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Study on reliability of punching shear of flat slabs without shear reinforcement according to NBR6118

Estudo da confiabilidade da punção em lajes lisas sem armadura de cisalhamento de acordo com a NBR6118

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Abstract

The frequent use of flat slabs in building constructions highlights the importance of improving the study of slab-column joints, particularly regarding the verification of the ultimate limit state of punching shear, given the complexity of this phenomenon. This article applies concepts of the Theory of Reliability in order to evaluate the safety of the formulation established by NBR6118: 2014 standard to check punching shear in flat slabs with centered columns. Twelve probabilistic model analysis for C and C' equations were developed, considering the influence that the variation of the slab thickness, the f_{ck} and the shear force eccentricity have on reliability index β and failure rate P_f. Results indicated that formulation of C boundary is reasonably safe, although C' boundary revealed β index below expectations.

Keywords: punching shear, flat slab, reliability, reinforced concrete.

Resumo

O uso frequente de lajes lisas na construção de edifícios destaca a importância do aprimoramento no estudo das ligações laje-pilar, principalmente quanto a verificação do Estado Limite Último de Punção, visto a complexidade envolvida neste fenômeno. Este artigo aplica conceitos da Teoria da Confiabilidade com o objetivo de avaliar a segurança existente na formulação proposta pela norma NBR6118: 2014 para verificação de punção em lajes lisas com pilares centrados. Foram elaborados 12 modelos de análise probabilística das equações dos contornos C e C', considerando a influência que a variação da espessura da laje, do f_{ck} e da excentricidade da força cortante exerce sobre o índice de confiabilidade β e a probabilidade de falha P_f. Os resultados indicaram que a formulação do contorno C possui segurança razoável, porém o contorno C' apresentou índice β abaixo do esperado.

Palavras-chave: punção, lajes lisas, confiabilidade, concreto armado.

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1. Introduction

The structural solution of flat slabs for buildings is used as a viable alternative to the conventional system (formed by columns, beams and slabs) due to greater simplicity and agility in the execution of forms, reduced labor, reduced interference with installations, standardization and rationalization of shoring, reducing the floor height (enabling a better use of the land occupation) besides an easier launch and densification of concrete (reducing the appearance of the concrete failure).
In spite of the benefits mentioned, this kind of structural system requires a more sophisticated analysis, especially in the design of its Ultimate Limit State because of the phenomenon known as punching shear, which arises from the concentration of forces applied to small areas of the slab, generally in slab-column joints, causing its perforation.
Punching failure is associated to the main tensile stress due to its shear and it is considered a brittle failure because it occurs in a sudden (without previous notice), which causes a rupture of the connection between slab and column and may also cause progressive collapse slabs on lower floors. The standard NBR 6118: 2014 presents the formulation to design the punching shear of flat slabs with and without punching shear reinforcement.
It is possible to evaluate the calculation model proposed by NBR 6118: 2014 through the Theory of Reliability and with the assistance of Monte Carlo simulation, a numerical analysis method widely used nowadays due to its robustness, simplicity and flexibility.

1.1 Theory of reliability

Structural safety is associated with the following factors:
- Reliability which is characterized by a low probability of failure in the ultimate and service ability limit states;
- Adaptability and warning structural (ductility of structural elements);
- Fidelity associated with the impossibility of false warning collapses;
- Durability, related to the ability of maintaining the previous three qualities throughout life, since the planned maintenance had been done.

The theory of reliability considers in the uncertainty associated with each variable involved in the safety and performance of the structure. Thereby, it is possible to evaluate its probability of fail by global collapse of a located structural element (Ultimate Limit State), or by cracking and excessive deformation (Serviceability State Limit). To evaluate the probability of failure, it is defined the failure function $G = R - S$, associated with random quantities that influence the resistant capacity R and internal forces level S:

$$G = R\left(Y_1,\ldots,Y_n;C_Y\right) - S\left(X_1,\ldots,X_m;C_x\right) \quad (1)$$

Where $Y_j\left(j=1,2,\ldots,n\right)$ and $X_i\left(i=1,2,\ldots,m\right)$ are the random variables that involve the internal forces and resistance, respectively, and C_Y and C_X symbolize sets of constants and deterministic functions relating to the random variables R and S. It is important to mention that if there are variables, even X_i and Y_j, significantly correlated, that is, in situations where it is not possible to ignore the interdependence among them, the complexity of the problem increases. In this work, the correlation among the variables is not significant, they will not be considered. This has been made in other similar cases cited in the bibliography.

