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SciELO

ORIGINAL ARTICLE

Evaluation of shear design criteria of beams according to NBR6118 applying the modified compression field theory

Avaliação dos critérios de verificação a cisalhamento segundo a NBR6118 aplicando a teoria do campo de compressão modificada

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Abstract: It is known that design deficiencies in shear design are more dangerous than bending, as shear failures can occur in a fragile way and without possibility of redistributing internal forces. Unlike bending design, designing for shear loads by different standards can generate significantly different results for the same element, as long as design models have been under discussion for many years. This paper analyses the evaluation of the behavior of the combined bending and shear loads in reinforced concrete beams for different pairs of these forces. For this purpose, the verification presented in ABNT NBR 6118 was used and compared to a more improved theory currently used, the Modified Compression Field Theory - MCFT. This theory is able to predict the relationships of specific loads and strains, as well as the shear strength of sections with great precision, being parameterized by several tested elements. As the use of this theory is not practical for manual calculations, the Response-2000 software, developed at the University of Toronto by Evan C. Bentz, was used. The program allows the analysis of beams and columns subject to moments, shear forces and axial loads, for any type of geometry, material properties and reinforcement arrangement, resulting in accurate responses of the behavior of the sections using MCFT as a basis.

Keywords: reinforced concrete, shear, modified compression field theory, Response 2000.

Resumo: Sabe-se que deficiências de projeto no dimensionamento à força cortante são mais perigosas que as de flexão, pois rupturas por cisalhamento podem ocorrer de forma frágil e sem possibilidade de redistribuição dos esforços internos. Diferentemente do dimensionamento à flexão, o dimensionamento a solicitações cisalhantes por diferentes normas, podem gerar resultados significativamente distintos para um mesmo elemento, visto que os modelos de dimensionamento seguem em discussão por muitos anos. Este trabalho considera a avaliação do comportamento da ação das solicitações combinadas de flexão e força cortante em vigas de concreto armado para diferentes pares de momentos fletores e forças cortantes. Para tanto foi utilizada a verificação definida na ABNT NBR 6118, comparada a uma teoria mais aprimorada utilizada atualmente, a Teoria do Campo de Compressão Modificada (*Modified Compression Field Theory - MCFT*). Essa teoria é capaz de prever as relações entre carregamentos e deformações específicas, bem como a resistência à força cortante de seções com grande precisão, sendo parametrizada por diversos elementos ensaiados. Como a utilização dessa teoria não é prática para os cálculos manuais, foi utilizado o *software* Response-2000, desenvolvido na Universidade de Toronto por Evan C. Bentz. O programa permite a análise de vigas e pilares sujeitos a momentos, forças cortantes e cargas axiais, para qualquer tipo de geometria, propriedades de materiais e arranjo de armaduras, resultando em respostas precisas do comportamento das seções utilizando como base a *MCFT*.

Palavras-chave: concreto armado, força cortante, teoria do campo de compressão modificada, Response-2000.

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1 INTRODUCTION

Although the subject of shear resistance of reinforced concrete beams is studied throughout the several last decades, this problem of the determination of the shear resistance determination is nowadays still in discussion. The different design standards still present very different recommendations for the shear design.

For the flexural design, the Navier-Bernoulli hypothesis of plane sections is universally accepted for the design of slender elements, and consequently the forecast of flexural resistance is very similar in the different standards. Differently, for the shear design, the part of the resistance due to the concrete resistance is based on empirical equations, and there is not a consensus on a universally theoretical basis for these equations (Bentz et al. [1]).

Usually, the shear resistance equations are based on the truss model developed by Ritter [2] and Mörsch [3] approximately a century ago. This model does not correspond to the sophistication of the numerical procedures developed internationally throughout the last decades, also considering the computational capacity available nowadays for the structural engineering.

Recent research on the shear resistance of structural concrete is much concentrated nowadays in the study of the rupture mechanisms. This research led to the development of the Compression Field Theory (CFT) and later to the Modified Compression Field Theory (MCFT), as described by Bentz et al. [1]. These theories have been developed from the analysis of a great number of tests in reinforced concrete elements subjected to pure bending and shear combined with axial forces.

The analysis of the of several conducted tests have shown great agreement with the theories, for a great diversity of structural members, such as beams subjected to bending, shear, torsion, deep beams, shear walls, columns, plates, and shells (Vecchio [4]).

The tests shown also that the shear rupture of concrete members presents a different behavior compared with the flexural rupture, are relatively fragile and without the possibility of redistribution of internal forces (Collins et al., [5]). In this way, the understanding of the mechanisms of shear behavior is of the utmost importance.

Another point analyzed herein is that the theories are supported mostly in experiments performed in simply supported beams of small dimensions, very different from the beams of actual structures. Actual continuous beams present points of inversion of the sign of the bending moment, i.e., points of null moment. In these points, shear forces can be high as is in the beam supports, but without the favorable effect of the high vertical compressive forces present in the supports (Kotsovou [6]).

This paper intends to evaluate the criteria for shear design defined in Brazilian Standard ABNT NBR 6118 [7] applying the MCFT for the several scenarios of combined action of shear forces and bending moments in a reinforced concrete section, using the software Response-2000 [8]. This software, developed in the University of Toronto by Evan C. Bentz, uses the MCFT for the analysis of beams and columns subjected to bending moments, shear forces and axial forces, leading to precise results.

The paper summarizes the results obtained in the M.Sc. Thesis of Sá [9].

2 METHODOLOGY

Regarding a better understanding of the behavior of beams near the rupture, when subjected to simultaneous action of bending moments and shear forces, as well as to analyze the safety of the criteria defined in the ABNT NBR 6118 [7], a standardized rection of a reinforced concrete beam has been analyzed, for different pairs of forces and different values of flexural reinforcement ratio. The design according ABNT NBR 6118 [7] is based on the generalized truss shown in Figure 1.

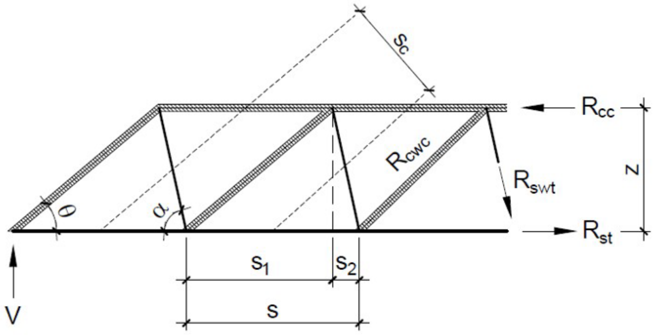

Figure 1. Generalized concrete strut

In this concrete truss, the considered variables are:
R_{swt}: resulting forces in the inclined tensioned ties (stirrups);
R_{cwc}: resulting forces in the compressed diagonal strut;
R_{cc}: resulting force in the horizontal compressed strut;
R_{st}: resulting force in the horizontal tensioned tie (main flexural reinforcement);
V: acting shear force;
z: level arm between the horizontal main struts and ties;
s: spacing between inclined tensioned ties;
s_c: width of the compressed diagonal struts;
α: angle of the inclined tensioned ties;
θ: angle of the compressed diagonal struts.

