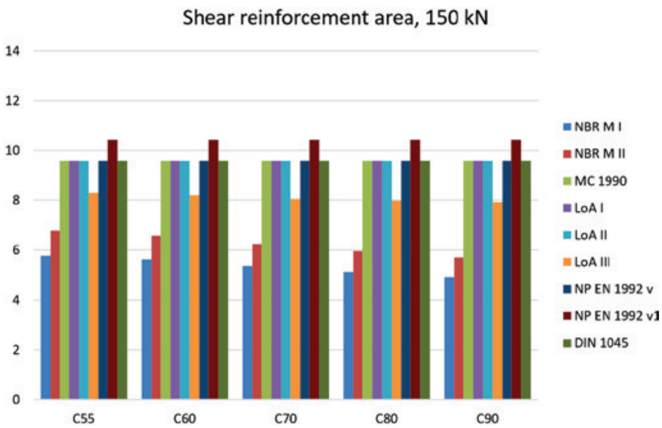

Figure 4
Comparative graph of the transverse reinforcement areas (cm²/m) of example 01 (beams of 12 cm by 40 cm), for applied shear force of 150 kN

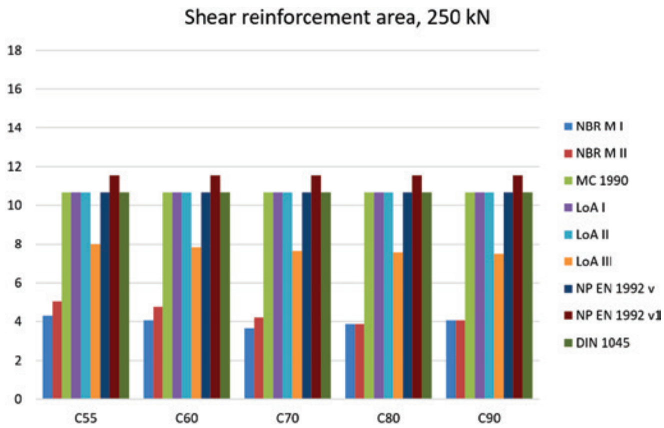

Figure 7
Comparative graph of the transverse reinforcement areas (cm²/m) of example 02 (beams of 20 cm by 60 cm), for applied shear force of 250 kN

3.4 Ultimate shear strength

Table 13 shows the ultimate shear strengths, as stresses (MPa), for each concrete class, obtained through the studied normative procedures' expressions.

3.5 Comparison of results

Based on the obtained results, the comparative Tables 14, 15 and 16, respectively related to examples 01, 02 and 03, are presented. Comparative graphs are presented in Figures 2, 3, 4, 5 (example 01); 6, 7, 8, 9 (example 02) and 10, 11, 12, 13 (example 03), for each shear force intensity and cross section situation. In the Tables, the resulting areas were considered as a percentage of the calculated areas by the Model Code 1990.

In Tables 14, 15 and 16 it can be verified that the international methodologies – apart from *LoA* III of MC 2010 [7] and the calculation procedure of the Portuguese code [10] which uses the parameter v_1 in the calculation (destined for situations in which the design stress of the shear reinforcement are below 80% of the characteristic yield stress) – generate the same transverse reinforcement

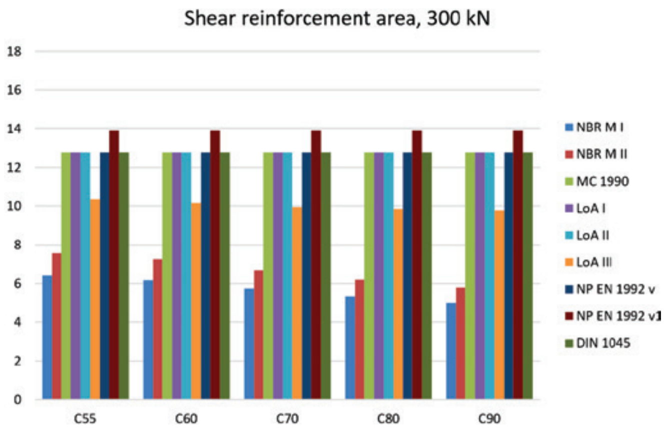

Figure 8
Comparative graph of the transverse reinforcement areas (cm²/m) of example 02 (beams of 20 cm by 60 cm), for applied shear force of 300 kN

