Volume 8, Number 6 (December 2015) p. 763-786 • ISSN 1983-4195
http://dx.doi.org/10.1590/S1983-41952015000600003

REVISTA IBRACON DE ESTRUTURAS E MATERIAIS
IBRACON STRUCTURES AND MATERIALS JOURNAL

Recommendations for verifying lateral stability of precast beams in transitory phases

Recomendações para verificação da estabilidade lateral de vigas pré-moldadas em fases transitórias



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Abstract

This paper presents recommendations for security check of precast beams in transitory phases, compare results of parametric analyzes with national and international code recommendations and confront the formulations used for the calculation of critical load of lateral instability. In transport and lifting phases, precast beams are susceptible to loss lateral stability because the established supports provides little restriction to the element rotate on its principal axis and move laterally. To recommend limits of slenderness, parametric analysis are performed using formulations based on bifurcacional instability, including eigenvalue problems with the finite element method. The results show that the safety limits for I beams and rectangular beams are different. For the analyzed cases and with reference to beam slenderness equation used by fib Model Code [13], the limit determined for rectangular beams would be 85 and for I beams 53, which could be taken as 50, as recommended by the code. Within the analyzed cases of I beams, only the fib Model Code [13] recommendation attend the slenderness limit for transitory phases.

Keywords: lateral instability of beams, precast concrete, lifting, transport, slenderness.

Resumo

Este artigo objetiva apresentar recomendações para a verificação da segurança de vigas pré-moldadas em fases transitórias, comparar resultados de análises paramétricas com recomendações de normas nacionais e internacionais e confrontar as formulações utilizadas para o cálculo da carga crítica de instabilidade lateral. Nas fases transitórias de transporte e içamento, as vigas pré-moldadas são suscetíveis à perda de estabilidade lateral, porque a vinculação estabelecida oferece pequena restrição ao elemento de girar em torno de seu eixo e deslocar-se lateralmente. Para recomendar limites de esbeltez são realizadas análises paramétricas utilizando formulações baseadas em instabilidade bifurcacional, incluindo problemas de autovalor com o método dos elementos finitos. Os resultados mostram que os limites de segurança para vigas I e retangular são diferentes. Para os casos analisados e tomando como referência a equação de esbeltez de viga utilizada pelo fib Model Code [13], o limite determinado para vigas retangulares seria de 85 e para vigas de seção I seria de 53, o que poderia ser tomado igual a 50, como recomendado pela norma. Dentre os casos analisados de vigas I, somente a recomendação do fib Model Code [13] atende o limite de esbeltez para fases transitórias.

Palavras-chave: instabilidade lateral de vigas, concreto pré-moldado, içamento, transporte, esbeltez.

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Received: 07 May 2015 • Accepted: 27 Aug 2015 • Available Online: 24 Nov 2015

1. Introduction

The increase in concrete strength, improvements in fabrication process and increase in capacity of transport and lifting equipments enable the production of long slender precast beams.
Precast concrete elements are subject to transport and lifting transitory phases. In these situations, the provisory supports do not restraint the element against twist rotation and lateral deflection, as occurs in permanent phase. In general, transitory situations are considered critical for lateral buckling of precast beams, as presented by Lima [1] and [2], El Debs [3] and Krahl [4].
Typically, beams have low lateral flexural stiffness and when undergo rotation about its longitudinal axis part of the self-weight acts laterally. Adding the prestressing effect, the stress state at specific points of the section (usually the top flange) can overcome the stress level that causes cracking in concrete.
Thus, it is important check lateral buckling in the design of precast beams. The verification can be performed by considering geometric and material nonlinearities in a complete nonlinear analysis or through safety limits, set by bifurcacional analysis (buckling load). The latter is the base for slenderness limits recommended by codes of concrete structures.
In this context, studies of beam stability which consider support flexibility are emphasized. According to Trahair [5], exact or analytical solutions of buckling load cannot be obtained for beams with flexible supports. Then, it is necessary to utilize numerical methods to obtain approximate solutions as in Lebelle [6], Trahair [5] and Lima [1]. Rayleigh-Ritz method, Galerkin, Runge-Kutta, Finite Difference Method and Finite Element Method are commonly used methods that can perform bifurcacional analysis.

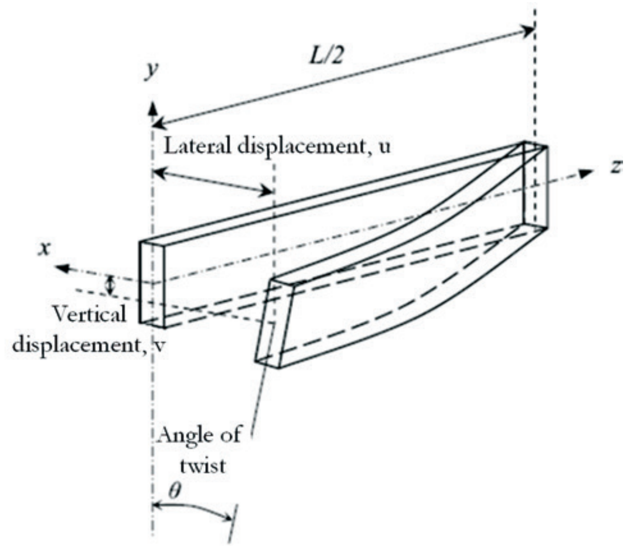
Table 1 – Code recommendations for lateral stability of concrete beams

Code	Slenderness limit	
	Permanent phase	Transitory phase
Eurocode 2 (11)	$\ell_{0f}\, h^{1/3} / b_f^{4/3} < 50$ $h / b_f < 2{,}5$	$\ell_{0f}\, h_{1/3} / b_f^{4/3} < 70$ $h / b_f < 3{,}5$
ABNT NBR 9062 (9)[2]	$\ell_{0f}\, h / b_f^2 < 500$ $\ell_{0f} / b_f < 50$	$h_m / a > 2$
ACI 318-02 (12)[1]	$\ell_{0f} / b_f < 50$	
fib Model Code (13)[1]	$\ell_{0f}\, h_{1/3} / b_f^{4/3} < 50$	
BS:8110-1 (14)[1]	$\ell_{0f}\, h / b_f^2 < 250$ $\ell_{0f} / b_f < 60$	
ABNT NBR 6118 (10)[1]	$h / b_f < 2{,}5$ $\ell_{0f} / b_f < 50$	

ℓ_{0f}: theoretical span or spacing between lateral restraints;
h: section height;
b_f: compressed flange width. For rectangular section change b_f for b_w;
h_m: distance between the center of gravity and the support point;
a: elastic beam lateral displacement, considering the self-weight acting laterally.

