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ORIGINAL ARTICLE

Evaluation of the reliability of optimized reinforced concrete beams

Avaliação da confiabilidade de vigas otimizadas de concreto armado

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Abstract: In the present research, the reliability of optimized reinforced concrete beams was evaluated in different design situations. Simply supported beams were optimized to find the dimensions and reinforcements of the cross-section that minimize costs, meeting the criteria of technical codes, through genetic algorithms. For each optimized beam, the reliability index was obtained in relation to the ultimate limit state of flexure with the iHLRF algorithm, considering the uncertainties of the resistance and load models, loads and resistances. It wa s verified that the reliability indexes, in general, were higher than the minimum value of 3.8, recommended by technical codes, in design situations with little live load. Through a parametric study, trends were identified for the reliability index according to the design parameters and characteristics of the beams.

Keywords: beams, reinforced concrete, structural optimization, structural reliability.

Resumo: Na presente pesquisa foi realizada a avaliação da confiabilidade de vigas otimizadas de concreto armado em diferentes situações de projeto. Vigas simplesmente apoiadas foram otimizadas para encontrar as dimensões e armaduras da seção transversal que minimizam os custos, atendendo aos critérios das normas técnicas, por meio dos Algoritmos Genéticos. Para cada viga otimizada, foi obtido o índice de confiabilidade em relação ao estado-limite último de flexão com o algoritmo iHLRF, considerando as incertezas dos modelos da resistência e da solicitação, das cargas e das resistências. Foi verificado que os índices de confiabilidade, de modo geral, foram maiores que o valor mínimo de 3,8, recomendado por códigos normativos, nas situações de projeto com pouca carga variável. Através de um estudo paramétrico, foram identificadas tendências para o índice de confiabilidade em função dos parâmetros de projeto e das características das vigas.

Palavras-chave: vigas, concreto armado, otimização estrutural, confiabilidade estrutural.

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INTRODUCTION

Nowadays several computational programs can be used to design structural projects, simulating structures with a high degree of complexity in a very realistic way, with integrated systems that cover all stages of the project [1]. Despite the various advances that already exist in structural engineering, the procedure for designing structures is still a process of trial and error. In the conventional procedure, the structure is pre-sized, where the dimensions of the structural elements are defined. Structural analysis and structure sizing are then performed. If the safety, construction and serviceability criteria are not met, a new pre-sizing is carried out and the criteria are rechecked. The procedure continues until a viable solution is found. The final solution adopted will not necessarily be the best solution, among all possible.

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To determine the best solution, optimization techniques can be incorporated into the structural design. Mathematically, an optimization problem aims to find variables that minimize a function, meeting the constraints imposed by the problem. Optimization variables are called design variables (describe the system), and the function to be minimized as objective function [2]. In structural design, design variables can be the dimensions of structural elements, and the objective function can minimize the cost for example. Thus, by transforming conventional structural design into an optimization problem, it is possible to find the best possible solution (optimal solution) to minimize (or maximize) some specific objective.

The conventional procedure for designing structures, in addition to being a trial and error process, is also a deterministic procedure. However, the design of structures involves uncertainties associated with the calculation models, loads and materials properties [3]. To cover these uncertainties, a deterministic response of the structure is obtained using safety factors. Therefore, at the end of the project, it is likely that there is an over-designed or under-designed structure, in the face of uncertainties [4].

The ideal approach to dealing with engineering projects involving uncertainties is the stochastic approach. In this approach, the statistical characteristics of variables that have uncertainties are considered in the system analysis procedure. In this way, a system response is obtained through analysis with statistical properties. At the end of the design, there is a robust system, which is safe against uncertainties [4]. To assess the reliability of structures, a stochastic approach is used to "quantify" a safety measure. This measure is known as a reliability index and can be determined through numerical methods.

Several studies were carried out to optimize reinforced concrete beams [5]–[9], with different formulations and methods, but without evaluating reliability. In studies of structural reliability of reinforced concrete beams, the reliability index was evaluated in some design situations [10]–[12]. Other studies presented calibration procedures, based on reliability, of the partial safety factors of technical codes [13].

Therefore, the present work evaluated the structural reliability of reinforced concrete beam optimized in different design situations, through the reliability index.

FORMULATION OF THE OPTIMIZATION PROBLEM

The reinforced concrete beam evaluated, presented in Figure 1, is a simply supported beam subject to a uniformly distributed load formed by the dead load g and live load q, and with span L. The cross-section of the beam was rectangular of width b and height h, with n_t tension bars with diameter $\emptyset_t$ and n_c compressed bars with diameter $\emptyset_c$. The steel of the longitudinal reinforcements was CA-50 and the stirrups was CA-60; the level of the environmental aggressiveness (CAA) was equal to II, with a cover of 30 mm; and the diameters of the vibrator and the large aggregate were equal to 25 mm and 19 mm, respectively.