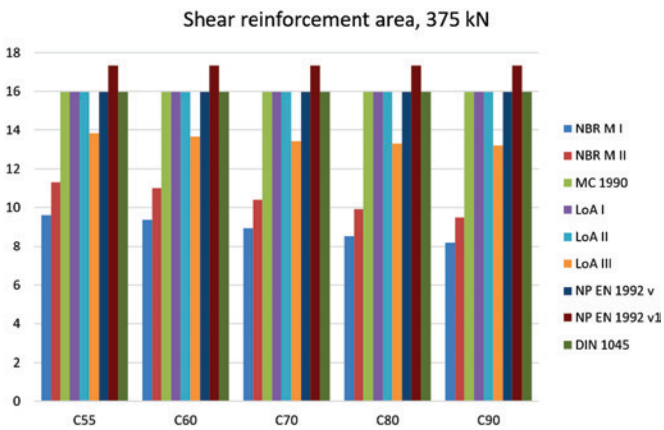

Figure 9
Comparative graph of the transverse reinforcement areas (cm²/m) of example 02 (beams of 20 cm by 60 cm), for applied shear force of 375 kN

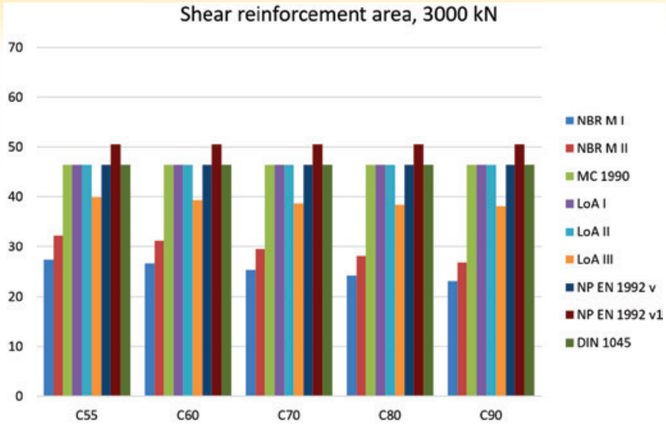

Figure 10
Comparative graph of the transverse reinforcement areas (cm²/m) of example 03 (beams of 60 cm por 165 cm), for applied shear force of 3000 kN

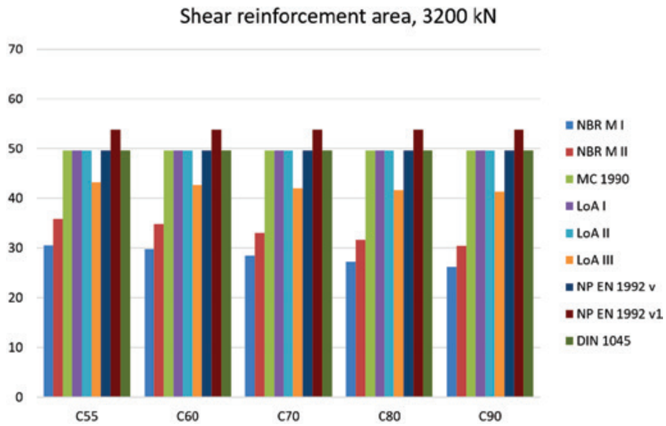

Figure 11
Comparative graph of the transverse reinforcement areas (cm²/m) of example 03 (beams of 60 cm por 165 cm), for applied shear force of 3200 kN

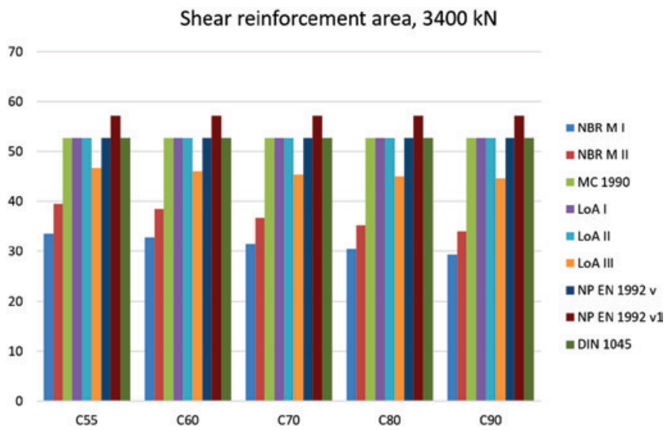

Figure 12
Comparative graph of the transverse reinforcement areas (cm²/m) of example 03 (beams of 60 cm por 165 cm), for applied shear force of 3400 kN

Table 17
Results of DIN 1045-1

Beam	f_c (MPa)	b (mm)	d (mm)	Shear reinforcement		Longitudinal reinforcement		$V_{failure}$ (kN)
				ϕ/s (mm)	ρ_w (MPa)	n ϕ	ρ_l	
H60/2	60.8	200	353	ϕ6/200	0.747	2ϕ32	2.28	179.74
H60/3	60.8	200	351	ϕ8/210	1.267	2ϕ32	2.29	258.78
H60/4	60.8	200	351	ϕ8/210	1.267	2ϕ32 + 1ϕ25	2.99	308.71
H75/2	68.9	200	353	ϕ6/200	0.747	2ϕ32	2.28	203.94
H75/3	68.9	200	351	ϕ8/210	1.267	2ϕ32	2.29	269.35
H75/4	68.9	200	351	ϕ8/210	1.267	2ϕ32 + 1ϕ25	2.99	255.23
H100/2	87.0	200	353	ϕ6/165	0.906	2ϕ32	2.28	225.55
H100/3	87.0	200	351	ϕ8/210	1.291	2ϕ32	2.29	253.64
H100/4	87.0	200	351	ϕ8/210	1.291	2ϕ32 + 1ϕ25	2.99	266.53

areas, for the same shear intensities, beam's cross sections and inclination of the struts of 45º.