Notes[1] do not distinguish between transitory and permanent phases [2] As the current version do not contemplate the subject, it is being done reference to previous version.

Figure 1 – Lateral buckling of beam



Stratford et al. [7] used the finite element method to perform prebuckling and postbuckling analysis to study all beam load-displacement path. Initial geometric imperfections, inclined supports that are not included in bifurcational analysis were considered. Based on the results, Stratford et al. [7] recommend simplified formulations for calculating critical load. Furthermore, the effect of initial imperfections is considered by Southwell [8] hypothesis.
The current Brazilian codes NBR 9062: 2006 [9] and NBR 6118: 2014 [10] for precast concrete structures and concrete structures do not present recommendations for verification of lateral stability of beams in transitory phases. Slenderness limits recommendations of some international codes are presented in Table 1. Currently, only Eurocode 2 [11] specifies slenderness limit for transitory situation.
As noted, it is necessary to verify beam stability in transitory phases to prevent possible damage to the elements which can compromise its structural performance. Furthermore, accidents have occurred in these construction phases. Some cases are presented in Krahl [4].
This article provides slenderness limits for precast beams in transitory phases. These limits are compared to code recommendations of Table 1. Besides, the results of analytical and numerical models will be compared.

2. Background

From classical theory of flexural-torsional buckling of beams, such as presented in Timoshenko and Gere [15], it is known that a beam in bending about its major axis may buckle sideways if its compressed region is not laterally restricted. The phenomenon is characterized by lateral displacement and twisting rotation, as shown in Figure 1.

The nonlinear behavior of beams is influenced by several factors that can be considered in a simplified manner in bifurcacional analysis. They are: type of load, load application point in relation to shear center, support conditions and geometric imperfections. In transitory phases, the load is the self-weight. Therefore, the first and second factors are constant in the problem.
Parametric analyzes will be performed to establish slenderness limits for precast beams. It will be used buckling load solutions of Lebelle [6], Stratford et al. [7] and eigenvalues using finite element method.
The eigenvalue analysis will be performed using free access computer program LTBeam [16][1]. The program calculates the lateral buckling load for beams with several support and load conditions. The background shall be presented based on Trahair [5].
Lebelle [6] presents buckling load solution for beams with torsion flexible supports. Thus, the beam is partially restricted to rotate by twisting at the supports. The restriction corresponds to the spring stiffness k_θ, equation (1).

$$p_{crit} = k\frac{16}{\ell^3}\sqrt{\alpha_{crit}}\sqrt{EI_y GI_t} \quad (1)$$

in which,
k: constant which depends on the flange stiffness in the case of I section (β coefficient) and of the distance of the support position relative to beam shear center, equation (2);

$$k = \sqrt{1 + 2{,}47\beta + 0{,}52\delta^2} - 0{,}72\delta \quad (2)$$

where,
β: coefficient that accounts for lateral flange stiffness, equation (3);

$$\beta = \frac{EI_{y,flanges}}{GI_t}\frac{2z}{\ell^2} \quad (3)$$

$I_{y,flanges}$: weighted average of flange inertias;
z: distance between flange centroids, z = 0 for rectangular section;
δ: coefficient that accounts for support and load positions, equation (4);

$$\delta = \frac{2y_{rot}}{\ell}\sqrt{\frac{EI_y}{GI_t}} \quad (4)$$

y_{rot}: distance between the loading and support positions;
ℓ: total beam span;
E: concrete modulus of elasticity;
I_y: minor-axis moment of inertia;
G: concrete shear modulus;
I_t: torsion constant;
α_{crit}: coefficient which estimates the support deformability effect.
Equation (1) can be used for lifting and transportation. The differences are the distance between the longitudinal axis of rotation position relative to the center of gravity (y_{rot}) and α_{crit} coefficient particular to each phase.
For lifting, α_{crit} depends on the attachment point of cables, y_{rot} and lateral flexural and torsional stiffness. The constant α_{crit} can be obtained with Table 2. According to Lebelle [6], this variable is related to the function g(α) expressed by Equation (5). Thus, g(α) is calculated and α_{crit} is obtained for a given ratio ϱ=a/ℓ. The overhang

Table 2 – Values for the coefficient α_{crit} based on results of function g(α)

g(α)	ϱ=a/ℓ					
	0,5	0,6	0,7	0,8	0,9	1
0,02	2,55	2,4	0,133	0,018	0,0043	0,0014
0,04	10,1	9,23	0,523	0,0716	0,0171	0,0056
0,08	40	31,8	1,95	0,278	0,0672	0,0222
0,16	150,1	83,5	6,28	1	0,253	0,0854
0,32	485,3	148,4	14,8	2,93	0,83	0,297
0,6	1079,5	193,1	23,8	5,76	1,89	0,751
1,2	1833	222,4	31,7	8,9	3,34	1,48
2,5	2396,5	238,4	36,7	11,2	4,57	2,19
5	2678,2	245,9	39,3	12,5	5,29	2,63
10	2817,1	249,7	40,7	13,2	5,68	2,88
20	2885,3	251,6	41,4	13,5	5,89	3,02
40	2919,7	252,5	41,7	13,7	6	3,09
100	2944,4	253,5	42	13,9	6,07	3,16

[1] *[available in https://www.cticm.com/content/ltbeam-version-1011. Accessed on March 28, 2015.]*

length is a and the total beam span ℓ.

$$g(\alpha) = \frac{4y_{rot}}{\ell}\sqrt{\frac{EI_y}{GI_t}} \quad (5)$$

In which y_{rot} is the distance between the loading point and the longitudinal axis of rotation. In transport, α_{crit} depends on the stiffness of the vehicle suspension and the beam torsional stiffness. Lebelle [6] presents a function that estimates α_{crit} and hence the critical load for a given value k_θ, equation (6).