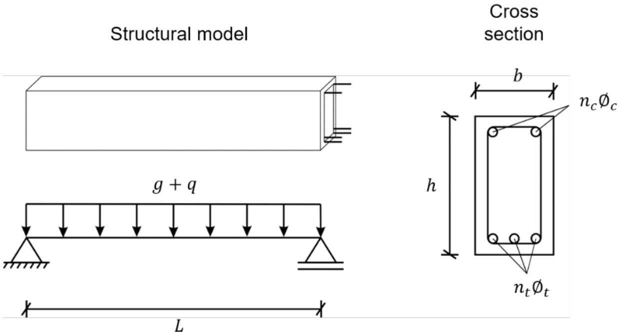

Figure 1. Reinforced concrete beam considered.

To obtain results in different design situations, the values of span (L), total loading ($g + q$), the relationship between live load and total load ($r = \frac{q}{g+q}$) and characteristic compressive strength of concrete (f_{ck}) were varied. The span varied from 3 to 7 m, in increments of 0.5 m. The total loading varied from 10 to 40 kN/m, in increments of 5 kN/m. This values of ($g + q$) do not include the self-weight of the beam. However, the self-weight was considered in the implementations. The r ratio was varied from 0.2 to 0.8, in increments of 0.2. The f_{ck} ranged from 25 to 35 MPa, in increments of 5 MPa. Thus, by combining the values of L, $(g + q)$, r, and f_{ck}, the reinforced concrete beam was optimized for 756 design situations. The following terminology will be used to identify the beams: (V - L - $(g + q)$ - r - f_{ck}).

In the optimization of the reinforced concrete beam in Figure 1, the width (b), the height (h), the number (n_t) and the diameter of the tension bars ($\emptyset_t$), and the number (n_c) and the diameter of the compressed bars ($\emptyset_c$) that compose the cross section were considered as design variables.

The objective function aims to minimize the beam costs (Equation 1), considering the costs of concrete (C_C), steel (C_A) and formwork (C_F):

$$C = C_C + C_A + C_F \quad (1)$$

In the cost of concrete (Equation 2), c_c is unit cost in R\$/m³. In the cost of steel (Equation 3), the costs of the tension bars, compressed bars, stirrup and skin reinforcement were considered, where $c_{\emptyset_t}$, $c_{\emptyset_c}$, $c_{\emptyset_e}$ and $c_{\emptyset_p}$ are their unit costs, respectively, in R\$/kg. In Equation 3, ρ, c and n_e are the mass density of steel (7850 kg/m³), the cover and the number of stirrups in the beam, respectively. In the cost of the formworks (Equation 4), c_f it is the unit cost in R\$/m².

$$C_C = bhLc_c \quad (2)$$

$$C_A = \left(\frac{\pi{\emptyset_t}^2}{4}Ln_t\rho c_{\emptyset_t}\right) + \left(\frac{\pi{\emptyset_c}^2}{4}n_cL\rho c_{\emptyset_c}\right) + \left\{\frac{\pi{\emptyset_e}^2}{4}[2(h+b)-8c]n_e\rho c_{\emptyset_e}\right\} + \left(\frac{\pi{\emptyset_p}^2}{4}L\rho c_{\emptyset_p}\right) \quad (3)$$

$$C_F = (b + 2h)Lc_f \quad (4)$$

The unit costs of concrete, steel and formwork were extracted from the Sistema Nacional de Pesquisa de Custos e Índices da Construção Civil (SINAPI) [14], from September of 2019 in the state of Pernambuco (Brazil). Table 1 lists the costs of concrete for types C25 to C35, the costs of steel bars for commercial diameters from 5 mm to 25 mm and the cost of the formwork. These unit costs consider, in addition to the material, costs associated with labor in its composition. In the unit cost of the formwork, it was also considered their reuse.

Table 1. Unit costs.

Concrete		
Source	Type	Cost (R\$/m³)
94965 - SINAPI PE 09/2019	C25	327.94
94966 - SINAPI PE 09/2019	C30	336.69
(interpolation between C30 and C40)	C35	353.99
CA-50 steel		
Source	Diameter (mm)	Cost (R\$/kg)
-	5	8.20
92760 - SINAPI PE 09/2019	6.3	8.20
92761 - SINAPI PE 09/2019	8	8.16
92762 - SINAPI PE 09/2019	10	6.67
92763 - SINAPI PE 09/2019	12.5	6.01
92764 - SINAPI PE 09/2019	16	5.68
92765 - SINAPI PE 09/2019	20	5.25
92766 - SINAPI PE 09/2019	25	5.82
Formwork		
Source		Cost (R\$/m²)
92448 - SINAPI PE 09/2019		78.07