Same as Models I and II of the NBR [5], Level III Approximation presents decreases on the required reinforcement areas for considering the concrete contribution in the design. The portions correspondent to these contributions increase while the concrete class escalates, and reduce with the rise of the applied shear forces. For all proposed situations, Models I and II of the Brazilian code generated the smallest areas.

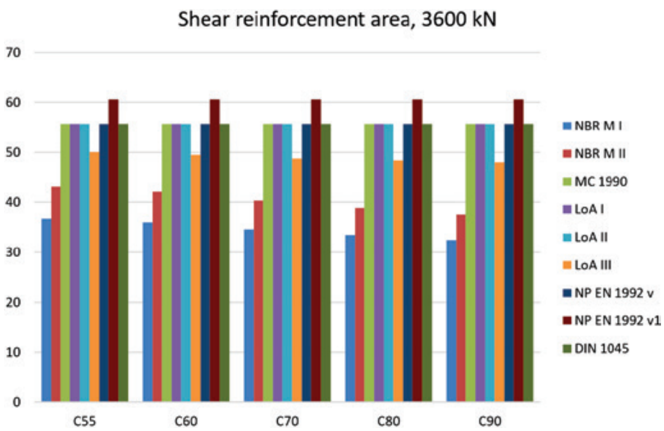

Figure 13
Comparative graph of the transverse reinforcement areas (cm_2/m) of example 03 (beams of 60 cm por 165 cm), for applied shear force of 3600 kN

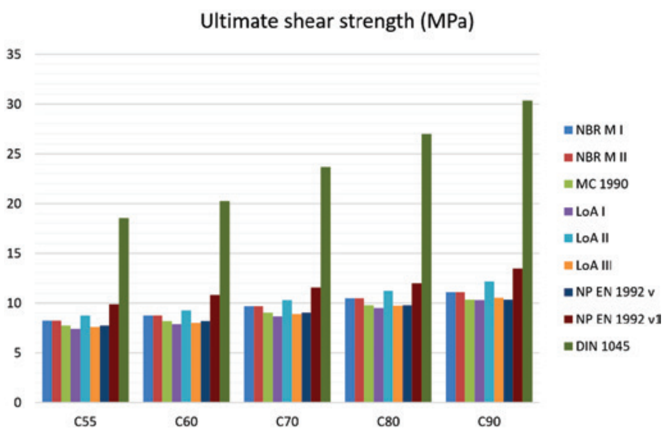

Figure 14
Comparative graph of the ultimate shear strengths (MPa)

Table 18
Properties of the transverse reinforcement bars

Diameter – Series	Area (mm²)	f_y (MPa)	f_u (MPa)
ϕ6 – H60 and H75	28.27	530	680
ϕ8 – H60 and H75	50.27	530	685
ϕ6 – H100	28.27	530	680
ϕ8 – H100	50.27	540	672

It is possible to verify that the German standard predicts shear resistances superior to the others, as presented in Figure 14. The other codes, including the Brazilian, comprehend higher values of reduction factors over the resistance, which is significantly penalized. It must be questioned whether this higher admissible resistance of the German code is justifiable by the higher rigor demanded in executing the concrete or by other factors unrelated to the calculation procedure, which are not contemplated in the design standard.

4. Experimental analysis

From the previous simulations, it is observed that the NBR calculation procedure produces smaller areas than the analyzed international procedures. Among them, only *LoA* III of the Model Code 2010 [7] adopts the complementary mechanisms of the concrete (pin effect, aggregate gearing and arch effect) contributions. The remaining international codes presented the inconsistency of generating equal shear reinforcement areas for the same cross section and applied force, even with increase of the compression

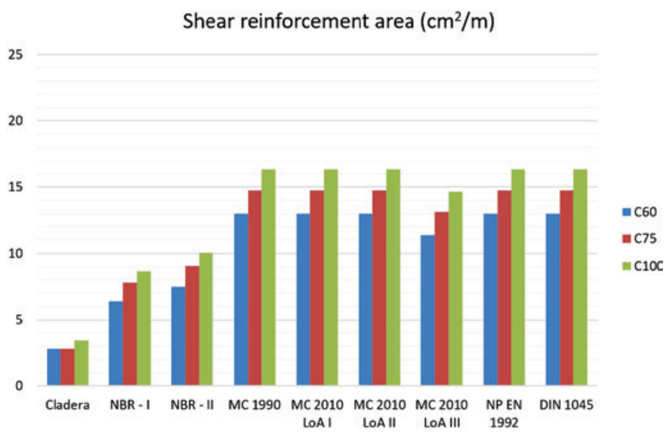

Figure 15
Comparative graph of the transverse reinforcement areas (cm²/m) of the beams of series 2