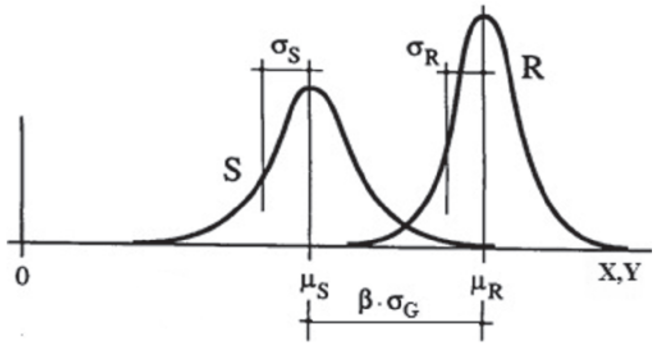

Figure 1
Representation of probability of failure and index β

The Reliability Theory purpose ensures that the event $(G = R - S > 0)$ occurs with high probability during the period of use of the structure, through analysis of probability of failure $P_f = P\left(G \le 0\right)$, associated with the event $\left(G \le 0\right)$.
Determined the random variables R and S and their respective statistical parameters, average value (μ_R e μ_S) and variance (σ_R^2 e σ_S^2), it is possible to obtain the average value μ_G and standard deviation σ_G of the variable G using the equations (2) and (3) described below:

$$\mu_G = \mu_R - \mu_S \quad (2)$$

$$\sigma_G = \sqrt{\sigma_R^2 + \sigma_S^2} \quad (3)$$

The function G is a function of random variables defined by the difference between other two R and S and generally its probability distribution is unknown. This is our case, although there are other alternatives we try to solve it with the Monte Carlo method. The case where R and S have normal distribution and G results also normal, it is interesting, because it defines the Reliability Index described hereafter.
In cases where there are random variables represented by probability distributions different from normal distribution, it is possible to obtain acceptable approximations through an equivalent normal distribution, that is, a normal distribution with the same probability density values and accumulated probability as the original distribution in design point.

1.1.1 Reliability index (β)

The reliability index β is a parameter associated with the

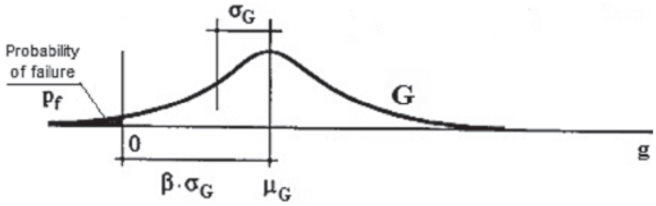

Figure 2
Reliability Index β (8)

probability of failure that is considered a reference in the safety assessment of structures. Given the independent random variables R and S with normal distribution, the reliability index β is given by:

$$\beta = \frac{\mu_G}{\sigma_G} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \quad (4)$$

And the probability of failure can be described as:

$$P_f = P(G \le 0) = \phi(-\beta) \quad (5)$$

A possible way to estimate an acceptable failure probability for a given structure consists in calibrating the reliability index β according to the relationship between the relative safety cost and its failure consequences, as seen in Table 1, recommended by the Eurocode [17].
However, this table does not separate structural elements of different responsibility as the ACI calibration work does (Nowak [13]). NBR 6118 has been adopted calibration values required for the ACI 318 calibration, i.e., 3,8 for columns, 3,3 for beams and 2,5 for bending in slabs. In the case of punching shear of flat slabs without shear reinforcement a value greater than 2,5 is desirable, if possible around 3,0.

1.2 Monte Carlo method

According to Stucchi and Moraes [7], Monte Carlo method consists in an approximate numerical simulation to solve equation of any limit state, in the case of this work the failure equation G:

$$G = G(S,R) = G(X_1, X_2, \ldots, X_i, \ldots, X_m; Y_1, Y_2, \ldots, Y_j, \ldots, Y_n; C_x, C_y) \quad (6)$$