For defining the different values of flexural reinforcement, nine values of the non-dimensional depth of the neutral axis/effective beam height ($k_x = x/d$) were considered, varying from 0.05 to 0.45, that is the limiting value defined in Brazilian Standard for usual concrete ($f_{ck} \leq 50$ MPa).

From each defined parameter k_x, the corresponding value of maximum resistant bending moment (M_d) can be evaluated. From the obtained values of these maximum bending moments, different pairs of moments and shear forces are evaluated, i.e., for obtaining the maximum resistant value of the shear force corresponding to a certain value of fraction of the maximum resistant bending moment.

Ten values of fraction of the maximum moment are analyzed, varying between 0.1 M_d to 1.0 M_d. For each value of moment fraction, the maximum simultaneous allowed shear force is evaluated, following the criteria of ABNT NBR 6118 [7], considering also its limits for verification of maximum compressive stresses in the inclined struts.

Concerning the inclination of the concrete struts (θ), three different values were investigated:
a) $\theta_1 = 45°$, according to Model I of resistance of ABNT NBR 6118 [7];
b) $\theta_2 = 30°$, minimum value according to Model II of ABNT NBR 6118 [7];
c) θ_3 - angle evaluated in each case according *fib* Model Code 2010 [10].

For evaluating the eventual contribution of secondary reinforcement in flexural resistance, present in all actual structures, three different situations of actual reinforcement were considered (see Figure 2):
a) only basic flexural and shear reinforcement (Model A);
b) Model A reinforcement plus top horizontal reinforcement (Model B);
c) Model B reinforcement plus skin reinforcement.

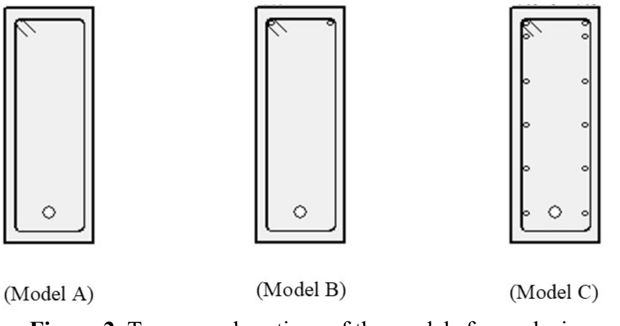

Figure 2. Transversal sections of the models for analysis

It is important to note that the flexural reinforcement of model A correspond to the conventional design defined in ABNT NBR 6118 [7]. In order to evaluate the influence of the secondary superior reinforcement (Model B) and this secondary reinforcement plus the skin reinforcement (Model C), these reinforcements were added to the flexural reinforcement according to the recommendations of this Standard.

3 PROPERTIES OF THE ANALYZED BEAM

For the several analyses, a rectangular beam has been considered, of 30 cm width and 80 cm height. Considered concrete cover is 3 cm. Concerning the materials, concrete class C25 (f_{ck} = 25 MPa), steel reinforcement CA 50 (f_{yk} = 500 MPa, f_{yd} = 435 MPa) and nominal coarse aggregate diameter 10 mm are considered.

The stress-strain relationship for the concrete, in its parabolic branch, as defined in ABNT NBR 6118 [7], is reproduced in Equation 1, in which, for concrete with $f_{ck} \leq 50$ MPa, the considered values are n = 2, ε_{c2} = 2.0 mm/m and ε_{cu} = 3.5 mm/m (in rupture).

$$\sigma_c = 0.85\, f_{cd} \left[1 - \left(1 - \frac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right] \tag{1}$$

The value for maximum tension of concrete is, according to ABNT NBR 6118 [7], $f_{ctd,inf}$, as defined in Equation 2, being:

$$f_{ctd,inf} = \frac{f_{ctk,inf}}{1.4} \tag{2}$$

and:

$$f_{ctk,inf} = 0.7\, f_{ct,m} \tag{3}$$

where:

$$f_{ct,m} = 0.3\, {f_{ck}}^{2/3} \tag{4}$$

Considering f_{ck} = 25 MPa, for input in the software, the tension concrete resistance is $f_{ctd,inf}$ = 1.28 MPa. The inferior value of the concrete tension resistance was adopted to be consistent with the ABNT NBR 6118 [7] criterion for the shear concrete resistance, that considers this value.

4 LOAD CASES TO BE ANALYZED

As previously stated, nine cases are analyzed, for different values of parameter $\underline{k_x}$ varying between 0.05 to 0.45, and for each of these values, maximum bending moments resisted by the section are evaluated, considering:

$$k_z = 1 - 0.4\, k_x \tag{5}$$

$$k_{md} = 0.68\, k_x\, k_z \tag{6}$$

Where k_z is the non-dimensional value of level arm/effective beam height (z/d) and k_{md} is a non-dimensional bending moment, as defined in Equation 7.

$$M_d = k_{md}\, b_w\, d^2\, f_{cd} \tag{7}$$

Equations 5 to 7 allows for the evaluation of the maximum bending moments M_d as a function of k_x.
From the values of M_d, the corresponding values of the flexural reinforcement A_s are evaluated, with Equation 8:

$$A_s = \frac{M_d}{k_z\, d\, f_{yd}} \tag{8}$$

Table 1 presents the numerical values associated with each analyzed case.

Table 1. Summary of numerical values of each case

Case	k_x	k_z	k_{md}	M_{sd} (kNm)	A_s (cm^2)
1	0.050	0.980	0.033	87.47	2.93
2	0.100	0.960	0.065	171.36	5.87
3	0.150	0.940	0.096	251.69	8.80
4	0.200	0.920	0.125	328.44	11.73
5	0.250	0.900	0.153	401.63	14.66
6	0.300	0.880	0.180	471.24	17.60
7	0.350	0.860	0.205	537.29	20.53
8	0.400	0.840	0.228	599.76	23.46
9	0.450	0.820	0.251	658.67	26.39

For each of the nine cases presented next, ten different pairs of maximum bending moments x shear forces were evaluated, each of them associated with a different fraction of the maximum moment M_d. These fractions correspond to values between 0.1 M_d to 1.0 M_d.

The maximum shear forces corresponding to each value of fraction of the maximum moment M_d are evaluated considering Equation 9 from ABNT NBR 6118 [7], which defines the forces $F_{sd,cor}$ in the flexural reinforcement, for the simultaneous action of bending moments and shear forces.

$$F_{sd,cor} = \left[\frac{M_{sd}}{z} + |V_{sd}|(\cot\theta - \cot\alpha)\frac{1}{2}\right] \leq \frac{M_{sd,máx}}{z} \tag{9}$$

With:

$$F_{sd,cor} = f_{yd}\, A_s \tag{10}$$

$$z = k_z\, d \tag{11}$$

Therefore, the maximum allowed shear force is as defined in Equation 12.

$$V_{sd} = \left(\mathrm{F_{sd,cor}} - \frac{M_{sd}}{z}\right)\frac{2}{(\cot\theta - \cot\alpha)} \tag{12}$$

For the angle α, the value 90° is taken (vertical stirrups). For the angle θ, three possibilities are analyzed, θ_1=45°, θ_2=30° e θ_3 evaluated according to *fib* Model Code 2010 [10]. The *fib* expression for the minimum value of θ is reproduced in Equation 13.

$$\theta_{min} = 20° + 10000\,\varepsilon_x \tag{13}$$

Where:

$$\varepsilon_x = \frac{\frac{M_{Ed}}{z} + V_{Ed} + 0.5\,N_{Ed}}{2\,E_S\,A_S} \leq 0.003 \tag{14}$$

and:

ε_x: specific stress at the center of the effective height

M_{Ed}: acting bending moment

N_{Ed}: acting axial force

E_s: steel Young modulus

If the value of ε_x results in negative values, it should be taken as zero.