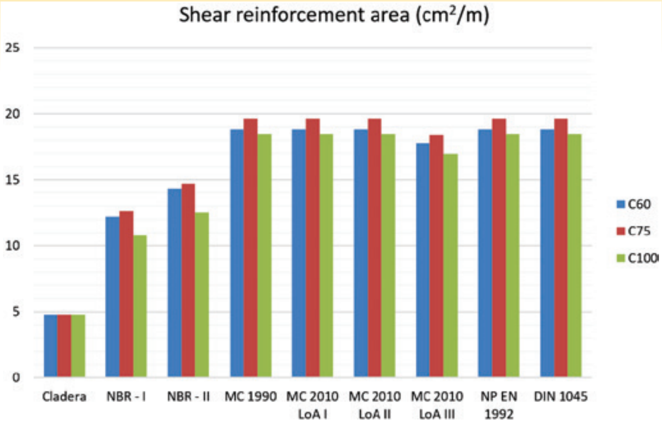

Figure 16
Comparative graph of the transverse reinforcement areas (cm²/m) of the beams of series 3

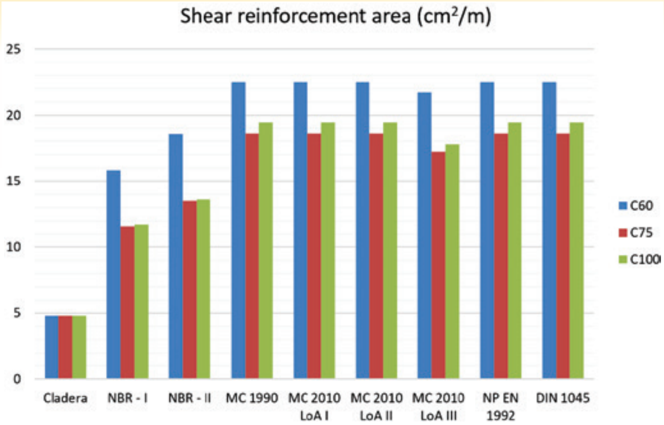

Figure 17
Comparative graph of the transverse reinforcement areas (cm²/m) of the beams of series 4

strength. Because of this, and with the intent to enrich the discussion, a comparison between the experimental results [2, 4] and the normative predictions will be performed.
Considering the results obtained by Cladera [2], the test-beams of series 2 (H60/2, H75/2 and H100/2), 3 (H60/3, H75/3 and H100/3) and 4 (H60/4, H75/4 and H100/4), which characteristics are expressed in Table 17 and illustrated in Figure 1, will be contemplated. These were selected by meeting the f_{ck} range of group II (between 55 MPa and 90 MPa), and for being transversely reinforced, allowing the desired comparisons.
Tables 19, 20 and 21 present the areas required by the studied codes for the experimental situations [2]. These were cal-

Table 19
Transverse reinforcement areas (cm²/m) for beams of series 2

Beam	f_{ck} (MPa)	b_w (cm)	d (cm)	V_{sd} (kN)	A_{sw} (cm²/m)
H60/2	60.8	20	35.3	179.74	2.82
NBR - I	60.8	20	35.3	179.74	6.38
NBR - II	60.8	20	35.3	179.74	7.47
MC 1990	60.8	20	35.3	179.74	13.01
MC 2010 LoA I	60.8	20	35.3	179.74	13.01
MC 2010 LoA II	60.8	20	35.3	179.74	13.01
MC 2010 LoA III	60.8	20	35.3	179.74	11.38
NP EN 1992	60.8	20	35.3	179.74	13.01
DIN 1045	60.8	20	35.3	179.74	13.01

Beam	f_{ck} (MPa)	b_w (cm)	d (cm)	V_{sd} (kN)	A_{sw} (cm²/m)
H75/2	68.9	20	35.3	203.94	2.82
NBR - I	68.9	20	35.3	203.94	7.77
NBR - II	68.9	20	35.3	203.94	9.06
MC 1990	68.9	20	35.3	203.94	14.76
MC 2010 LoA I	68.9	20	35.3	203.94	14.76
MC 2010 LoA II	68.9	20	35.3	203.94	14.76
MC 2010 LoA III	68.9	20	35.3	203.94	13.14
NP EN 1992	68.9	20	35.3	203.94	14.76
DIN 1045	68.9	20	35.3	203.94	14.76