Where X_i, Y_j, C_x and C_y are random variables and deterministic functions described in item 1.1 of this work.
Then N simulations are generated ($l = 1, 2, \ldots, N$) of the limit equation G, with the use of random number generators $0 \le a_{i,l} \le 1$ and $0 \le b_{j,l} \le 1$, whose probability densities remain constant in the range. The result of $a_{i,l}$ and $b_{j,l}$ is associated with the cumulative probability of each random variable $X_{i,l}$ and $Y_{j,l}$, respectively:

$$X_{i,l} = F_{(X_i)}^{-1}(a_{i,l})$$

$$Y_{j,l} = F_{(Y_j)}^{-1}(b_{j,l})$$

Where: $F_{()}^{-1}(\;)$ is the inverse function of cumulative distribution of each random variable involved in the problem. Each simulation provides individual results of random variables $X_{i,l}$ and $Y_{j,l}$, that applied to the limit equation G generate values g_l:

$$g_l = G(X_{1l}, X_{2l}, \ldots, X_{il}, \ldots, X_{ml}; Y_{1l}, Y_{2l}, \ldots, Y_{jl}, \ldots, Y_{nl}; C_x, C_y) \quad (7)$$

Once finished the simulations, the N_f failure events associated with condition $g(x) < 0$ are accounted, and the average failure probability P_f can be estimated by:

$$\overline{P_f} = \frac{N_f}{N} \cong P_f \quad (8)$$

It should be noted that the greater the number of simulations N, the more $\overline{P_f}$ converges to the probability of failure value P_f.
Based on the g_l results obtained, it is possible to generate the graph of cumulative probability density function F_g, as illustrated in Figure 3.
It is worth emphasizing the importance of reliability index β as a parameter used to evaluate the existing safety of the normative formulations. Therefore, despite the possible lack of knowledge on the probability distribution of the limit equation G, it is acceptable to determine the value of β_{eq} equivalent to equation (9):

$$\beta eq \approx -\phi^{-1}(P_f) \quad (9)$$

Where ϕ^{-1} is the inverse cumulative normal distribution function.

Table 1
Reference values to the parameter β. (17)

Relative cost of safety measure	Ultimate limit state			
	Expected consequences given a failure			
	Low	Some	Moderate	High
High	β = 0	β = 1,5	β =2,3	β = 3,1
Moderate	β = 1,3	β = 2,3	β = 3,1	β = 3,8
Low	β =2,3	β = 3,1	β = 3,8	β = 4,3

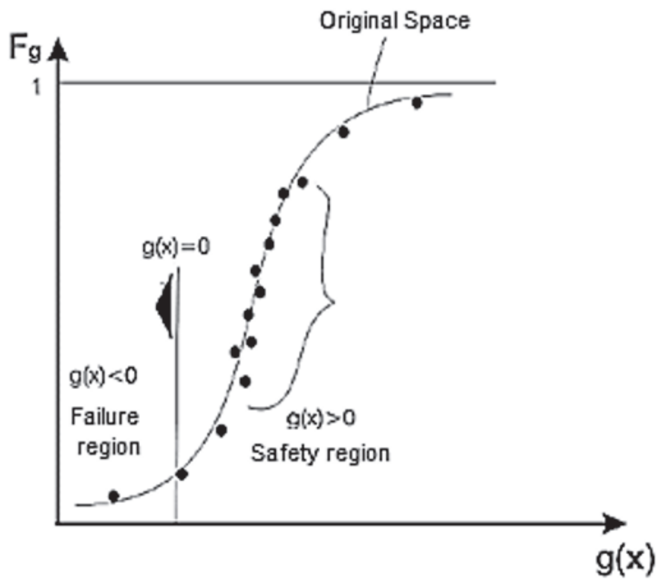

Figure 3
Simulation of Monte Carlo method (8)

2. Methodology and formulation

2.1 Formulation

The calculation model proposed by NBR6118: 2014 for punching shear design of internal columns corresponds to the of check two critical surfaces, C and C', according to item 19.5.1 of the respective standard. Figure 4 presents the critical perimeter for different geometries for internal columns.

After validating the prescribed conditions, in principle, shear reinforcement for slab-columns joints is not necessary. Nevertheless, according to item 19.5.3.5 of the same standard, in cases where the global stability depends on the punching shear resistance of the slab, it should be provided reinforcement to resist at least 50% of the internal forces, even if the conditions meet the requirements.