In the beam optimization problem (Equations 5 to 21), the constraints are criteria associated to the ultimate and serviceability limit states, details and limitations for the design variables, according to the specifications of NBR 6118 [15]. The variables b, h, n_t and n_c should be limited (Equations 6, 7, 8, 9, 10, 11 and 12). The bending moment (M_{sd}) should be less than or equal to the resistant moment (M_{Rd}) (Equation 13) to ensure safety to the ultimate limit state of flexure. The ratio of the neutral line position to effective depth (x/d) must comply with the limit of 0.45 to ensure good ductility conditions (Equation 14). The design value of the requesting shear force (V_{sd}) should be limited to resistant shear force (V_{Rd2}) to prevent the ruin of compressed concrete diagonals (Equation 15). The final displacement (a_t) and the characteristic crack opening (w) should be limited (Equations 16 and 17). The reinforcement areas (A_s e A'_s) must meet minimum and maximum values (Equations 18 and 19). In the detailing, the distance from the center of gravity of the bars to the center of the farthest bar (a) should be less than 10% of the height (Equation 20) and the spacing of the stirrups (s) must meet a minimum value (Equation 21).

Therefore, the optimization problem aims to find the values of b, h, n_t, $\emptyset_t$, n_c, and $\emptyset_c$ of the cross section (Figure 1) to minimize beam costs (Equation 5), meeting the imposed constraints (Equations 6 to 21). The optimization problem was written as:

Find the vector $\mathbf{x} = \{b, h, n_t, \emptyset_t, n_c, \emptyset_c\}^T$, to minimize the cost:

$$C(\mathbf{x}) = C_C + C_A + C_F \tag{5}$$

Subject to:

$$12 \text{ cm} \leq b \leq 25 \text{ cm} \tag{6}$$

$$25 \text{ cm} \leq h \leq 100 \text{ cm} \tag{7}$$

$$2 \leq n_t, n_c \leq 20 \tag{8}$$

$$\emptyset_t, \emptyset_c = \{5, 6.3, 8, 10, 12.5, 16, 20\,, 25\} \text{ mm} \tag{9}$$

$$3h \leq L \tag{10}$$

$$n_t \geq n_c \tag{11}$$

$$\emptyset_t \geq \emptyset_c \tag{12}$$

$$M_{sd} \leq M_{Rd} \tag{13}$$

$$\frac{x}{d} \leq 0.45 \tag{14}$$

$$V_{sd} \leq V_{Rd2} \tag{15}$$

$$a_t \leq a_{tlim} = \frac{L}{250} \tag{16}$$

$$w \leq w_{lim} = 0.3 \text{ mm} \tag{17}$$

$$A_s \geq A_{smin} \tag{18}$$

$$(A_s + A'_s) \leq 0.04bh \tag{19}$$

$$a \leq 0.1h \tag{20}$$

$$s \geq s_{min} \tag{21}$$

The optimization problem (Equations 5 to 21) was implemented in MATLAB (version R2016a). Thus, there was a constrained optimization problem that involves discrete variables and non-differentiable functions. In this case, the Genetic Algorithms (GA) of the MATLAB Global Optimization Toolbox [16] were used because it is an appropriate method for these situations [17]. The GA, in fact, are commonly used in the optimization of reinforced concrete beams, as verified in Govindaraj and Ramasamy [6], Alexandre [7], Oliveira [8] and Bezerra [9].

The performance of GA depends mainly on their parameters, such as population size and crossover and mutation rates [18]. Thus, after the implementation of the optimization problem in MATLAB, the parameters of the AG [16] "populationsize", "crossoverfraction" (fraction of individuals produced by the crossover operators) and "elitecount" (fraction of individuals that survive) were calibrated [19], resulting in 1000, 0.7 and 0.05, respectively. After calibration, the implementation of the optimization of reinforced concrete beams was validated through examples of Oliveira [8] and Bezerra [9].

FORMULATION OF THE RELIABILITY PROBLEM

In the evaluation of the reliability for the optimized reinforced concrete beams, the following random variables were considered (Equation 22):

$$X = \{\theta_R, \theta_S, g, q, f_c, f_y\}^T \tag{22}$$

where θ_R is the resistance model error; θ_S is the load model error; g is the dead load; q is the live load; f_c is the compressive strength of concrete and f_y is yield strength of the steel reinforcement. The statistical characteristics of the random variables (probability distribution, mean and standard deviation) are found in Table 2 and were extracted from Scherer et al. [12].

Table 2. Statistical characteristics of $\boldsymbol{X}$.