Beam	f_{ck} (MPa)	b_w (cm)	d (cm)	V_{sd} (kN)	A_{sw} (cm²/m)
H100/2	87.0	20	35.3	225.55	3.42
NBR - I	87.0	20	35.3	225.55	8.66
NBR - II	87.0	20	35.3	225.55	10.03
MC 1990	87.0	20	35.3	225.55	16.33
MC 2010 LoA I	87.0	20	35.3	225.55	16.33
MC 2010 LoA II	87.0	20	35.3	225.55	16.33
MC 2010 LoA III	87.0	20	35.3	225.55	14.64
NP EN 1992	87.0	20	35.3	225.55	16.33
DIN 1045	87.0	20	35.3	225.55	16.33

culated considering the failure shear $V_{failure}$ (Table 17), experimentally achieved, as an applied shear force and by using the effectively observed compression strength of the concrete (60.8 MPa, 68.9 MPa and 87 MPa). It must be noted that on the standard's

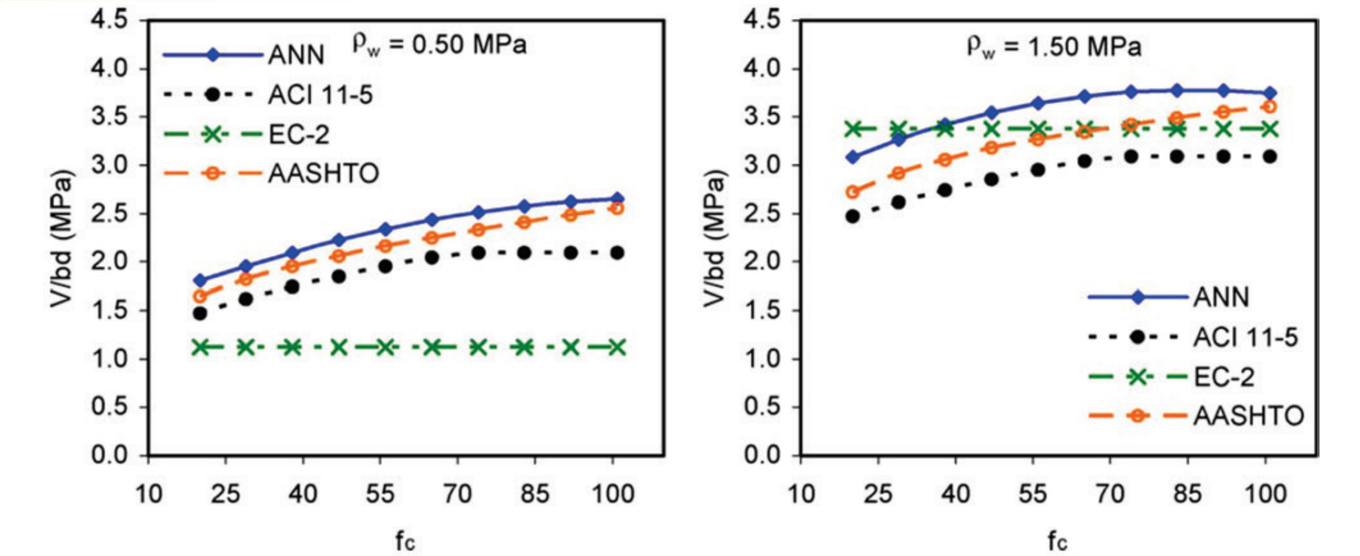

Figure 18
ANN results as compared to the ACI 11-5, EC-2 and AASHTO predictions for beams with web reinforcement. Influence of the concrete compressive strength in relation to the amount of transverse reinforcement. (Cladera & Marí [4])

Table 20
Transverse reinforcement areas (cm²/m) for beams of series 3

Beam	f_{ck} (MPa)	b_w (cm)	d (cm)	V_{Sd} (kN)	A_{sw} (cm²/m)
H60/3	60.8	20	35.1	258.78	4.78
NBR - I	60.8	20	35.1	258.78	12.20
NBR - II	60.8	20	35.1	258.78	14.29
MC 1990	60.8	20	35.1	258.78	18.84
MC 2010 LoA I	60.8	20	35.1	258.78	18.84
MC 2010 LoA II	60.8	20	35.1	258.78	18.84
MC 2010 LoA III	60.8	20	35.1	258.78	17.75
NP EN 1992	60.8	20	35.1	258.78	18.84
DIN 1045	60.8	20	35.1	258.78	18.84
Beam	**f_{ck} (MPa)**	**b_w (cm)**	**d (cm)**	**V_{Sd} (kN)**	**A_{sw} (cm²/m)**
H75/3	68.9	20	35.1	269.35	4.78
NBR - I	68.9	20	35.1	269.35	12.62
NBR - II	68.9	20	35.1	269.35	14.71
MC 1990	68.9	20	35.1	269.35	19.61
MC 2010 LoA I	68.9	20	35.1	269.35	19.61
MC 2010 LoA II	68.9	20	35.1	269.35	19.61
MC 2010 LoA III	68.9	20	35.1	269.35	18.41
NP EN 1992	68.9	20	35.1	269.35	19.61
DIN 1045	68.9	20	35.1	269.35	19.61
Beam	**f_{ck} (MPa)**	**b_w (cm)**	**d (cm)**	**V_{Sd} (kN)**	**A_{sw} (cm²/m)**
H100/3	87.0	20	35.1	253.64	4.78
NBR - I	87.0	20	35.1	253.64	10.80
NBR - II	87.0	20	35.1	253.64	12.51
MC 1990	87.0	20	35.1	253.64	18.47
MC 2010 LoA I	87.0	20	35.1	253.64	18.47
MC 2010 LoA II	87.0	20	35.1	253.64	18.47
MC 2010 LoA III	87.0	20	35.1	253.64	16.94
NP EN 1992	87.0	20	35.1	253.64	18.47
DIN 1045	87.0	20	35.1	253.64	18.47