2.1.1 Perimeter *C*

At the critical boundary C, determined by the column perimeter, resistance is indirectly verified by concrete diagonal compression using the apparent shear stress τ_{Rd2}:

$$\tau_{Rd2} = 0{,}27\,\alpha_v f_{cd} \tag{10}$$

$$\tau_{sd} = \frac{F_{sd}}{ud} + \frac{k_x M_{sdx}}{W_{px} d} + \frac{k_y M_{sdy}}{W_{py} d} = \frac{F_{sd}}{d}\left(\frac{1}{u} + \frac{k_x e_y}{W_{px}} + \frac{k_y e_x}{W_{py}}\right) \tag{11}$$

$$\tau_{sd} \leq \tau_{Rd2} \tag{12}$$

Where:
u : critical perimeter;
$d = (h - d')$: effective height of the slab;
d': average distance between the upper face of the slab and the center of gravity of the superior flexural reinforcement in orthogonal directions;
h : slab thickness;
F_{sd} : design value for a reaction or a concentrated force;
$M_{sd} = eF_{sd}$: design value of the moment resulted of unbalanced tension in the boundary, represented by eccentricity e of critical perimeter in relation to the column center of gravity multiplied by F_{sd};
W_p : plastic resistance module of the critical perimeter;
k : coefficient that provides part of M_{sd} transmitted to the column in the punching shear;

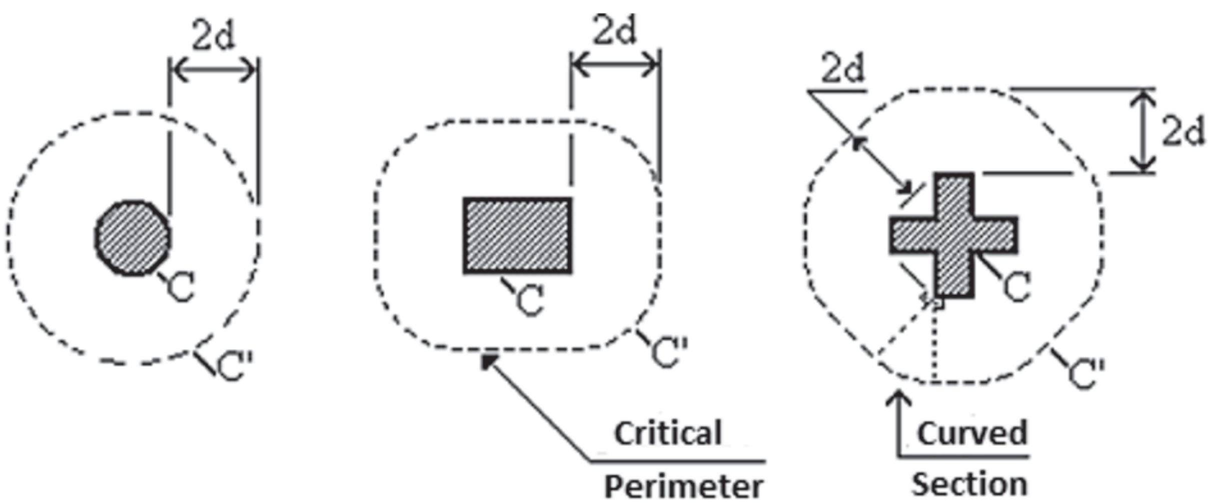

Figure 4
Perimeters C and C′ to different geometries of internal columns.(1)