Variables	Distribuition	Mean ($\boldsymbol{\mu}$)	Standard deviation ($\boldsymbol{\sigma}$)
θ_R	Lognormal	1	0.05
θ_S	Lognormal	1	0.05
g	Normal	$1.05g$	0.10μ
q	Gumbel	$\frac{q}{(1 + 0.35 \times 0.25)}$	0.25μ
f_c	Normal	$\frac{f_{ck}}{(1 - 1.645 \times 0.10)}$	0.10μ
f_y	Normal	$\frac{f_{yk}}{(1 - 1.645 \times 0.05)}$	0.05μ

The ultimate limit state considered is related to flexion, being calculated according to Equation 23, by the difference between resistance and load effects. In the portion of the load effects, the bending moments are considered due to the total loading $(g + q)$ and the self-weight of the beam $(\rho_c bh)$, where ρ_c is the specific weight of the reinforced concrete equal to 25 kN/m^3. In Equation 23, M is the resistant moment.

$$g(X) = \theta_R M - \theta_S \left[\frac{(g+q)L^2}{8} + \frac{(\rho_c bh)L^2}{8}\right] \tag{23}$$

The random variables θ_R, θ_S, g, q, f_c and f_y are parameters that have uncertainties and were considered in other reliability research of reinforced concrete beams [10]–[12]. The variability of θ_R is due to the approximations of the resistance calculation model, in this case being the model of NBR 6118 [15]. The variable θ_S is related to the inaccuracies of the action model. Loading g is an uncertain value and q varies in space and time. The variability of the strength of f_c concrete is due to the microstructural non-homogeneity of the material, formed by cement slurry and aggregate, and the non-homogeneity of the mixture. And the variability in the strength of f_y steel is a consequence of its production process and bars [20].

Thus, the reliability problem consists in finding the reliability index of the optimized beam, associated with the probability of failure when the resistance is less than the load effects ($g(X) < 0$) in the bending, in the face of the uncertainties of X.

In structural reliability, the probability of failure (Equation 24) is given as the integral of the joint density function of the random variables ($f_X(X)$) over the failure domain ($X|\ g(X) \leq 0$) [20]:

$$p_f = \int_{X|\ g(X)\leq 0} f_X(X)dX \tag{24}$$

Through transformation methods, the reliability index (β) can be associated with p_f by Equation 25 [20]:

$$p_f \cong \Phi(-\beta) \tag{25}$$

where $\Phi(.)$ is the standard normal cumulative distribution function.

To find the reliability index of the optimized beams, the improved algorithm of Hasofer, Lind, Rackwitz and Fiessler, the iHLRF [21], [22], which presents improvements over the original HLRF algorithm, was used. In iHLRF, the step size in the algorithm was adjusted to ensure convergence, presenting better performance than other algorithms [23]. As MATLAB does not have an iHLRF function for reliability analysis, iHLRF was implemented and validated through examples of Scherer et al. [12] and Nogueira and Pinto [11]. The implementation was carried out according to Beck's formulation [20], using the Armijo rule presented in Zhang and Der Kiureghian [21].

RELIABILITY INDEX OF OPTIMIZED BEAMS

After the optimization of reinforced concrete beams for the 756 design situations, varying the parameters L, $(g + q)$, r and f_{ck}, the reliability index β of each optimized beam was determined (V - L - $(g + q)$ - r - f_{ck}).

Figure 2 shows the main effects chart for the reliability index. In the graph, for each parameter L, $(g + q)$, r and f_{ck}, the averages of β are displayed in each parameter value. It was verified a change in β values as L was varied. Similarly, the values of β change with the variation of $(g + q)$. With the variation of the r ratio, a change is also observed in β. The r graph, when compared to the L and $(g + q)$ results, shows a large slope, which indicates a great effect of this parameter in β. On the other hand, the variations f_{ck} do not present significant changes in β, when compared to the changes due to the variations of the other parameters.

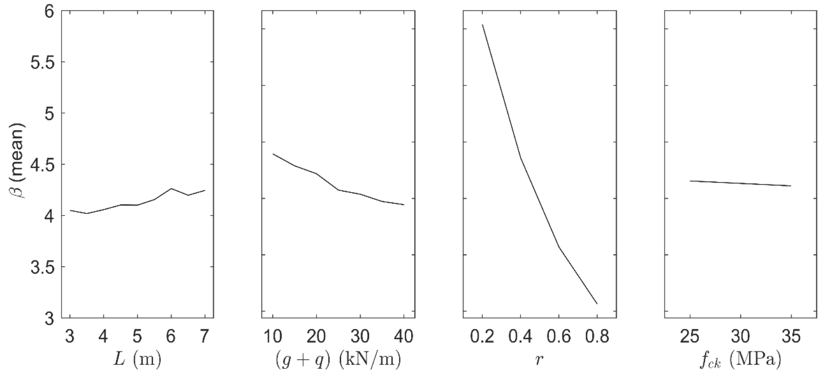

Figure 2. Main effects chart for reliability index.