Since for the evaluation of the shear force as defined in Equation 12 is necessary to know the value of θ, and for the evaluation of θ_{min}, as defined in Equation 13 in necessary to know the value of the shear force, a interactive process is necessary until the values of the two angles be coincident.

Then, it is possible to proceed with the evaluation of the shear reinforcement, according to Equation 15.

$$\mathrm{V_{sd}} \leq \mathrm{V_{Rd3}} = \mathrm{V_c} + \mathrm{V_{sw}} \tag{15}$$

In order to fix one of the variables of the design, the stirrup spacing is defined as 20 cm, and for defining the necessary shear reinforcement per meter, (A_{sw}/s), a fictitious stirrup "leg" area $\mathrm{A}_ø$ is defined as:

$$\mathrm{A}_ø = \mathrm{s_{adot}}\, A_{sw}/s\ \frac{1}{n} \tag{16}$$

Where:
$\mathrm{s_{adot}}$: adopted stirrup spacing;
A_{sw}/s : necessary shear reinforcement per meter;
n: number of stirrup "legs", fixed in this study as two;
$\mathrm{A}_ø$: area of one stirrup "leg".

For the evaluation of the eventual contribution of the superior secondary reinforcement in the flexural resistance, two bars of 8 mm diameter were considered in the analyses.

Besides this secondary reinforcement, the eventual contribution of the skin reinforcement is also considered. According to ABNT NBR 6118 [7], skin reinforcement is necessary on beams with height superior to 60 cm, with area per meter equal to 0.10% of the concrete section in each vertical face, not superior to 5 cm^2/m per face and with spacing not superior to 20 cm.

The defined data can be then introduced in the software Response-2000 [8]. The main screen of the software is shown in Figure 3.

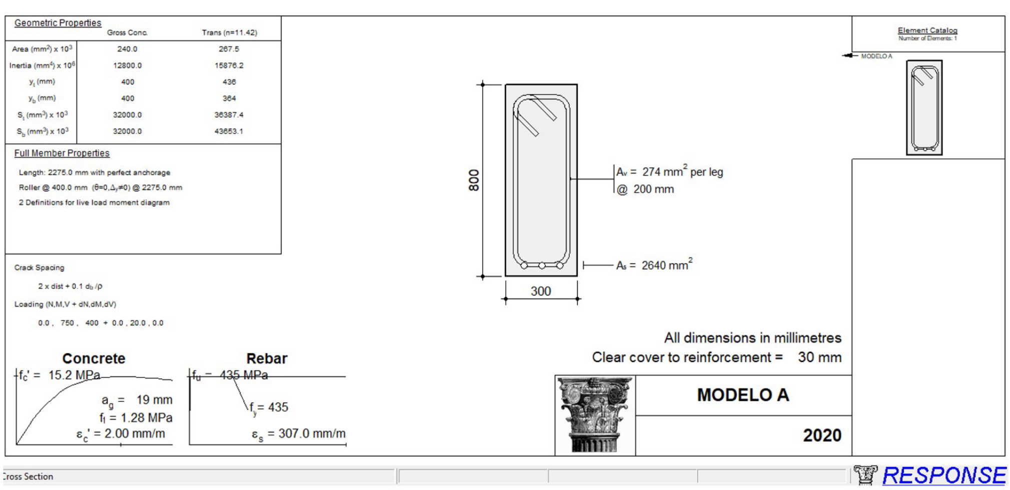

Figure 3. Main screen of Response-2000

For the Response-2000 [8] runs it is necessary to point out that the concrete resistance f_c' considered in the program, correspond to the maximum concrete stress 0.85 f_{cd} defined in the Brazilian Standard.

5 RESULTS

Complete results of the performed analysis can be found in Sá [9]. For the sake of concision, results presented herein are only for the value $k_x = 0.45$ which correspond to the maximum flexural reinforcement without compression reinforcement.

The presented results correspond to the three considered angles of the diagonal compression struts. For each of them, results corresponding to three reinforcement models are presented: flexural and shear reinforcement (Model A); Model A plus top secondary reinforcement (Model B); Model B plus skin reinforcement (Model C).

Each set of results, initially are presented tables in which: "Case" refers to a fraction, between 1.0 and 0.1 of the maximum moment $M_{sd,max}$ resisted with the reinforcement evaluated with $V_{sd} = 0$ in Equation 9; "M_{sd}" correspond to $M_{sd,max}$ times the fraction corresponding to the analyzed case; "V_{sd}" is the shear force evaluated according to Equation 12; "V_{sd}^{final}" is "V_{sd}" limited to V_{rd2} which is the maximum shear force defined in ABNT NBR 6118 [7] corresponding to the maximum compression stress in the diagonal strut (in the analyzed case $V_{rd2} = 911.25\ kN$); "V_c" is the part of the shear force resisted by the complimentary mechanisms in concrete according to ABNT NBR 6118 [7]; "V_{sw}" is the part of the shear force resisted by the shear reinforcement. This is the shear reinforcement that will be considered in Response-2000 [8] runs, also considering the minimum shear reinforcement of 3.08 cm^2/m.

Then, Tables 2 to 5 and Figures 4 and 5 are presented with the relationships between bending moments and allowable shear forces, obtained with the equations of ABNT NBR 6118 [7] and with Response-2000 [8] for reinforcement Models A, B and C.

- Results for $\theta_I = 45°$ (according to Model I of resistance of ABNT NBR 6118 [7])

Table 2. Shear design for θ_1.

Case	M_{sd} (kNm)	V_{sd} (kN)	V_{sd}^{final} (kN)	V_c (kN)	V_{sw} (kN)
1	658.67	0.00	0.00	161.59	0.00
2	592.80	229.50	229.50	161.59	67.91
3	526.93	459.00	459.00	161.59	297.41
4	461.07	688.50	688.50	161.59	526.91
5	395.20	918.00	911.25	161.59	749.66
6	329.33	1147.50	911.25	161.59	749.66
7	263.47	1377.00	911.25	161.59	749.66
8	197.60	1606.50	911.25	161.59	749.66
9	131.73	1836.00	911.25	161.59	749.66
10	65.87	2065.50	911.25	161.59	749.66

Table 3. Shear forces obtained with θ_I.