predictions CA-50 steel was used. On the other hand, Cladera [2] adopted the experimentally obtained yield stress, presented in Table 18, for determining the area of the transverse reinforcement. Furthermore, to make the comparison viable, like the author, no majoring factors on the applied forces or reduction coefficients on resistances were used. The areas predicted by the codes and the experimental ones correspondent to the failure shear forces are shown in Figures 15, 16 and 17. It is noticed that the predicted areas by the codes are superior to the experimentally required, indicating a "reserve" of resistance. According to the aforementioned by the numerical simulations of section 3, the international standards generate greater transverse reinforcement areas then the national. The expected decrease via *LoA* III procedure of the Model Code 2010 [7] is highlighted, differing from the predictions of the other European codes. These last do not consider the concrete contribution, which in fact observed, as identified by Cladera & Marí [4] (Figure 18).

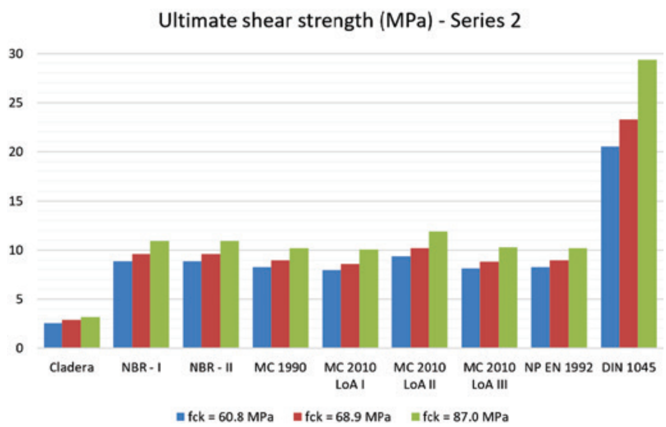

Figure 19
Comparative graph of the ultimate shear strengths (MPa) of series 2

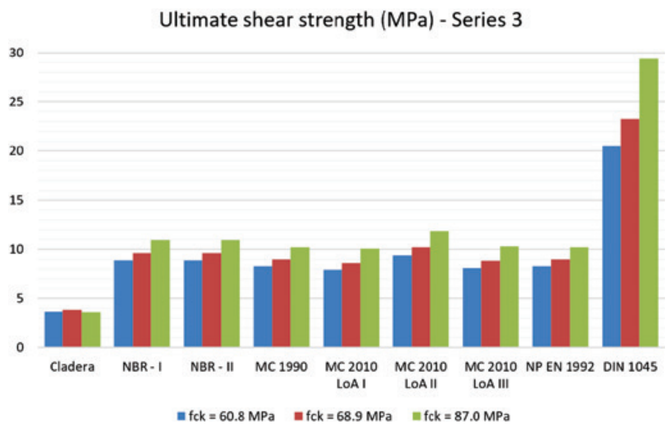

Figure 20
Comparative graph of the ultimate shear strengths (MPa) of series 3

Table 21
Transverse reinforcement areas (cm²/m) for beams of series 4

Beam	f_{ck} (MPa)	b_w (cm)	d (cm)	V_{Sd} (kN)	A_{sw} (cm²/m)
H60/4	60.8	20	35.1	308.71	4.78
NBR - I	60.8	20	35.1	308.71	15.84
NBR - II	60.8	20	35.1	308.71	18.55
MC 1990	60.8	20	35.1	308.71	22.48
MC 2010 LoA I	60.8	20	35.1	308.71	22.48
MC 2010 LoA II	60.8	20	35.1	308.71	22.48
MC 2010 LoA III	60.8	20	35.1	308.71	21.72
NP EN 1992	60.8	20	35.1	308.71	22.48
DIN 1045	60.8	20	35.1	308.71	22.48

Beam	f_{ck} (MPa)	b_w (cm)	d (cm)	V_{Sd} (kN)	A_{sw} (cm²/m)
H75/4	68.9	20	35.1	255.23	4.78
NBR - I	68.9	20	35.1	255.23	11.59
NBR - II	68.9	20	35.1	255.23	13.51
MC 1990	68.9	20	35.1	255.23	18.58
MC 2010 LoA I	68.9	20	35.1	255.23	18.58
MC 2010 LoA II	68.9	20	35.1	255.23	18.58
MC 2010 LoA III	68.9	20	35.1	255.23	17.29
NP EN 1992	68.9	20	35.1	255.23	18.58
DIN 1045	68.9	20	35.1	255.23	18.58