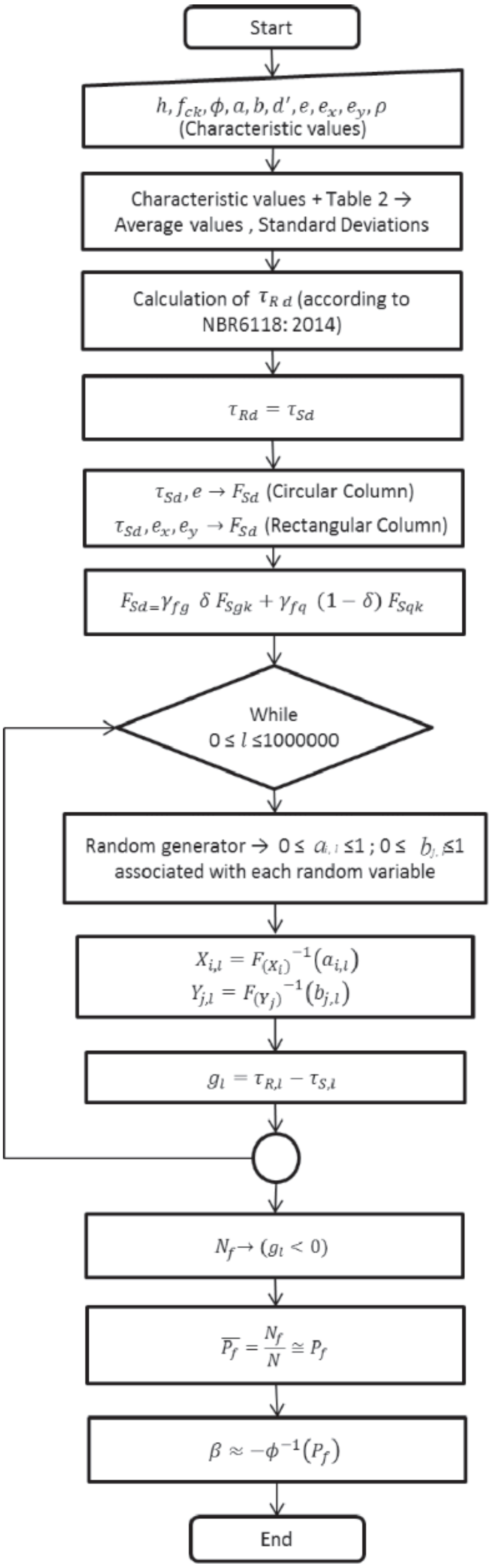

Figure 5
Flowchart of calculation routine

$\alpha_v = \left(1 - \frac{f_{ck}}{250}\right)$: coefficient of effectiveness of the concrete;

$f_{cd} = f_{ck} / \gamma_c$: Design value of concrete compressive strength;
It is possible to write equation (13) as follows:

$$\tau_{Rd2} - \tau_{sd} \geq 0 \qquad (13)$$

The existing $0,27$ constant of resistant stress equation τ_{Rd2} is derived from the apparent shear stress, with inclination of $tg(2/3) \cong 33,7°$, as shown in Fusco [9].This value considers $k_{mod} = 0,85$ coefficient, corresponding to the named Rüsch effect. Since this equation is used for permanent and variable loads, the coefficient $k_{mod} = 0,85$ should not be removed, it belongs to the design criteria.
This way, removing the implicit security inside the formulation, probability failure function G becomes:

$$G = \tau_{R2} - \tau_s \leq 0 \qquad (14)$$

$$G = 0,27\alpha_v f_c - \frac{F_s}{d}\left(\frac{1}{u} + \frac{k_x e_y}{W_{px}} + \frac{k_y e_x}{W_{py}}\right) \leq 0 \qquad (15)$$

For circular columns, the equation G may be simplified to:

$$G = 0,27\alpha_v f_c - \frac{F_s}{d}\left(\frac{1}{u} + \frac{ke}{W_p}\right) \leq 0 \qquad (16)$$

2.1.2 Perimeter *C'*

At the critical perimeter C', punching shear resistance for flat slabs without shear reinforcement is ensured by equation [17]:

$$\tau_{sd} \leq \tau_{Rd1} = 0,13\left(1 + \sqrt{\frac{20}{d}}\right)(100\rho f_{ck})^{\frac{1}{3}} \qquad (17)$$

Where:
$\rho = \sqrt{\rho_x \rho_y}$: geometric rate of adhering bending reinforcement;
It is important to note that this work does not include the portion related to the compressive stress due to prestressing σ_{cp}.
To obtain the limit equation G, it is necessary to remove the security inside equation (16). However, the formulation proposed by NBR6118 does not expose the safety factors implicit in the design model, this (equation 18) is explicit in Eurocode 2: 2004 standard, that considers the same theoretical model for punching shear checking in C' perimeter.

Table 2
Statistical parameters and probability distributions of random variables

Random variables	Type of probability distribution	Characteristic value	Average value	Coefficient of variation (%)	Source of coefficient of variation
Calculation model - perimeter C	Normal	0,27	0,27	11	(13)
Calculation model - perimeter C'	Normal	0,13	0,18	11	(13)
Compressive strength of concrete (f_{ck})	Normal	f_{ck}	$f_{cm}=f_{ck}/(1-1{,}645CV)$	15	(12)
Tensile strength of concrete (f_{ctk})	Normal	f_{ctk}	$(f_{cm})^{⅓}$	20	(12)
Diameter column (φ, a, b)	Normal	Design value	Design value	4	(12)
Slab thickness (h)	Normal	Design value	Design value	4	(12)
Distance between the CG of the upper flexural reinforcement and the concrete face (d')	Normal	Design value	Design value	12,5	Value adopted by the authors
Load eccentricity (e, ex, ey)	Normal	Design value	Design value	10	(13)
Coefficient K	Normal	Design value	Design value	10	(13)
Mean reinforcement ratio for longitudinal reinforcement (ρ)	Normal	Design value	Design value	5	(12)
Permanent loading (G)	Normal	S_{gk}	$S_{gm}=1{,}05\ S_{gk}$	10	(14)
Variable loading (Q)	Gumbel type 1	S_{qk}	$S_{qm}=0{,}934\ S_{qk}$	20	(14)

$$\tau_{sd} \le \tau_{Rd} = \frac{0{,}18}{\gamma_c}\left(1+\sqrt{\frac{20}{d}}\right)(100\,\rho\, f_{ck})^{\frac{1}{3}} \quad \textbf{(18)}$$