Case	NBR 6118		Model A		Model B		Model C	
	M_{sd} (kNm)	V_{sd}^{final} (kN)	V_{Rd} (kN)	V_{Rd}/V_{sd}	V_{Rd} (kN)	V_{Rd}/V_{sd}	V_{Rd} (kN)	V_{Rd}/V_{sd}
1	658.67	0.00	0.00	-	0.00	-	0.00	-
2	592.80	229.50	131.05	0.57	142.18	0.62	173.06	0.75
3	526.93	459.00	374.62	0.82	392.18	0.85	444.02	0.97
4	461.07	688.50	640.57	0.93	653.22	0.95	696.78	1.01
5	395.20	911.25	838.44	0.92	845.87	0.93	840.64	0.92
6	329.33	911.25	817.53	0.90	813.25	0.89	850.21	0.93
7	263.47	911.25	704.44	0.77	757.79	0.83	798.83	0.88
8	197.60	911.25	581.64	0.64	713.25	0.78	735.30	0.81
9	131.73	911.25	433.88	0.48	588.94	0.65	698.29	0.77
10	65.87	911.25	306.19	0.34	381.61	0.42	591.55	0.65

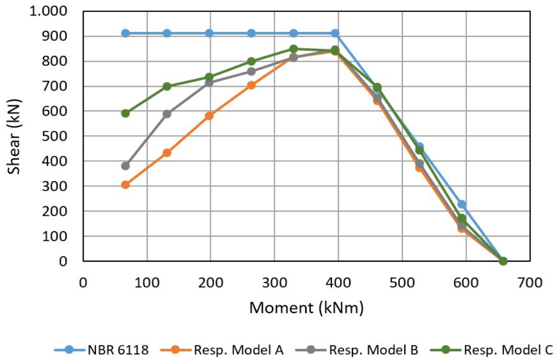

Figure 4. Shear forces obtained with θ_1.

- Results for $\theta_2 = 30°$ (according to Model II of resistance of ABNT NBR 6118 [7])

Table 4. Shear design for θ_2.

Case	M_{sd} (kNm)	V_{sd} (kN)	V_{sd}^{final} (kN)	V_{c1} (kN)	V_{sw} (kN)
1	658.67	0.00	0.00	161.59	0.00
2	592.80	132.50	132.50	161.59	0.00
3	526.93	265.00	265.00	134.97	130.04
4	461.07	397.51	397.51	100.85	296.66
5	395.20	530.01	530.01	66.73	463.28
6	329.33	662.51	662.51	32.61	629.90
7	263.47	789.17	789.17	0.00	789.17
8	197.60	789.17	789.17	0.00	789.17
9	131.73	789.17	789.17	0.00	789.17
10	65.87	789.17	789.17	0.00	789.17

Table 5. Shear forces obtained with θ_2.

Case	NBR 6118		Model A		Model B		Model C	
	M_{sd} (kNm)	V_{sd}^{final} (kN)	V_{Rd} (kN)	V_{Rd}/V_{sd}	V_{Rd} (kN)	V_{Rd}/V_{sd}	V_{Rd} (kN)	V_{Rd}/V_{sd}
1	658.67	0.00	0.00	-	0.00	-	0.00	-
2	592.80	132.50	131.05	0.99	142.18	1.07	173.06	1.31
3	526.93	265.00	170.74	0.64	176.86	0.67	186.88	0.71
4	461.07	397.51	314.43	0.79	317.08	0.80	329.62	0.83
5	395.20	530.01	464.19	0.88	466.90	0.88	471.03	0.89
6	329.33	662.51	603.87	0.91	603.09	0.91	602.99	0.91
7	263.47	789.17	628.72	0.80	624.69	0.79	664.25	0.84
8	197.60	789.17	520.28	0.66	563.75	0.71	597.17	0.76
9	131.73	789.17	388.99	0.49	509.99	0.65	557.52	0.71
10	65.87	789.17	277.25	0.35	342.70	0.43	496.57	0.63

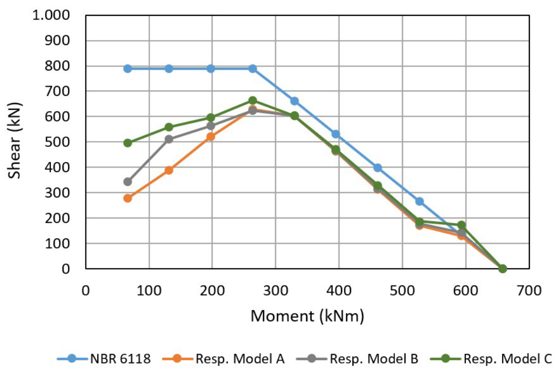

Figure 5. Shear forces obtained with θ_2

- Results for θ_3 (according to *fib* Model Code 2010 [10])

For obtaining the maximum allowable shear force, it is necessary firstly to define the strut angle θ. For determining θ_{min} is necessary to know the maximum shear force. Therefore, an interactive process is necessary for equaling the two angles. Tables 6 to 8 present the determination of the angles θ for each considered case.

Table 6. Values of inclination angles θ according to *fib* Model Code 2010 [10]

Case	c	M_{sd} (kNm)	Θ (°)	V_{sd} (kN)	ε_x	θ_{min} (°)
1	1.0	658.67	30.35	0.00	0.00104	30.35
2	0.9	592.80	30.54	135.39	0.00105	30.54
3	0.8	526.93	30.74	273.02	0.00107	30.74
4	0.7	461.07	30.97	413.28	0.00110	30.97
5	0.6	395.20	31.23	556.69	0.00112	31.23
6	0.5	329.33	31.53	703.91	0.00115	31.53
7	0.4	263.47	31.86	855.83	0.00119	31.86
8	0.3	197.60	32.25	1013.63	0.00122	32.25
9	0.2	131.73	32.71	1179.02	0.00127	32.71
10	0.1	65.87	33.26	1354.47	0.00133	33.25

Table 7. Shear design for θ_3.

Case	M_{sd} (kNm)	V_{sd} (kN)	Θ (°)	V_{rd2} (kN)	V_{sd}^{final} (kN)	V_{c1} (kN)	V_{sw} (kN)
1	658.67	0.00	30.35	794.70	0.00	161.59	0.00
2	592.80	135.39	30.54	797.59	135.39	161.59	0.00
3	526.93	273.02	30.74	800.74	273.02	133.42	139.60
4	461.07	413.28	30.97	804.21	413.28	98.30	314.98
5	395.20	556.69	31.23	808.04	556.69	62.83	493.86
6	329.33	703.91	31.53	812.31	703.91	26.92	676.99
7	263.47	855.83	31.86	817.09	817.09	0.00	817.09
8	197.60	1013.63	32.25	822.48	822.48	0.00	822.48
9	131.73	1179.02	32.71	828.64	828.64	0.00	828.64
10	65.87	1354.47	33.26	835.74	835.74	0.00	835.74

Table 8. Shear forces obtained with θ_3.