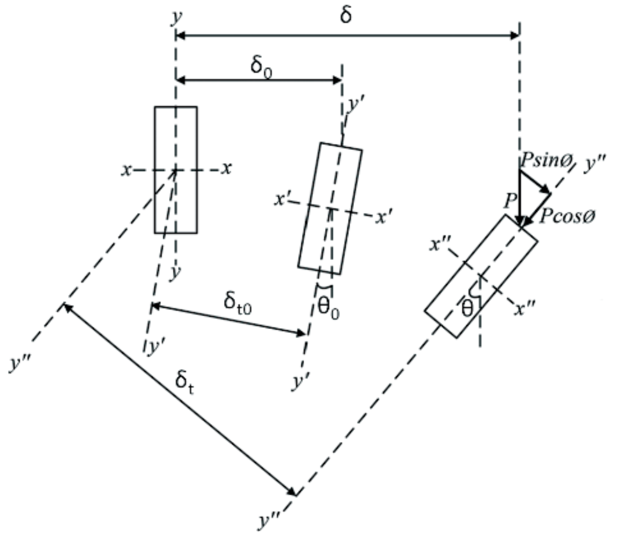
$$\frac{k_\theta \ell}{2GI_t} = f(\alpha) = \frac{\frac{8}{15}\alpha - \frac{356}{10395}\alpha^2}{1 - \frac{11}{30}\alpha + \frac{6617}{415800}\alpha^2} \quad (6)$$

For transport, Stratford et al. [7] recommend for buckling load solution the equation (7).

$$p_{crit} = 16{,}9\frac{\sqrt{EI_y GI_t}}{\ell^3} \quad (7)$$

To consider geometric imperfections, Stratford et al. [7] recommend to utilize the Southwell [8] hypothesis, equation (8).

Figure 2 – Lateral displacement and twist rotation of the section



$$\delta_t = \frac{\delta_0}{1 - \left(\frac{p_{\lim}}{p_{crit}}\right)^2} \quad (8)$$

In which δ_0 is the initial lateral displacement. Considering torsional stiffness, the relationship between the limit angle θ_{lim} and the corresponding displacement δ_t, according to Stratford et al. [7], the equation (9) may be adopted.

$$\frac{\theta_{\lim}}{\delta_t} = \frac{1{,}68}{0{,}36\ell\sqrt{\frac{GI_t}{EI_y}} + y_{rot}} \quad (9)$$

Thus, with equations (8) and (9) it has three unknowns θ_{lim}, δ_t and p_{lim}. To solution is usually adopted a value for θ_{lim}. Whereas the curves on highways have averaged 8% of superelevation or 4,57 degrees, θ_{lim} of 6 degrees or 0,105 rad is conservatively adopted. For two points lifting case by vertical cables, Stratford et al. [7] recommend the equation (10) which estimate the buckling load of a perfect.

$$p_{crit} = \frac{12EI_y y_{rot}}{\frac{\ell^4}{10} - a\ell^3 + 3a^2\ell^2 - 2a^3\ell - a^4} \quad (10)$$

where,
EI_y: elastic lateral flexural stiffness;
y_{rot}: distance between loading point and the longitudinal axis of rotation;
a : overhang length;
ℓ: total beam span.
The Southwell [8] hypothesis is utilized to consider geometric imperfections, equation (11).

$$\delta_t = \frac{\delta_0\left[1 - sen\left(\frac{\pi a}{\ell}\right)\right]}{1 - \frac{p_{\lim}}{p_{crit}}} \quad (11)$$

In which p_{lim} is the limit load that account for initial geometric imperfection.
Considering the relation between initial lateral displacement δ_o, final displacement δ_t and the limit twist rotation θ_{lim}, it is known that a load component $p_{lim}sen\theta_{lim}$ will act laterally (Figure 2) causing the displacement $(\delta_t - \delta_o)$ expressed by equation (12).

Table 3 – Geometric properties of analyzed precast elements

Rectangular beam			I-section beam		
Width (cm)	Height (cm)	Span (m)	Flange width (cm)	Height (cm)	Span (m)
15 a 50	150	30	40 a 80	150	30
20	150	20 a 30	80	150	30 - 40

$$\delta_t - \delta_o = \frac{g_{sw} \operatorname{sen} \theta_{lim}}{384EI_y}\left(5\ell^2 - 20a\ell - 4a^2\right)\left(\frac{6}{5}a^5 - \ell\right)^2 \quad (12)$$

where,
g_{sw}: self-weight;
θ_{lim}: limit twist angle.
Substituting the equation (12) in equation (11), it remains the unknowns p_{lim} and θ_{lim} in the resulted expression. To obtain p_{lim} is utilized the recommendation by Mast [17] to limit twist angle. Mast [17] performed experiments with a real scale beam PCI BT-72 and stablished a limit angle of 23 degrees for lifting.
The computational program LTBeam [16] is utilized in parametric analysis. The buckling load is obtained by calculating the smaller eigenvalue for a beam discretized in 100 finite elements. Trahair [5] presents a procedure to implement the eigen-problem with finite element method (FEM).
To obtain the eigenvalues λ_{cr} and eigenvectors $\{\delta\}$ using FEM is necessary to obtain first the stability matrix [G] for each element, besides the stiffness matrix [K]. The stability matrix is obtained from energy portion correspondent to the work variation of external loads. The eigen-problem can be represented by equation (13).

$$([K] - \lambda_{cr}[G])\{\delta\} = 0 \quad (13)$$

The load path of the model is set on stability matrix [G]. To solve equation (13) the matrix [G] must be inverted by utilizing a numerical method to obtain λ_{cr}. Other possibility is invert the stiffness matrix and get $1/\lambda_{cr}$. To obtain critical values just multiply λ_{cr} by the load path adopted.
The program enables to insert discrete flexible supports. Springs can be insert to partially restraint lateral displacement, rotation by lateral flexure, twist rotation and warping. For transitory phases, the torsional stiffness of the supports is the major parameter.

3. Results and discussion

The results of parametric analysis are presented for beams in transitory phases by utilizing the formulation of bifurcational analysis. The study of rectangular and I-section beams are performed separately. The smaller slenderness ratio obtained from buckling analysis will be adopted as safety limit.

The graphs present results of lifting phase along with transport phase. Geometric relations to obtain slenderness ratios were determined in accordance with Eurocode 2 [11]. The code limits for transitory phases are expressed in equation (14).