Figure 3 shows the interaction graph for the reliability index. In the graph, one can observe that the interaction between the parameters L, $(g+q)$, r and f_{ck}, through the averages of the reliability index in each parameter value. It was verified an interaction between L - $(g+q)$, L - r and $(g+q)$ - r, since the lines of the graphs were not parallel. This interaction between the parameters affects the values of β, as observed in Figure 3. The interaction of f_{ck} with the other parameters was less expressive, once that the lines of the graphs were almost parallels.

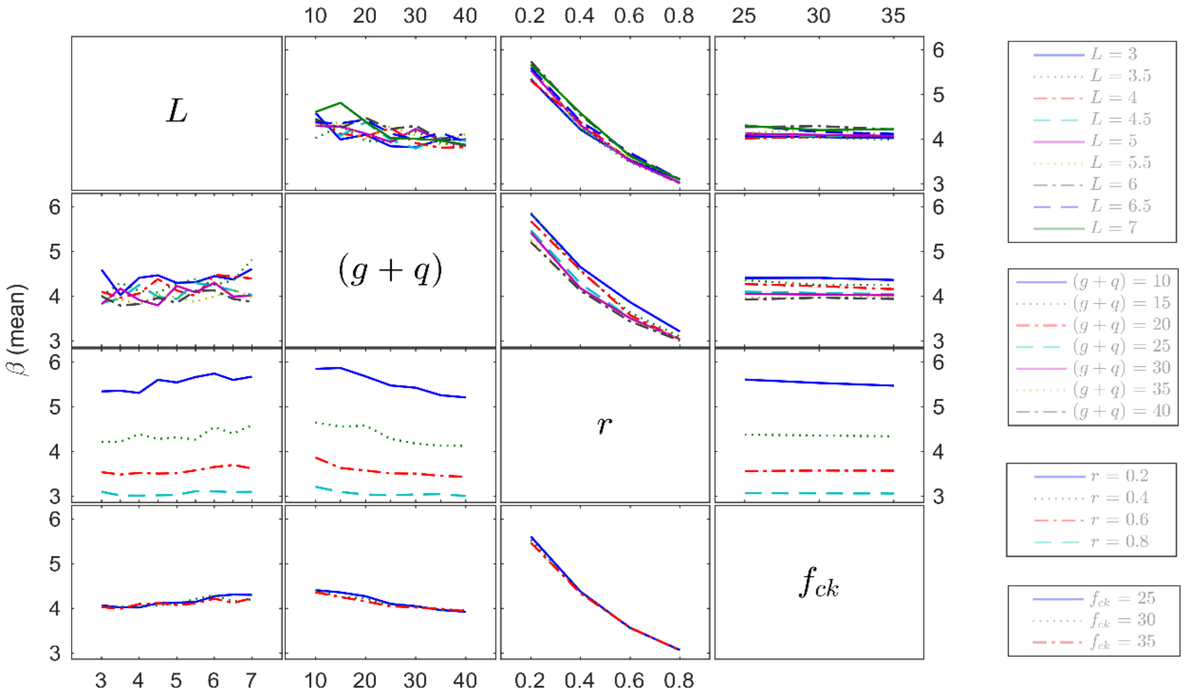

Figure 3. Interaction graph for reliability index.

The interaction and main effects graphs indicate that there was a main effect on the reliability index due to the variation of the L, $(g+q)$ and r parameters, where the most significant effect in β occurs due to variations of r, and that the interactions of f_{ck} with the other parameters has no significant influence on the values of β. The great influence of r on the reliability index of reinforced concrete beams was also observed by Santos et al. [10], Nogueira and Pinto [11] and Scherer et al. [12].

Moreover, to verify whether the previous observations are valid, an ANOVA analysis was performed for the reliability index. The ANOVA is a methodology that allows comparing the means of several groups and determining whether these means are different. This analysis can be done through an indicator called a p-value. A sufficiently small p-value indicates that at least one group mean was significantly different from the other means. It is common to consider the p-value small enough if it is less than 0.05 [24].

The results of ANOVA are presented in Table 3. Since the p-values of L, $(g+q)$ and r were less than 0.05, the mean β values are significantly different due to variation in these parameters. The p-value of f_{ck} was greater than 0.05, indicating that the mean values of β in the variations of f_{ck} were not significantly different. The p-values of the f_{ck} interactions with the other parameters were greater than 0.05, indicating that these interactions were not statistically significant in β. Furthermore, the interaction between L - r was not significant since it presented a p-value greater than 0.05. When analyzing Figure 3 again, it was verified that, in fact, the interaction between L - r was not significant, since the lines of the graph present a certain degree of parallelism.

Table 3. Analysis of Variance (ANOVA) for reliability index.