Case	NBR 6118		Model A		Model B		Model C	
	M_{sd} (kNm)	V_{sd}^{final} (kN)	V_{Rd} (kN)	V_{Rd}/V_{sd}	V_{Rd} (kN)	V_{Rd}/V_{sd}	V_{Rd} (kN)	V_{Rd}/V_{sd}
1	658.67	0.00	0.00	-	0.00	-	0.00	-
2	592.80	135.39	131.05	0.97	142.18	1.05	173.06	1.28
3	526.93	273.02	170.74	0.63	176.86	0.65	186.88	0.68
4	461.07	413.28	336.91	0.82	337.12	0.82	352.40	0.85
5	395.20	556.69	501.69	0.90	504.68	0.91	509.20	0.91
6	329.33	703.91	653.14	0.93	651.32	0.93	651.91	0.93
7	263.47	817.09	654.76	0.80	653.61	0.80	699.52	0.86
8	197.60	822.48	534.64	0.65	605.11	0.74	638.77	0.78
9	131.73	828.64	398.11	0.48	549.75	0.66	604.55	0.73
10	65.87	835.74	295.46	0.35	356.29	0.43	538.47	0.64

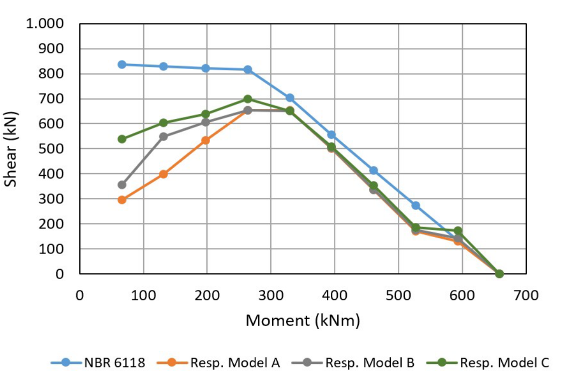

Figure 6. Shear forces obtained with θ_3

6 DISCUSSION OF RESULTS

Observing the results illustrated in Figures 4 to 6, it can be observed that shear design criteria defined in ABNT NBR 6118 [7] are in good agreement with results obtained with the Response-2000 [8] analyses.

However, for low values of bending moments, the allowable shear forces predicted by ABNT NBR 6118 [7] are not attained in the Response-2000 [8] runs. This is because for low values of bending moments, shear forces cause tension forces in the superior part of the beams, where there is not enough flexural reinforcement.

This can occur, as already pointed out by Kotsovou [6], in continuous beams, in spans where there is a change of sign of the bending moments, particularly in points where the bending moments are equal to zero and an important value of shear force is present.

Clearly, the safety in this point can only be achieved with the consideration of an adequate compression reinforcement. For showing this, another situation is analyzed, considering a compression reinforcement equal to the main flexural reinforcement. This is done for the already defined Model A. The analyzed section is presented in Figure 7.

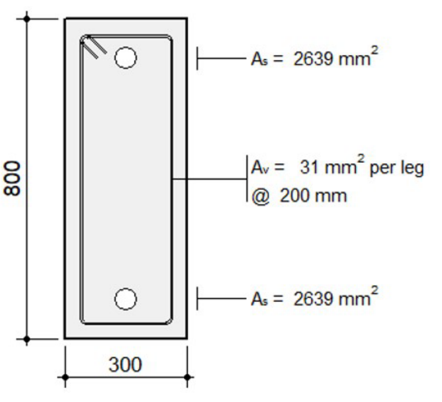

Figure 7. Transversal section with symmetrical reinforcement

The section is processed again, considering the compression reinforcement, being the results presented in Tables 9 to 11 and in Figures 8 to 10. The tables also present the relationship between allowable shear forces obtained with Response-2000 [8] and the ones predicted by ABNT NBR 6118 [7].

Considering the scope of this study, values of this relationship smaller than 1.00 would indicate the cases in which that ABNT NBR 6118 [7] is not safe enough. These can be considered as the final results of the study.

Table 9. Relationship between allowable shears, NBR6118/Response-2000, angle θ_I

Case	NBR 6118		Model A	
	M_{sd} (kNm)	V_{sd}^{final} (kN)	V_{Rd} (kN)	V_{Rd}/V_{sd}
1	658.67	0.00	0.00	-
2	592.80	229.50	180.24	0.79
3	526.93	459.00	411.22	0.90
4	461.07	688.50	656.28	0.95
5	395.20	911.25	840.72	0.92
6	329.33	911.25	851.88	0.93
7	263.47	911.25	853.98	0.94
8	197.60	911.25	820.61	0.90
9	131.73	911.25	817.65	0.90
10	65.87	911.25	818.31	0.90

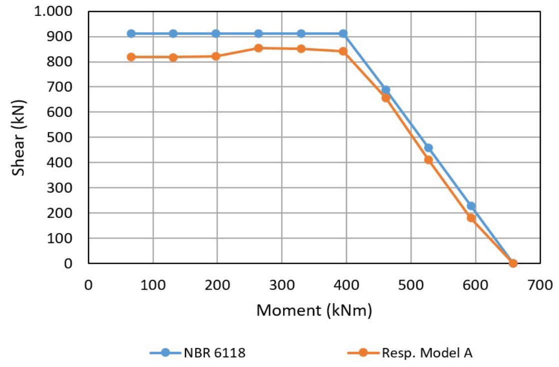

Figure 8. Shear forces obtained with θ_1. symmetrical reinforcement

Table 10. Relationship between allowable shear, NBR6118/Response-2000, angle θ_2.

Case	NBR 6118		Model A	
	M_{sd} (kNm)	V_{sd}^{final} (kN)	V_{Rd} (kN)	V_{Rd}/V_{sd}
1	658.67	0.00	0.00	-
2	592.80	132.50	180.24	1.36
3	526.93	265.00	191.63	0.72
4	461.07	397.51	337.01	0.85
5	395.20	530.01	480.50	0.91
6	329.33	662.51	597.44	0.90
7	263.47	789.17	687.46	0.87
8	197.60	789.17	676.36	0.86
9	131.73	789.17	653.55	0.83
10	65.87	789.17	653.82	0.83

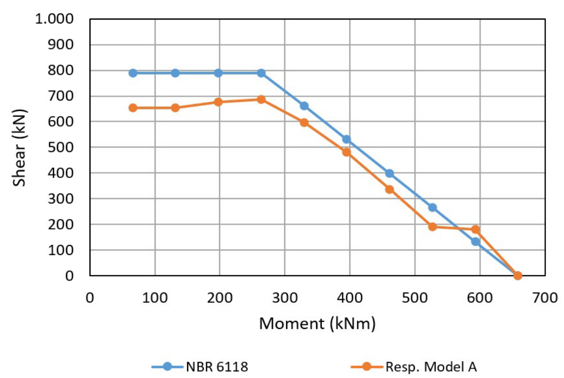

Figure 9. Shear forces obtained with θ_2. symmetrical reinforcement

Table 11. Relationship between allowable shear, NBR6118/Response-2000, angle θ_3.

Case	NBR 6118		Model A	
	M_{sd} (kNm)	V_{sd}^{final} (kN)	V_{Rd} (kN)	V_{Rd}/V_{sd}
1	658.67	0.00	0.00	-
2	592.80	135.39	180.24	1.33
3	526.93	273.02	191.63	0.70
4	461.07	413.28	358.75	0.87
5	395.20	556.69	515.81	0.93
6	329.33	703.91	646.83	0.92
7	263.47	817.09	722.71	0.88
8	197.60	822.48	721.19	0.88
9	131.73	828.64	709.27	0.86
10	65.87	835.74	720.36	0.86

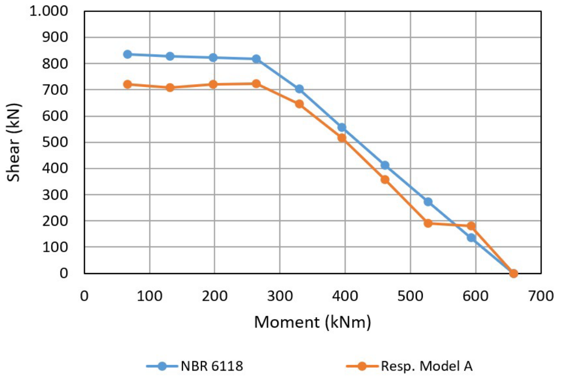

Figure 10. Shear forces obtained with θ_3. symmetrical reinforcement

Another important issue is the value of V_c, part of the shear force resisted by the complimentary mechanisms of concrete, to be considered in the design. Some standards, such as Eurocode 2 even neglect this contribution. Only for angle θ_2, a comparison is made, as shown in Figures 11 and 12, with/ without V_c,, in which is clear that, without the consideration of V_C, better results are achieved.