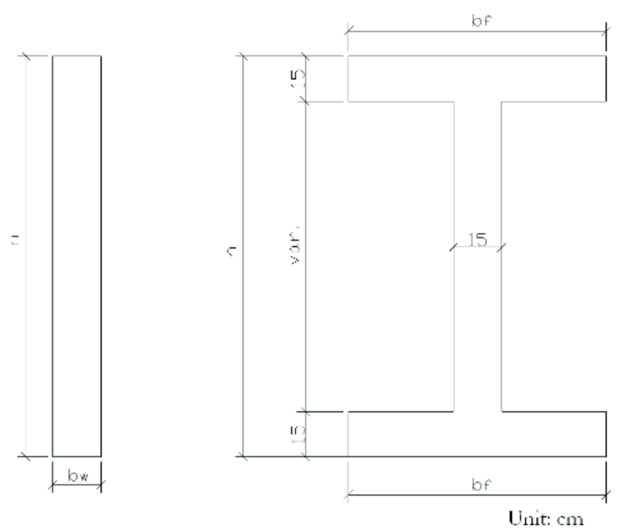
$$\frac{h}{b_f} \leq 3{,}5 \text{ and } \frac{\ell_{0f} h^{1/3}}{b_f^{4/3}} \leq 70 \quad (14)$$

To obtain slenderness limits the safety criterion p_{crit} / pp > 4 is considered which is adopted due to the difficulty in predicting how the transitory phases will be performed. Krahl [4] presented a smaller limit by utilizing the formulation of Mast [17]. However, the oldest value will be utilized to obtain slenderness limits.
Increase in compressive strength of concrete has a positive effect on lateral buckling of precast beams. Thus, in a conservative way it is considered a compressive strength of 30MPa in all analysis which is slightly smaller than the strength required for permanent phase.
Geometric imperfections and deviations in positioning the beam supports as lifting cables or truck suspension system can significantly reduce the safety against buckling. The influence of these factors is evaluated in Krahl [4].
The slenderness ratios presented in equation (14) are not affected by geometric imperfection variation and concrete modulus of elasticity as well. To consider them is necessary an expression that utilize buckling load. However, the codes of concrete structures recommend limits as equation (14). Thus, in a conservative way, the geometric slenderness limits are obtained for the imperfection recommended by Eurocode 2 [11] that is ℓ/300 as initial lateral displacement. In all analysis an overhang of 2,5m is considered.
Some of the formulations enable the use of flexible supports. According to Mast [17], it is recommended as torsion spring stiffness for beam in transport a value between 360 to 680kN.m/rad per dual-tire axel. In this article, it is adopted four dual-tire axel plus one simple-tire axel for the tractor resulting in 1530 kN.m/rad (360 kN.m/rad per dual-tire). The same is adopted for the trailer. The considered torsion spring for lifting is 1200 kN.m/rad.
Table 3 shows the geometric relations utilized in parametric analysis. The flange and web thickness of I-section beams are fixed in 15 cm. Figure 3 shows the generic representation of the section.

3.1 Width variation of rectangular beam

Table 4 presents the buckling load results for rectangular beam

Figure 3 – Generic representation of the utilized sections for parametric analysis



with widths of 15, 20, 30, 40 e 50 cm. The section height is 150 cm and beam span 30 m, with ℓ / h = 20.

In Figure 4, the dash-dotted line represents the self-weight and the dashed line represents four times this value. The latter being the safety criterion. The graphs for transport and lifting are presented separately in Figure 4 in which the buckling load is related to geometric slenderness from equation (14).

According to Figure 4, as the beam width increase the buckling load increase, tending to exceed the safety limit. In lifting phase with slenderness $\ell_{0f}\, h^{1/3} / b_w^{4/3} \leq 85$ and $h / b_w \leq 3$ the safety is verified. The first limit is bigger than Eurocode 2 [11] and *fib* Model Code [13] recommendations. The second is smaller than Eurocode 2 [11] limit, so the code limit is unsafe.

To $\ell_{0f}\, h / b_w^2$ slenderness, it is obtained the limit of 180 that corresponds to a safe buckling load that is smaller than BS:8110-1 [14] and ABNT NBR 9062:2006 [9] recommendations, as shown in Table 1. Thus, the Britain and Brazilian codes recommend unsafe limits for rectangular beams in transitory phases.

The slenderness ℓ_{0f} / b_w results 60 which coincides with the BS:8110-1 [14] recommendation. ACI 318-02 [12] and ABNT NBR9062:2006 [9] recommendations are conservative therefore safe.

In transport, the formulation of Lebelle [6] does not achieve the determined limit in lifting, as shown in Figure 4. However, the other formulations checked the safety in transport for the same limit in lifting.

In the graphs of Figure 4, it is verified that the formulation of Stratford et al. [7] tends to present high buckling load as the slenderness is decreased. In the lifting case, this formulation presents high sensibility to geometric imperfections, wherein for slenderness $\ell_{0f}\, h^{1/3} / b_w^{4/3} = 86{,}53$ the reduction in buckling load is 48%.

In lifting, the formulation of Stratford et al. [7] that considers geometric imperfections had results that agree with those obtained by finite element method (LTBeam). For the smaller slenderness considered the difference is 1,12% and 6,75% in the first two cases. However, as the slenderness ratio increases the difference is increased to 64%.

In the case of transport phase, the formulation of Lebelle [6] had results that agree with those obtained by the computational program LTBeam [16] for high slenderness, but as the slenderness decreases the difference increases. For the range of slenderness ratio studied the extreme differences are 7,7% and 28,4%. For lifting, the formulation of Lebelle [6] presents large variation in buckling load as the slenderness decreases.

3.2 Span variation of rectangular beam

For evaluating the influence of span variation in buckling load the present spans are adopted 20, 25 and 30 m. Table 5 and Figure 5 present the results. The rectangular section is 20 cm wide and 150 cm high, thus the relation h / b_w has a constant value of 7,5. According to the limit obtained in item 3.1, this value do not verify the safety limit $h / b_w \leq 3$. The ℓ / h relations are 13, 17 and 20.

According to Figure 5, in the case of beams with slenderness $\ell_{0f}\, h^{1/3} / b_w^{4/3} \leq 200$ all formulations present buckling load results that verify the safety for lifting and transport. In this point, it is clear the limitation of the geometric slenderness limits recommended by codes.

On the recommendation of item 3.1 ($\ell_{0f}\, h^{1/3} / b_w^{4/3} \leq 85$ e $h / b_w \leq 3$),

Table 4 – Buckling load of rectangular beams for width variation

h / b_w	$\ell_{0f}\, h^{1/3} / b_w^{4/3}$	Buckling load (kN/m)							
		Lebelle		Stratford et al.		Stratford et al.[1]		LTBeam	
		I[2]	T	I	T	I	T	I	T
10,00	430,89	4,07	10,62	3,82	10,24	3,73	9,87	10,45	11,51
7,50	293,62	9,55	18,43	9,14	24,28	8,61	23,40	17,61	19,64
5,00	171,00	31,5	35,37	31,13	81,96	25,78	79,00	34,14	38,38
3,75	116,52	72,75	53,63	74,13	194,30	49,70	187,20	53,32	60,09
3,00	86,53	138,30	60,48	145,17	379,45	74,06	365,63	74,9	84,50

Notes [1] Formulation which considers geometric imperfection [2] The letters I and T represent lifting and transport, respectively.