Parameter	*p*-value
L	0
$(g+q)$	0
r	0
f_{ck}	0.3390
L - $(g+q)$	0
L - r	0.1411
L -f_{ck}	0.9431
$(g+q)$ - r	0.0001
$(g+q)$ -f_{ck}	0.9837
r -f_{ck}	0.7228

Figure 4 shows the surface of β as a function of $(g+q)$ and r. In addition to the interaction $(g+q)$ - r, the interaction L - $(g+q)$ also resulted statistically significant according to the ANOVA. However, this interaction does not appear to result in a trend to β, as observed in Figure 3. In addition, the effect of this interaction was smaller when compared to $(g+q)$ - r. Moreover, there was a well-defined trend of increasing the reliability index as r and $(g+q)$ decrease. The influence of r on the increase β was more significant than the influence of $(g+q)$.

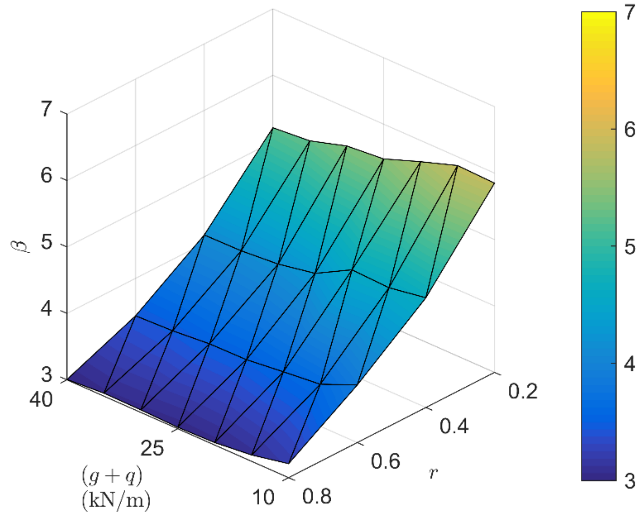

Figure 4. Relationship between reliability index and design parameters.

In addition to the trends for β as a function of the design parameters, the trends as a function of the characteristics of the optimized beams were also investigated. Figure 5 shows the β with the tension reinforcement ratio (ρ_{A_s}), concrete area (A_C), and with the relationships $\frac{h}{L}$ and $\frac{b}{h}$. The behavior is clearly defined by the r ratio, as observed in the previous results. It was not observed a trend of β as a function of ρ_{A_s}, A_C and $\frac{b}{h}$. For the $\frac{h}{L}$ ratio, however, there was a downward trend in the reliability index with the increase of $\frac{h}{L}$. This trend was more evident for low values of r.

In Figure 5, the reliability index equal to 3.8 represents a minimum reliability value suggested by the *fib* 2010 model code [25] for ultimate limit state, considering a representative period of 50 years and failures with average consequences. This target reliability index was also considered in the evaluations of Santos et al. [10], Nogueira and Pinto [11] and Scherer et al. [12]. It was observed that, in general, regardless of the characteristics of the optimized beams, the reliability index resulted less than 3.8 for higher values of r (0.6 and 0.8). Thus, in these situations, optimized beams do not have a minimum reliability index, following the criterion of code *fib* 2010 [25].

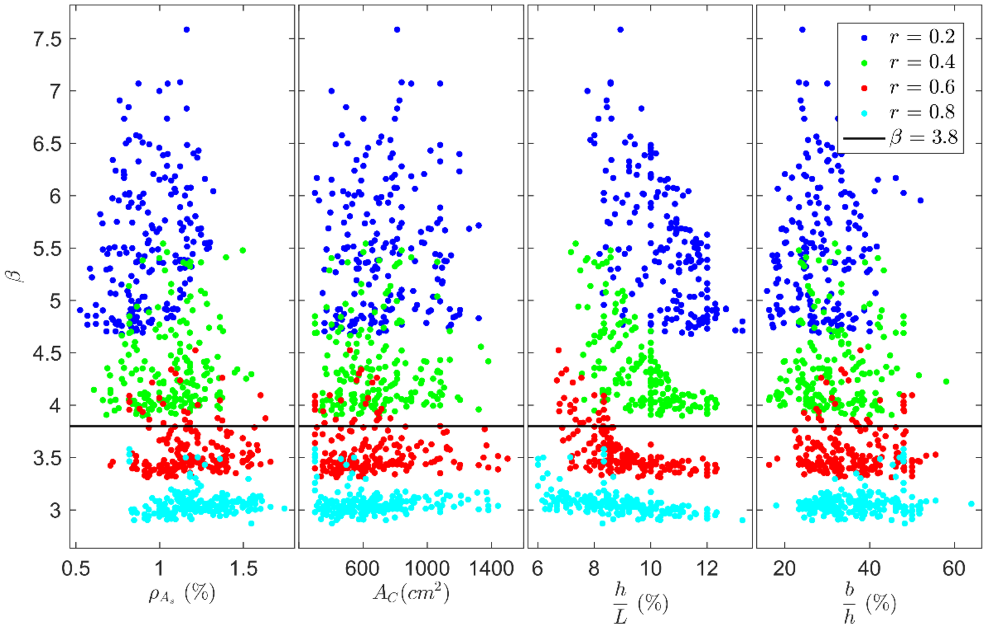

Figure 5. Relationship between reliability index and beam parameters.