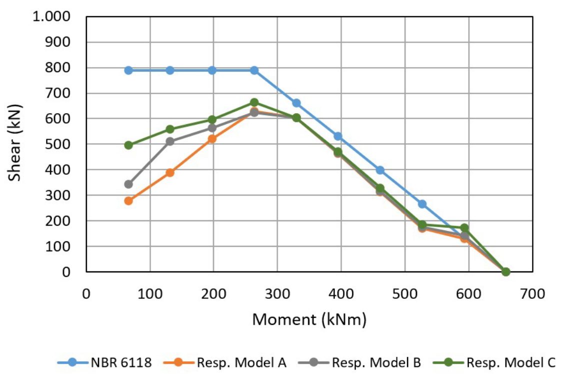

Figure 11. Comparison of results for θ_2 – With V_c

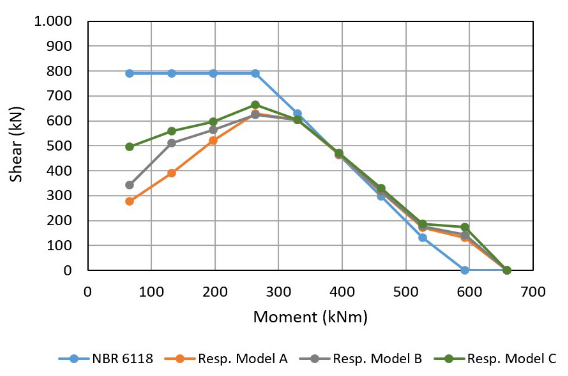

Figure 12. Comparison of results for θ_2 – Without V_c

7 PRACTICAL EXAMPLE

To evaluate the eventual consequence in the current design of beams of the presented results, an example is presented, taken from an actual project.

For the analysis and design of the beam, the system TQS [11] has been used. This system of complete analysis and design of building structures is presently one of the most used in Brazil.

Figures 13 and 14 show, respectively, formwork and perspective drawings of the analyzed beam.

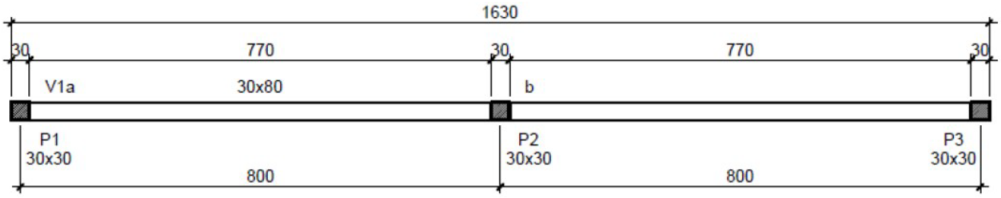

Figure 13. Formwork drawing of the analyzed beam

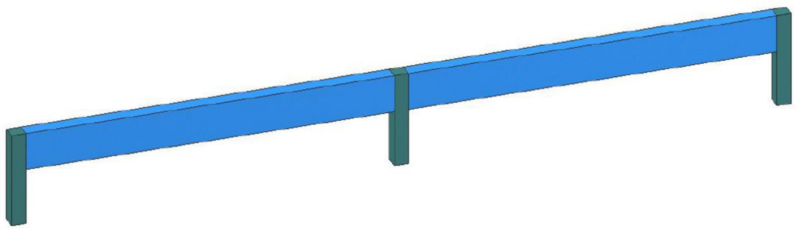

Figure 14. Perspective of the model.

Applied distributed loads in the beam are of 50 kN/m, besides the self-weight, automatically determined by the software. This loading has been adequately chosen in order that the limit of k_x = 0.45 be attained. The beams are considered as simply supported in the extreme supports.

Figure 15 shows the bending forces and shear forces diagrams, as well as the selected sections in which the analyses are made: Sections S1 and S4 are a distance *d* of the supports, Section S2 is in the point of maximum positive bending moment and Section S3 is in the point of zero moment.

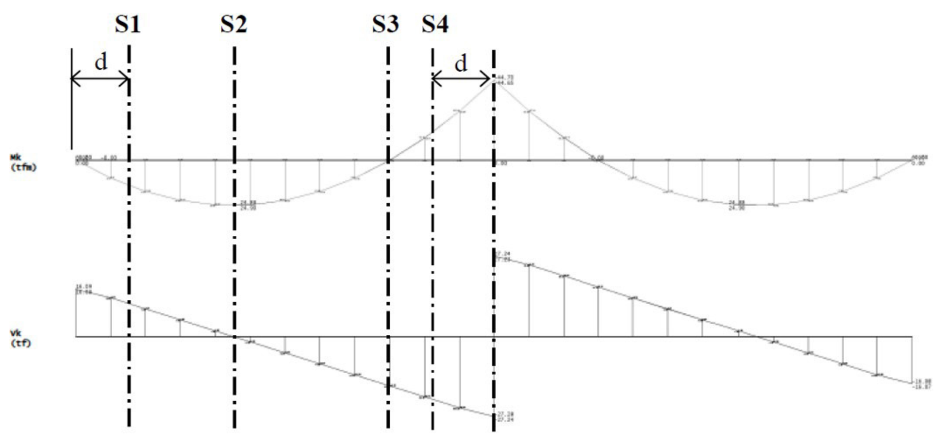

Figure 15. Diagrams of bending moments and shear forces and selected sections.

The shear design is done according to Model I of ABNT NBR 6118 [7]. Table 12 present bending moments and shear forces in the selected sections, with their characteristic and design values, considering γ_f = 1.4.

Table 12. Characteristic and design forces

Section	TQS Characteristic		TQS Design	
	M_k (kNm)	V_k (kN)	M_d (kNm)	V_d (kN)
S1	108.44	126.23	151.82	176.72
S2	249.00	0.00	348.60	0.00
S3	0.00	-167.90	0.00	-235.06
S4	-254.74	-237.84	-356.64	-332.98

Figure 16 presents the complete reinforcement detailing, automatically performed by the software TQS [11].