Figure 4 – Buckling load of rectangular beams for width variation

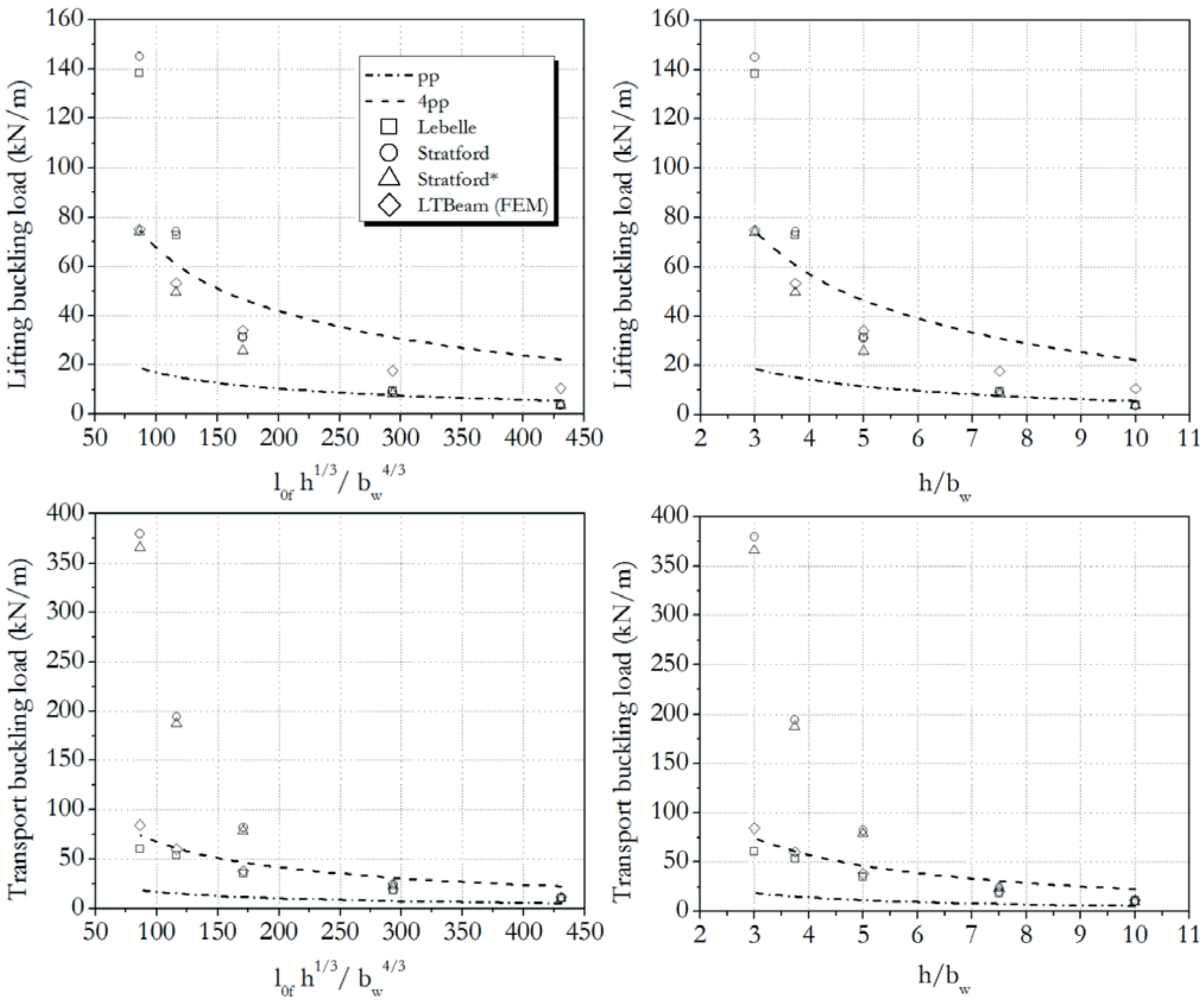
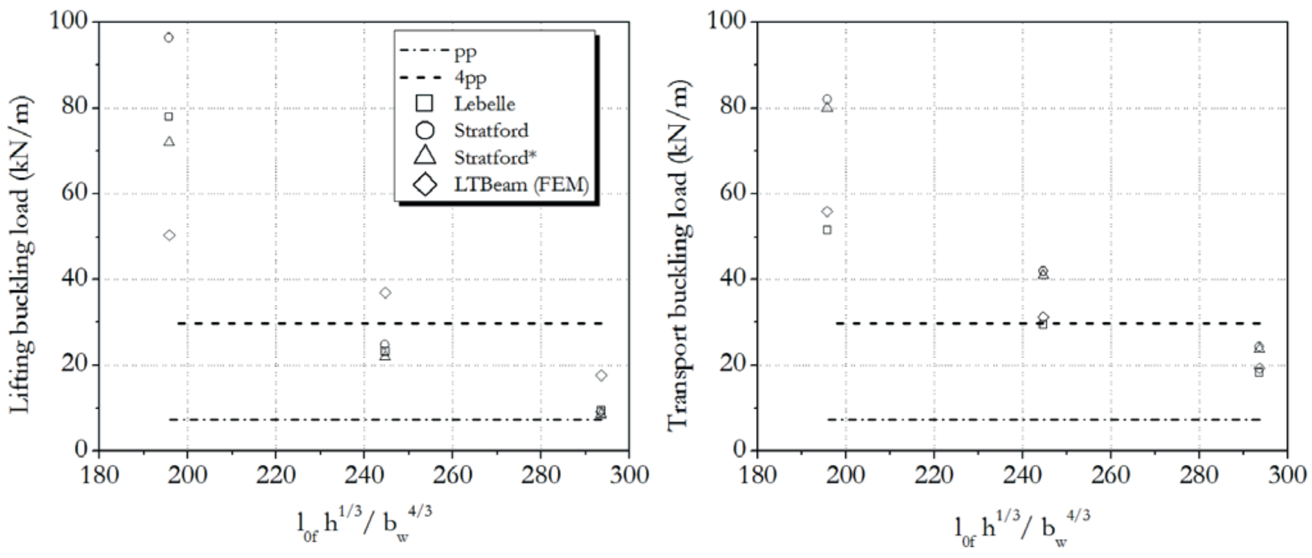


Table 5 – Buckling load of rectangular beams for span variation

Vão (m)	ℓ_{0f} h[1/3] / b_w[4/3]	Buckling load (kN/m)							
		Lebelle		Stratford et al.		Stratford et al.[1]		LTBeam	
		I[2]	T	I	T	I	T	I	T
20	293,62	9,55	18,43	9,14	24,28	8,61	23,79	17,61	19,34
25	244,68	23,20	29,49	24,83	41,96	22,03	41,03	37,00	31,20
30	195,74	77,80	51,50	96,30	81,96	71,94	79,93	50,40	55,78

Notes [1] Formulation which considers geometric imperfection [2] The letters I and T represent lifting and transport, respectively.