Beam	f_{ck} (MPa)	b_w (cm)	d (cm)	V_{Sd} (kN)	A_{sw} (cm²/m)
H100/4	87.0	20	35.1	266.53	4.78
NBR - I	87.0	20	35.1	266.53	11.73
NBR - II	87.0	20	35.1	266.53	13.60
MC 1990	87.0	20	35.1	266.53	19.41
MC 2010 LoA I	87.0	20	35.1	266.53	19.41
MC 2010 LoA II	87.0	20	35.1	266.53	19.41
MC 2010 LoA III	87.0	20	35.1	266.53	17.94
NP EN 1992	87.0	20	35.1	266.53	19.41
DIN 1045	87.0	20	35.1	266.53	19.41

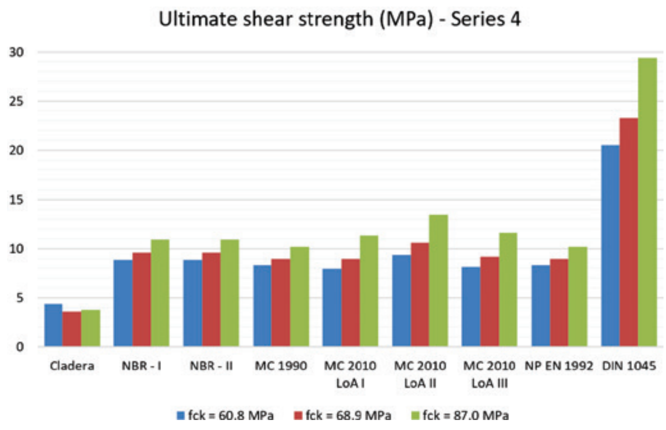

Figure 21
Comparative graph of the ultimate shear strengths (MPa) of series 4

The ANN curve – relative to the experimental results of beams of 350 mm of effective depth, 330 mm width, relation a/d = 3, and longitudinal reinforcement ratio of ρ_l = 3% - indicates the growth of shear strength with the increase of the concrete class. Despite contemplated by the American codes ACI 318-02 and AASHTO LRDF – not studied in the present work – in a conservative manner, this behavior is not considered by the Eurocode 2 [10], which admits that variations on shear resistance are due only to the transverse reinforcement, indicated by the translation of the EC-2 curve, with the increase in the transverse reinforcement rate from ρ_w = 0.50 MPa to ρ_w = 1.50 MPa.
If on one hand, most normative predictions do not consider the concrete contribution in the transverse reinforcement design, on the other, all predict ultimate shear strengths superior to those experimentally observed by Cladera [2], as presented in Table 22, which data are illustrated in Figures 19, 20 and 21. As verified in section 3.5, the German procedure predicted the highest resistances.
With the analysis of the ultimate shear strength, it is again noticeable

Table 22
Ultimate shear strengths (MPa)

Beam	f_{ck} (MPa)	V_{Ru} (MPa)		
		Série 2	Série 3	Série 4
Cladera	60.8	2.55	3.69	4.40
NBR - I	60.8	8.87	8.87	8.87
NBR - II	60.8	8.87	8.87	8.87
MC 1990	60.8	8.28	8.28	8.28
MC 2010 LoA I	60.8	7.93	7.93	7.93
MC 2010 LoA II	60.8	9.37	9.37	9.37
MC 2010 LoA III	60.8	8.11	8.11	8.11
NP EN 1992	60.8	8.28	8.28	8.28
DIN 1045	60.8	20.52	20.52	20.52
Beam	**f_{ck} (MPa)**	**V_{Ru} (MPa)**		
		Série 2	**Série 3**	**Série 4**
Cladera	68.9	2.89	3.84	3.64
NBR - I	68.9	9.63	9.63	9.63
NBR - II	68.9	9.63	9.63	9.63
MC 1990	68.9	8.98	8.98	8.98
MC 2010 LoA I	68.9	8.62	8.62	8.98
MC 2010 LoA II	68.9	10.18	10.18	10.62
MC 2010 LoA III	68.9	8.82	8.82	9.19
NP EN 1992	68.9	8.98	8.98	8.98
DIN 1045	68.9	23.25	23.25	23.25
Beam	**f_{ck} (MPa)**	**V_{Ru} (MPa)**		
		Série 2	**Série 3**	**Série 4**
Cladera	87.0	3.19	3.61	3.80
NBR - I	87.0	10.94	10.94	10.94
NBR - II	87.0	10.94	10.94	10.94
MC 1990	87.0	10.21	10.21	10.21
MC 2010 LoA I	87.0	10.07	10.07	11.34
MC 2010 LoA II	87.0	11.90	11.90	13.41
MC 2010 LoA III	87.0	10.30	10.30	11.61
NP EN 1992	87.0	10.21	10.21	10.21
DIN 1045	87.0	29.36	29.36	29.36