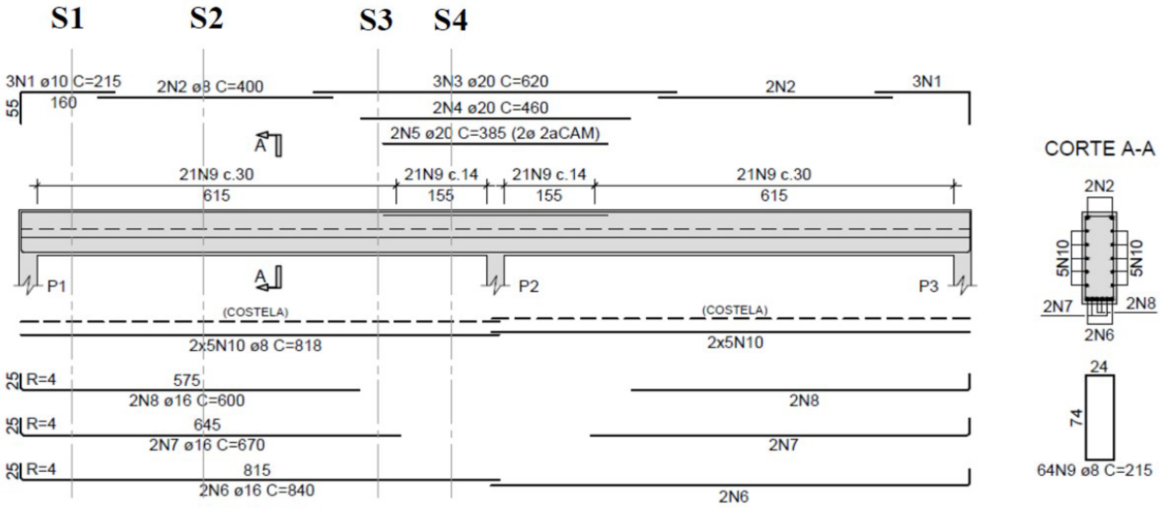

Figure 16. Reinforcement detailing

Tables 13 and 14 show the manual shear design of the selected sections, in order to check the design done by TQS [11].

Table 13. Shear design. Forces.

Section	$\|V_d\|$ (kN)	b_w (m)	d (m)	V_{Rd2} (kN)	V_c (kN)	V_{sw} (kN)
S1	176.72	0.30	0.75	976.34	173.14	3.58
S2	0.00	0.30	0.75	976.34	173.14	0.00
S3	235.06	0.30	0.75	976.34	173.14	61.92
S4	332.98	0.30	0.75	976.34	173.14	159.84

Table 14. Shear design. Reinforcement.

Section	A_{sw}/s^{calc} (cm^2/m)	A_{sw}/s^{min} (cm^2/m)	A_{sw}/s (cm^2/m)	N_{pernas}	ø (mm)	s (cm)
S1	0.12	3.08	3.08	2	8.0	32.66
S2	0.00	3.08	3.08	2	8.0	32.66
S3	2.11	3.08	3.08	2	8.0	32.66
S4	5.45	3.08	5.45	2	8.0	18.46

The shear design of the central section is presented in Tables 15 and 16. It can be observed that the shear reinforcement designed by TQS [11] is correct.

Table 15. Shear design. Central support. Forces.

$\|V_{d,máx}\|$ (kN)	b_w (m)	d (m)	V_{Rd2} (kN)	V_c (kN)	V_{sw} (kN)
381.36	0.30	0.75	976.34	173.14	208.22

Table 16. Shear design. Central support. Reinforcement.

A_{sw}/s^{calc} (cm^2/m)	A_{sw}/s^{min} (cm^2/m)	A_{sw}/s (cm^2/m)	N_{pernas}	ø (mm)	s (cm)
7.10	3.08	7.10	2	8.0	14.17

The obtained reinforcement is input for analyses with the software Response-2000 [8].

Table 17 shows a comparison between maximum shear forces obtained with TQS [11] and the ones obtained with Response-2000 [8].

Table 17. Comparison between maximum allowable shear forces.

Section	Response-2000		TQS	V_{Rd}/V_{sd}
	$M_{S,d}$ (kNm)	$V_{R,d}$ (kN)	$V_{S,d}$ (kN)	
S1	151.82	253.05	176.72	1.43
S2	348.60	191.14	0.00	-
S3	0.00	-236.18	-235.06	1.00
S4	-356.64	-421.49	-332.98	1.27

In all the analyzed sections, the effective resistant shear forces are equal or superior to the acting shear force, showing the safety of the criteria of ABNT NBR 6118 [7].

Special attention is given to the point of zero moment. For a better understanding of the behavior in S3, some results obtained with Response-2000 [8] are presented in Figures 17 to 19.