Figure 5 – Buckling load of rectangular beams for span variation



the beam with relation $h / b_w = 7{,}5$ and span of 20 m do not verify safety against buckling $\ell_{0f} h^{1/3} / b_w^{4/3} = 293{,}62$. However, the result obtained in this item shows that the criterion $p_{crit} / pp > 4$ is verified, as presented in Figure 5.
For lifting, the formulations of Lebelle [6] and Stratford et al. [7] present closed results with the biggest difference of 7,5%. Comparing the results of the formulation of Stratford et al. [7] with those of the program LTBeam [16], the biggest difference is 51,1%.
In transport, this occurs for the results of Lebelle [6] and LTBeam [16], with the maximum difference of 7,7% and minimum of 4,7%. The formulation of Stratford et al. [7] shows small sensibility to geometric imperfections in transport, because the biggest difference between buckling loads considering and not geometric imperfection is 2,5%.
In Table 6, the results of this item are compared to those of item 3.1. For this item the beam with 20 m span is considered. From this comparison, it can be recommended the slenderness ratio obtained in item 3.1.
The slenderness $\ell_{0f} h / b_w^2$ limit obtained for beams with rectangular sections is 180 whereas in ABNT NBR 9062:2006 [9] is 500, thus the code recommendation is unsafe.
To exemplify the obtained limit, consider for example a beam 40 cm wide, 150 cm high and a span of 20 m. Its slenderness is $\ell_{0f} h^{1/3} / b_w^{4/3} = 7{,}7$. The *fib* Model Code [13] recommends a value of 50 as slenderness limit, thus the beam in question do not verify this criterion. However, this precast element verifies the safety criterion obtained in the item 3.1 which is 85. It should be pointed out that the limits obtained in this article are based on the safety criterion $p_{crit} / pp > 4$.

Table 6 – Comparison of slenderness limit results for rectangular beams

Slenderness ratio	Item 3.1	Item 3.2
$\ell_{0f} h_{1/3} / b_w^{4/3}$	85	195
$\ell_{0f} h / b_w^2$	180	750
ℓ_{0f} / b_w	60	100
h / b_w	3	7,5

3.3 Flange width variation of I-section beam

Table 7 and Figure 6 present the buckling load results for simultaneous variation of top and bottom flanges width of I-section beam in transitory phases. The admitted widths are 40, 60 and 80 cm and the height, thickness of web and flanges and the beam span are fixed in 150 cm, 15 cm and 30 m, respectively, with the relation $\ell / h = 20$.
According to Figure 6, the results show that for slenderness ratios $\ell_{0f} h^{1/3} / b_f^{4/3} \leq 70$ and $h / b_f \leq 2{,}5$ all formulation present buckling loads that verify the safety criterion for transport and lifting. Comparison with codes will be done in item 3.4.
The formulation of Stratford et al. [7] presents similar behavior as described in item 3.1. In lifting, the author's formulation agree well with the results of LTBeam [16], being the maximum difference of 31,5%. In transport, the results of the formulation of Lebelle [6] are similar to those of LTBeam [16], being the maximum difference of 10,1%.

3.4 Span variation of I-section beam

The importance of span variation in the stability of I-section beams is verified in this item. Table 8 and Figure 7 present the results.

Table 7 – Buckling load of I-section beams for flange width variation

h / b_f	$\ell_{0f}\, h^{1/3} / b_f^{4/3}$	Buckling load (kN/m)							
		Lebelle		Stratford et al.		Stratford et al.[1]		LTBeam	
		I[2]	T	I	T	I	T	I	T
3,75	116,52	15,56	23,97	17,89	27,18	16,3	26,25	23,80	25,96
2,50	67,86	51,2	41,4	53,3	50,61	41,5	48,04	41,24	45,65
1,88	46,24	105,97	62,29	122,5	82,19	78,6	76,4	63,24	69,29

Notes [1] Formulation which considers geometric imperfection [2] The letters I and T represent lifting and transport, respectively.