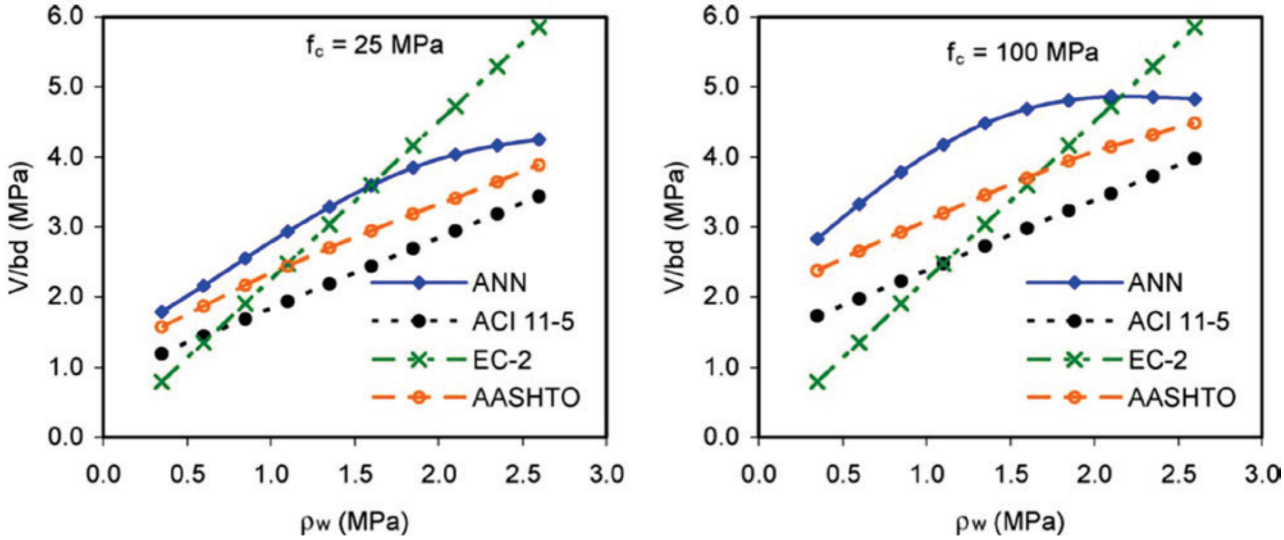

Figure 22
ANN results compared to the ACI 11-5, EC-2 and AASHTO predictions for beams with web reinforcement. Influence of the amount of shear reinforcement in relation to the concrete compressive strength (Cladera & Marí [4])

that, despite the growth of the shear resistance with the increase of the concrete class, in accordance to the normative procedures and experimental results, this behavior is not translated into advantage in design by the European procedures (apart from *LoA* III).

5. Conclusions

Due to the diffusion of high strength concretes, it is necessary to study the normative design procedures – specifically on shear design – which encompass concretes of classes C55 to C90. This work, therefore, aimed to analytically compare the usual normative methodologies in light of experimental results [2, 4].
From the analyses, one concludes that the NBR procedure produces areas inferior to the studied international codes. Unlike the Brazilian standard [5], they do not consider (apart from *LoA* III) the complementary concrete mechanisms contribution (pin effect, aggregate gearing and arch effect), despite experimentally observed.
According to the data shown in Tables 14, 15 and 16, one verifies that the MC 1990 [6] and MC 2010 [7] (*LoA* I and *LoA* II) calculation procedures and the Portuguese [10] (considering parameter v) and German codes [11] give the same areas for the same cross sections, shear intensities and inclination of the struts.
As verified at the 50th Brazilian Congress on Concrete [12] for concretes of group I, the use of Model II of NBR [5] in concretes of group II for given shear force, cross section and compressed diagonal inclination of 45°, results in areas superior to those obtained by Model I, when these are greater than the normative minimum.
The procedures that adopt a concrete contribution present reductions in the transverse reinforcement areas with the increase of the concrete class for a same applied shear force and cross section. In general, for the same class the areas increase with the loads.
Despite of not incorporating the concrete contribution in the design, the analyzed international procedures – as well as the national – predicted an increase in the ultimate shear force with the growth of the concrete class. In the performed comparisons it was detected that this same increase is superior to that experimentally obtained by Cladera [2], which reinforces the inconsistency and conservatism of these codes.
The lack of consideration of the concrete portion by part of the analyzed international codes leads to very conservative results, given that, regardless of class, for a same applied force, the areas are equal. Cladera & Marí [4] confirm this behavior when comparing the results of the ANN with the areas predicted by the Eurocode 2, as demonstrates Figure 22.

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