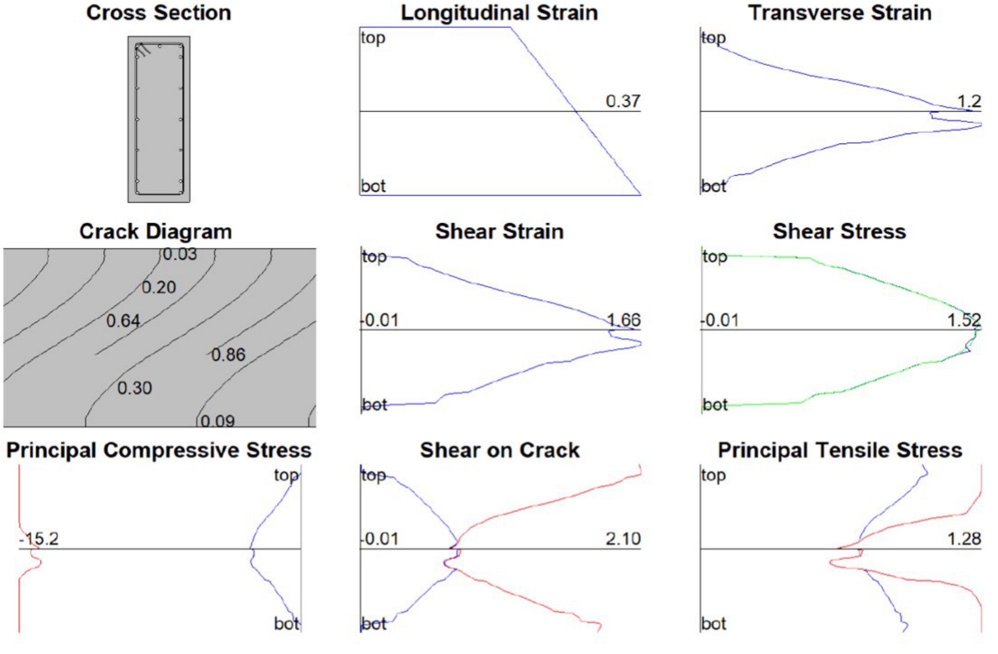

Figure 17. General results of the section – Response-2000

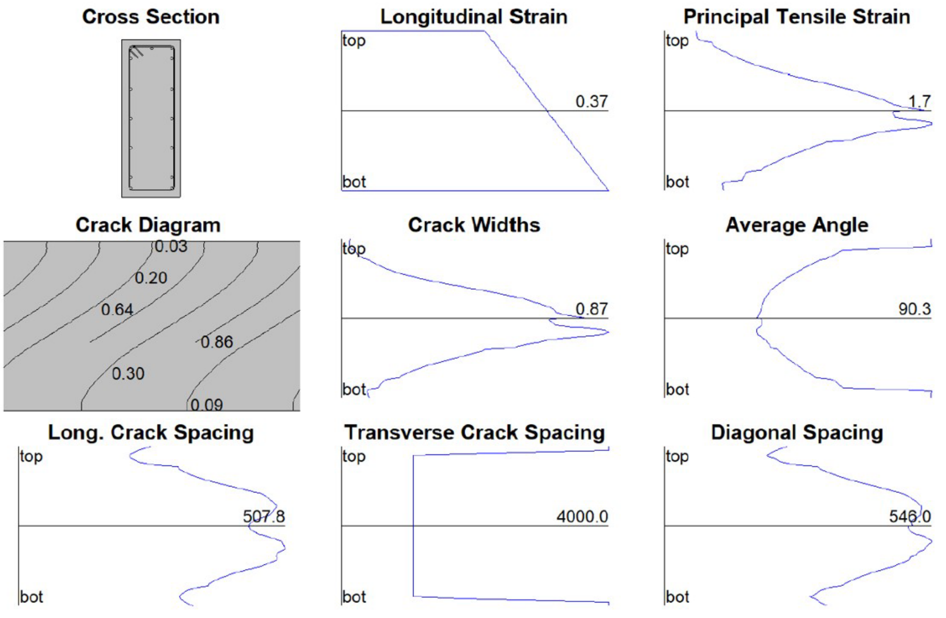

Figure 18. Results related to the cracks – Response-2000

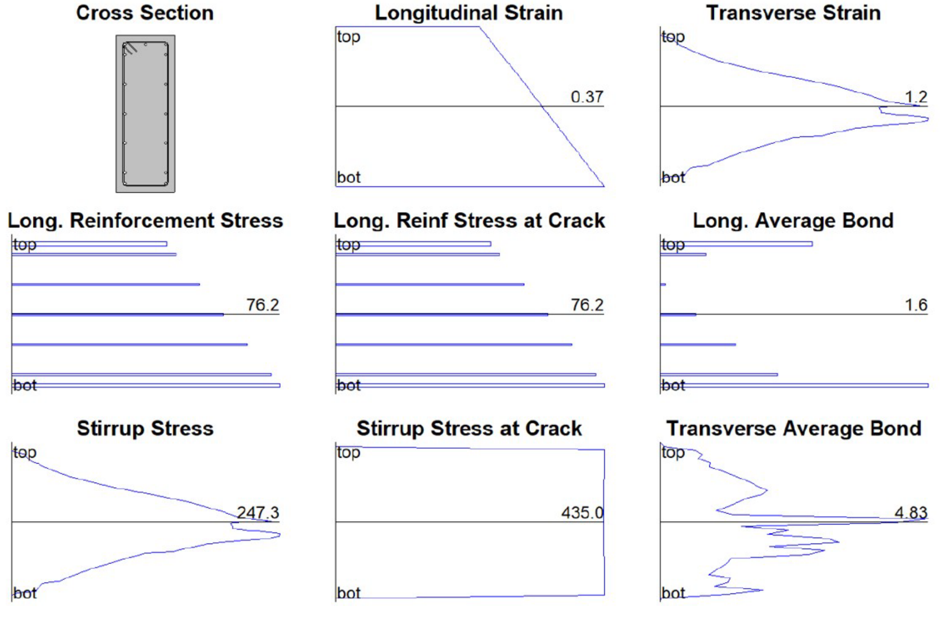

Figure 19. Results related to the reinforcement – Response-2000

It can be observed that the section is fully cracked throughout is height. This is due to the fact that, since the moment is null, the compression due to the flexural binary does not exist, being then the beam totally tensioned by the force due to the compressed diagonal, making all the section more fragile.

Evaluating the results related to the reinforcement, the rupture of the section is due to the yielding of the transversal reinforcement in the cracks, which is a verification made in the MCFT, not done in the usual shear design.

Another observation is that the MCFT considers the presence of all the reinforcement present in the section, including the skin reinforcement, not considered in the usual design. For evaluate the importance of this detail, the beam is verified also without the skin reinforcement. Results are presented in Table 18.

Table 18. Comparison between maximum allowable shear forces. No skin reinforcement.

Section	Response-2000		TQS	V_{Rd}/V_{sd}
	$M_{S,d}$ (kNm)	$V_{R,d}$ (kN)	$V_{S,d}$ (kN)	
S1	151.82	264.79	176.72	1.50
S2	348.60	80.51	0.00	-
S3	0.00	-218.04	-235.06	0.93
S4	-356.64	-415.95	-332.98	1.25

As seen in Table 18, without the skin reinforcement, the relationship resisting/ acting shear force is now 0.93, i.e. the beam is not able to resist to the acting shear force according to the MCFT.

Another analysis is done, considering skin reinforcement, but not respecting the minimum reinforcement of ABNT NBR 6118 [7]. The shear design is shown in Table 19.

Table 19. Shear design. No minimum reinforcement.

Section	A_{sw}/s^{calc} (cm^2/m)	N_{pernas}	ø (mm)	s (cm)
S1	0.12	2	5.0	321.48
S2	0.00	2	5.0	-
S3	2.11	2	6.3	29.55
S4	5.45	2	8.0	18.46

Since the design of section S1 led to a great reinforcement spacing a value of 30 cm is adopted for the Response-2000 [8] runs. For section S2 the same spacing is considered.

Comparison between allowable shear forces is presented in Table 20.

Table 20 Comparison between maximum allowable shear forces. No minimum shear reinforcement

Section	Response-2000		TQS	V_{Rd}/V_{sd}
	$M_{S,d}$ (kNm)	$V_{R,d}$ (kN)	$V_{S,d}$ (kN)	
S1	151.82	191.29	176.72	1.08
S2	348.60	100.33	0.00	-
S3	0.00	-224.68	-235.06	0.96
S4	-356.64	-347.03	-332.98	1.04

In this case, the results of the previous analytical results are confirmed, and a safety relationship is usually slightly superior to 1.00.

However, in section S3, although not being the one of maximum shear force, present insufficient safety. The TQS design, strictly in accordance with ABNT NBR 6118 [7], presents a resisting shear force of 235.06kN. The shear force evaluated with Response-2000 [8] is 224.68kN, showing that the section of null moment could be the critical one in a continuous beam, for the design following the criteria of ABNT NBR 6118 [7].

8 CONCLUSIONS

From the extensive studies summarized herein, some conclusions can be drawn.

Considering the sophisticated design criteria of MCFT, the state-of-the-art criteria for the combined bending moments-shear forces design according to the *fib* Model Code 2000 [10], the results obtained applying the criteria of ABNT NBR 6118 [7] led to a safe and economical design.

Nevertheless, in situations of small bending moments and high shear forces, such as points of zero moments in continuous beams, the actual criteria can lead to results against safety. As shown in the text, in these regions the situation of the concrete section presents great fragility, being tensioned throughout its high. Critical design situations can be presented in these regions, not considered in the usual design.

Another point that deserves future attention is that better results are obtained with the shear forces being totally resisted by the shear reinforcement ($V_c = 0$), indicating that perhaps the value of V_c could be overestimated by ABNT NBR 6118 [7].

Regarding the secondary reinforcement, is has been shown that the superior horizontal reinforcement can be considered in the usual evaluation of the resistance, but its contribution is small. Inversely, skin reinforcement is not usually considered in the evaluation of the resistance, but its contribution can be relatively important.

As pointed out by Schulz [12] shear rupture mechanisms in reinforced concrete sections are very complex, of fragile character and without possibility of redistribution of forces. This opens a vast field of research in this subject.

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