Figure 6 – Buckling load of I-section beams for flange width variation

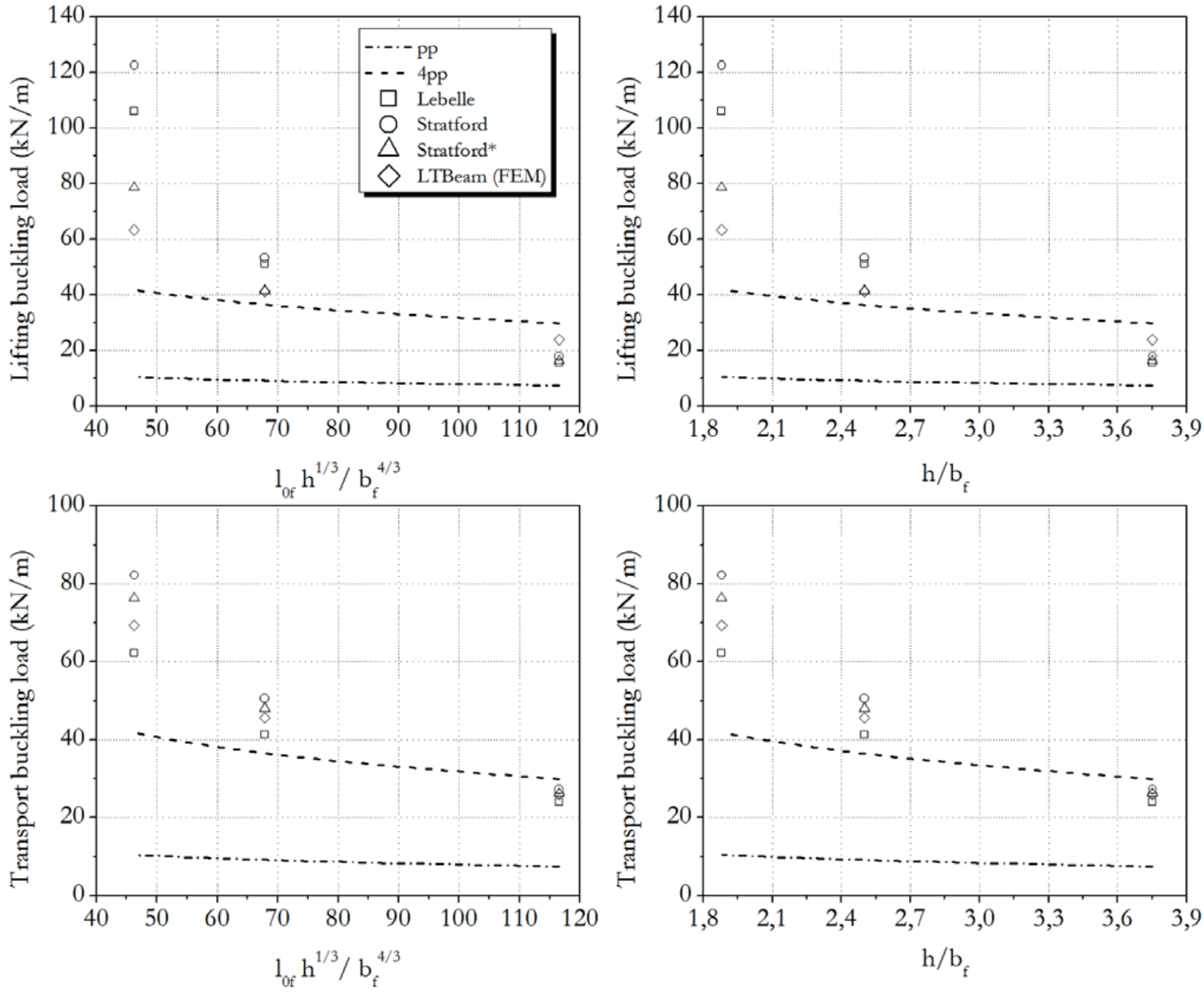


Table 8 – Buckling load of I-section beam for span variation

Vão (m)	$\ell_{0f}\ h^{1/3}\ /\ b_f^{4/3}$	Buckling load (kN/m)							
		Lebelle		Stratford et al.		Stratford et al.[1]		LTBeam	
		I[2]	T	I	T	I	T	I	T
30	46,24	105,97	62,3	121,84	82,2	78,14	76,36	63,24	69,29
35	53,95	45,40	42,10	55,40	51,75	42,20	48,25	42,25	46,12
40	61,66	23,73	29,86	28,93	34,67	24,27	32,4	29,7	32,31

Notes [1] Formulation which considers geometric imperfection [2] The letters I and T represent lifting and transport, respectively.

The admitted spans are 30, 35 and 40 m, being the relation ℓ / h of 20, 23 and 26, respectively. The flange width, flange and web thickness and section height are fixed in 80 cm, 15 cm e 150 cm, respectively. Thus, the relation h / b_f is constant with value of 1,88. In Figure 7, safety is checked for slenderness of $\ell_{0f}\ h^{1/3}\ /\ b_f^{4/3} \leq 53$ in lifting and transport. This result is close to the limit recommended by *fib* Model Code [13]. The slenderness $h / b_f \leq 1{,}8$ can only be recommended if simultaneously verify the slenderness $\ell_{0f}\ h^{1/3}\ /\ b_f^{4/3}$.

Imposing these safe limits, some cases of beams that verify stability in transitory phases by buckling load shall not be accepted. For example, the beam with slenderness $\ell_{0f}\ h^{1/3} / b_f^{4/3} = 67{,}8$ presented in item 3.3 which verifies the criterion p_{crit} / pp > 4.

The results patterns are equal to those presented in the preceding items. In lifting, the formulation of Stratford et al. [7] presented results whose maximum difference is 18,3% to formulation of LTBeam [16]. In transport, the maximum difference between the formulations of Lebelle [6] and LTBeam [16] is 7,6%.

In Table 9, the slenderness limits obtained in item 3.3 and item 3.4 are compared with code recommendations.

In accordance with Table 9, the only code that presents safe slenderness limit, when compared with the results obtained, is the *fib* Model Code [13]. Therefore, for I-section beams the recommended limits are those obtained in item 3.4. The slenderness $\ell_{0f}\ h^{1/3}\ /\ b_f^{4/3}$ can be assumed as the value of *fib* Model Code [13].

As did in item 3.2 for exemplification, it is considered now an I-section beam with flange widths, web and flange thickness, height and span of 50 cm, 15 cm, 150 cm and 20 m, respectively. Its slenderness is $\ell_{0f}\ h^{1/3}\ /\ b_f^{4/3} = 57{,}7$. This slenderness does not verify the verification of *fib* Model Code [13] nor the limit obtained in this article.

4. Conclusions

Based on results of parametric analysis, it can be concluded:

a) From parametric analysis, for lifting phase one states that the

Figure 7 – Buckling load of I-section beam for span variation

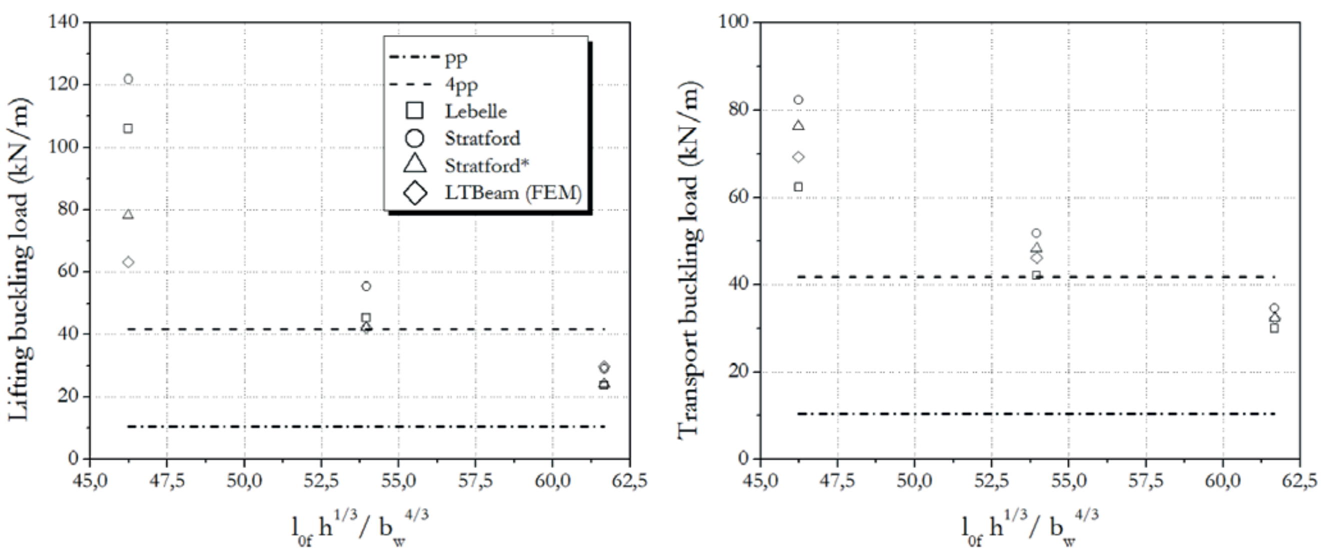


Table 9 – Comparison of slenderness limit results for I-section beams

Esbeltez	fib Model Code (13)	NBR9062 (9)	ACI 318-02 (12)	BS:8110-1 (14)	Eurocode 2 (11)	Item 3.3	Item 3.4
$\ell_{0f}\, h^{1/3} / b_f^{4/3}$	50	–	–	–	70	70	53
$\ell_{0f}\, h / b_f^2$	–	500	–	250	–	125	82
ℓ_{0f} / b_f	–	50	50	60	–	50	43
h / b_f	–	–	–	–	3,5	2,5	1,88

formulation of Stratford et al. [7] presented results that agree with those obtained by the computational program LTBeam [16] which was taken as reference. For the slenderness range considered for rectangular and I-section beams, the minimum difference between buckling load curves was 1,1% and maximum 64%.

b) The buckling load results for transport from formulation of Lebelle [6] have approached the results of LTBeam [16]. Between the considered slenderness for rectangular and I-section beams, the minimum difference obtained in the buckling load curves was 4,7% and maximum of 28,4%.

c) The slenderness limit determined for rectangular beams is different from the limit encountered for I-section beams. It is important to note that none code does this distinction. Thereby, the rectangular beams usually result excessively robust, wherein the buckling load calculation shows that the elements could be more slender.

d) Taking *fib* Model Code [13] as reference, the limit of the slenderness $\ell_{0f}\, h^{1/3} / b_f^{4/3}$ for rectangular beams would be 85, for the analyzed cases.

e) For I-section beams, the limit determined for $\ell_{0f}\, h^{1/3} / b_f^{4/3}$ is 53 that could be taken equal 50, as recommended by *fib* Model Code [13].

f) In the studied cases for I-section beams, only the recommendation of *fib* Model Code [13] meets the slenderness limits for the analyzed cases.

Note the conclusions were obtained for a study that involved the following situations: f_{ck} of 30 MPa, geometric imperfection of $\ell/300$ (when considered), overhangs of 2,5 m, vertical lifting cables, safety criterion $p_{crit} / pp > 4$. The torsional spring stiffness adopted in lifting is 1200 kN.m/rad and transport 1530 kN.m/rad.

The geometric relations utilized were presented in Table 3. Remembering that the flange and web thickness are fixed in 15 cm.

When the code recommendations are not met, it can be appealed to a more rigorous analysis, for example, buckling load calculation. This type of analysis considers the effect of geometric nonlinearities that is characteristic in slender beams. The safety is then verified by comparing the buckling load to the beam self-weight. The relation between the two quantities must meet always to a safety criterion, usually it is adopted a value of 4.

5. Acknowledgments

The authors would like to thank to the Department of Structural Engineering of University of São Paulo and to the Brazilian research funding agency CAPES for the master's degree scholarship.

6. References

[1] LIMA, M. C. V. Instabilidade lateral das vigas pré-moldadas em serviço e durante a fase transitória, São Carlos, 1995, Dissertação (Mestrado), Escola de Engenharia de São Carlos – USP, 146p.

[2] LIMA, M. C. V. Contribuição ao estudo da instabilidade lateral de vigas pré-moldadas, São Carlos, 2002, Tese (Doutorado), Escola de Engenharia de São Carlos – USP, 179 p.

[3] EL DEBS, M. K. Concreto pré-moldado: fundamentos e aplicações. São Carlos, EESC-USP - Projeto Reenge, 2000.

[4] KRAHL, P. A. Instabilidade lateral de vigas pré-moldadas em situações transitórias, São Carlos, 2014, Dissertação (Mestrado), Escola de Engenharia de São Carlos - USP, 209 p.

[5] TRAHAIR, N. S. Flexural-Torsional Buckling of Structures. London: E. & F. N. Spon, 1993, 360 p.

[6] LEBELLE, P. Stabilité élastique des poutres en béton précontraint a l'égard de déversement latéral. Ann. Batiment et des Travaux Publics, v. 141, p. 780–830, 1959.

[7] STRATFORD, T. J.; BURGOYNE, C. J.; TAYLOR, H. P. J. Stability design of long precast concrete beams. Proceedings of the Institution of Civil Engineers – Structures and Bridges, v.134, p.159-168, 1999.

[8] SOUTHWELL, R. V. On the analysis of experimental observations in problems of elastic stability. Proceedings of the Royal Society, v. 135, p. 601–616, 1932.

[9] ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. Projeto e execução de estruturas de concreto pré-moldado. - NBR 9062, Rio de Janeiro, 2001.

[10] ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. Projeto de estruturas de concreto - Procedimento. NBR 6118, Rio de Janeiro, 2014.

[11] EUROPEAN COMMITTEE OF STANDARDIZATION. Design of Concrete Structures - Part 1-1: General rules and rules for buildings. - EUROCODE 2, Brussels, 2004.

[12] ACI COMMITTEE 318, "Building Code Requirements for Structural Concrete (ACI 318-02) and Commentary (318R-02)," American Concrete Institute, Farmington Hills, Mich., 2002, 443 pp.

[13] FÉDÉRATION INTERNATIONALE DU BÉTON, *fib* Model Code 2010 - final draft, Vol. 2, Bulletin 66, Lausanne, Switzerland, 2012.

[14] BRITISH STANDARDS INSTITUTION, BS 8110. Code of practice for structural use of concrete. London; 1997.

[15] TIMOSHENKO, S.; GERE, J. Theory of Elastic Stability. McGraw Hill, New York, 1988, 541p.

[16] LTBeam 1.0.10 [Computer software]. Saint-Aubin, France, Centre Technique Industriel Construction Metallique (CTICM).
[17] MAST, R. F. Lateral stability of long prestressed concrete beams, part 1. PCI Journal, v. 34, p. 34 53, 1989.