Figure 6 shows the reliability indexes as a function of the $\frac{h}{L}$ ratio and the load $(g + q)$. The results in the figure indicate that a tendency of β to decrease with the increase of $\frac{h}{L}$, being it related to $(g + q)$. As the $\frac{h}{L}$ ratio becomes higher, along with $(g + q)$, the reliability index tends to decrease. Thus, optimized beams with high $\frac{h}{L}$ values and subject to a large load tend to have lower reliability indexes.

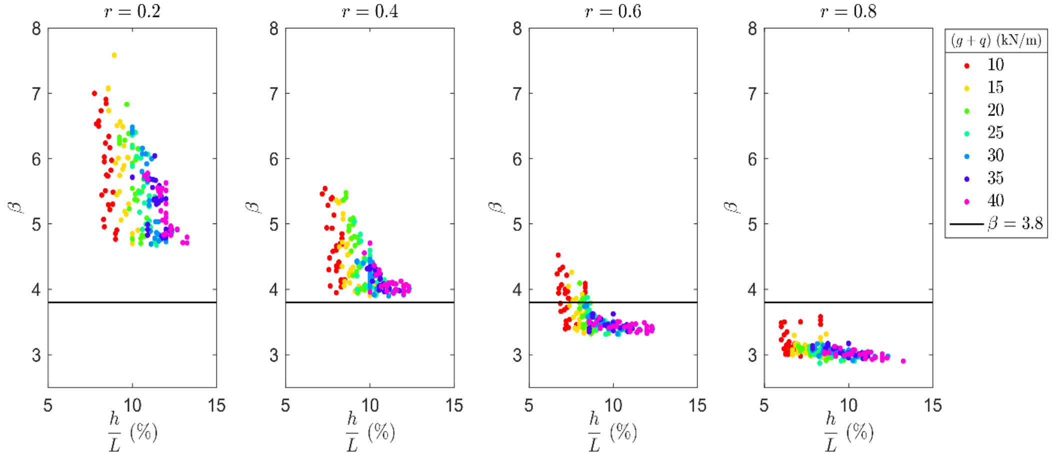

Figure 6. Relationship between the reliability index, $\frac{h}{L}$ and $(g + q)$.

To find a justification for the trend of decrease in the reliability index with the increase of $\frac{h}{L}$ and $(g + q)$, the resistant moment (M_R) and the bending moment (M_S) of the optimized beams were calculated, considering the mean values of the random variables X (Table 2). Through the relationship $\frac{M_R}{M_S}$, it was possible to infer about the resistance and load effects on the beam. Figure 7 shows the reliability indexes as a function of $\frac{h}{L}$ and $\frac{M_R}{M_S}$. A general trend of $\frac{M_R}{M_S}$ to decrease with the increase of $\frac{h}{L}$ was observed, indicating a greater load effect in the beams in these situations. Thus, optimized beams with high $\frac{h}{L}$ values and loading tend to present a higher load effect and, consequently, lower reliability indexes.

A statistical summary of the reliability index of optimized beams is shown in Figure 8. The mean reliability indexes were 4.13. Both the average and median were higher than the minimum index of 3.8. From the 756 indexes obtained, 404 were higher than 3.8, representing 53.44% of the cases.

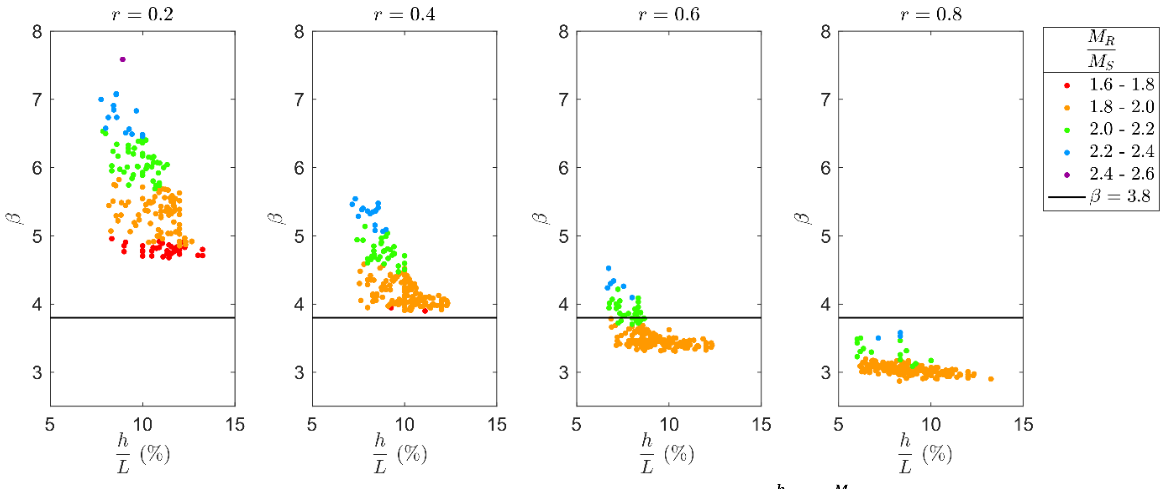

Figure 7. Relationship between reliability index, $\frac{h}{L}$ and $\frac{M_R}{M_S}$.

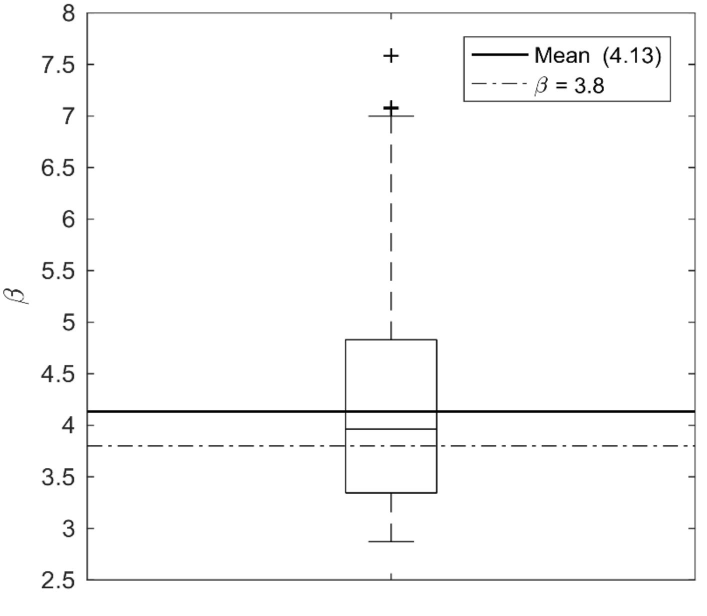

Figure 8. Reliability index variation.

CONCLUSION

Through the stochastic approach it was possible to determine the reliability index of optimized reinforced concrete beams. The reliability index is associated with the probability of failure of the beams in relation to the ultimate limit state of flexure, when uncertainties of the resistance and load models, loads and material strength were considered.

For the various design situations analyzed, it was observed a well-defined trend of the reliability index of optimized beams, as loading and the relationship between loads were changed. As r and $(g+q)$ decrease, the reliability index increases, and this increase is caused mostly by the r ratio. In the design of optimized beams, a small value of r $\left(r=\frac{q}{g+q}\right)$ represents a small portion of live load at total loading. Thus, the observed trend indicates that the reliability index of optimized beams was higher in design situations with small live load. This great influence of r on the reliability index is justified since, among the random variables considered, the live load was the one with the greatest variability in space and time.

While regarding the variation of the span in the design of the optimized beams, it was verified an effect on the reliability index of the beams due to this variation. However, there was no well-defined trend or interaction with the other design parameters. On the other hand, the variations of f_{ck} did not result in significant changes in the reliability index. The analysis of the main effects, interactions graphs and ANOVA indicated no influence of f_{ck}.

The results indicate that there was no trend to the reliability index due to the reinforcement ratio, concrete area and the width/height ratio of optimized beams. However, there was a tendency to decrease the reliability index with the increase in the height/span ratio, depending on the r ratio. This trend is also related to loading, once that the height/span ratios with higher values occur in high loading situations (Figure 6). This behavior was justified by the increased load effects on the beams in these high height/span and load situations (Figure 7). Then, it was possible to affirm that the beams optimized for high loads, and that they have a high height/span ratio, tend to have lower reliability indexes.

The average reliability indexes of the optimized beams was 4.13, being higher than the minimum value recommended by the model code *fib* 2010 [25] that is 3.8. Despite the variations in the reliability index between approximately 3 and 7 (Figure 8), more than half of the indexes obtained (53.44%) were greater than 3.8, indicating an acceptable reliability for the optimized beams in these cases. The beams that presented a reliability index lower than 3.8, in general, were those that were designed with high live load. In these cases, as observed in Figure 6, the beams had reliability indexes lower than acceptable, regardless of their characteristics (reinforcement ratio, concrete area, etc.).

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