This way, applying the same procedure proposed for the perimeter C, we obtain the equations (19) and (20) to punching resistance verification of perimeter C' at rectangular and circular columns, respectively:

$$G = 0{,}18\left(1+\sqrt{\frac{20}{d}}\right)(100\,\rho\, f_c)^{\frac{1}{3}} - \frac{F_s}{d}\left(\frac{1}{u}+\frac{k_x e_y}{W_{px}}+\frac{k_y e_x}{W_{py}}\right) \le 0 \quad \textbf{(19)}$$

$$G = 0{,}18\left(1+\sqrt{\frac{20}{d}}\right)(100\,\rho\, f_c)^{\frac{1}{3}} - \frac{F_s}{d}\left(\frac{1}{u}+\frac{ke}{W_p}\right) \le 0 \quad \textbf{(20)}$$

2.2 Methodology

The calculation routine to obtain the probability of failure P_f and its corresponding reliability index β is shown in Figure 5, and it is valid for both the critical perimeters C and C'.

- Initially, the input data are entered, which consist of the average and characteristic values, besides coefficients of variation for the random variables involved in the problem, as well as the percentage of permanent (g) or variable (q) loads. The values considered are shown in Tables 2, 3 and 4.
- Determine the design value of resistant strain τ_{Rd} prescribed in standard and based on the characteristic values of random variables, appropriately weighted by the respective factors. In this work it was considered the respective partial factors: $\gamma_c = 1{,}40; \gamma_s = 1{,}15; \gamma_{fg} = \gamma_{fq} = 1{,}40$;
- Determinate the applied shear stress design value τ_{Sd} using the design limit condition $\tau_{Sd} = \tau_{Rd}$.
- F_{sd} is established from τ_{Sd} and the eccentricity e;
- The relationship $F_{sk} = \delta F_{sgk} + (1-\delta) F_{sqk}$ generates permanent (g) and variable (q) portions of the actions, where δ measures the action proportion;
- For each simulation were generated $N = 1000000$ (1 million) of failure equations g_l, determining the random numbers $a_{i,l}$ and $b_{j,l}$, and consequently $X_{i,l}$ and $Y_{j,l}$, from the following relations:

$$X_{i,l} = F_{(X_i)}^{-1}(a_{i,l})$$

$$Y_{j,l} = F_{(Y_j)}^{-1}(b_{j,l})$$

Table 3
Considered data in the models for circular column

Considered data – circular column	
h (cm)	16
d' (cm)	3,5
ρ (%)	0,5
ø (cm)	50
f_{ck} (MPa)	35
e (cm)	10

- In each l-th iteration, the set of random variables values result in applied tensions values $\tau_{S,l} = \tau\left(X_{i,l}\right)$ and resistant $\tau_{R,l} = \tau(Y_{j,l})$ and a result g_l to the failure function G through the following expression:

$$g_l = \tau_{R,l} - \tau_{S,l}$$

- After completing the iterations, the N_f failure occurrences are accounted, it means, that the action value exceeded the corresponding resistance $(g_l < 0)$;
- Finally, the probability of failure P_f is determined and therefore the β_{eq} index.

Table 2 shows the random variables considered in the formulation of perimeters C and C', their respective parameters and statistical probability distributions.
Note that in equation (16) the portion $\left(f_{ck}\right)^{\frac{1}{3}}$ is related with the tensile concrete strength, so the coefficient of variation indicated in Table 2 is applied to the result of the cube root of f_{cm} for its application in Monte Carlo method.

Table 4
Considered data in the models for rectangular column

Considered data – rectangular column	
h (cm)	16
d' (cm)	3,5
ρ (%)	0,5
a (cm) – parallel to the *x* axis	40
b (cm) – parallel to the *y* axis	30
f_{ck} (MPa)	35
e_x (cm)	8
e_y (cm)	6

Table 5
Combinations of considered actions

Permanent (g)	Variable (q)	Values of (δ)
30%	70%	0,3
50%	50%	0,5
70%	30%	0,7

2.3 Application

Different models were developed to perform probabilistic analysis

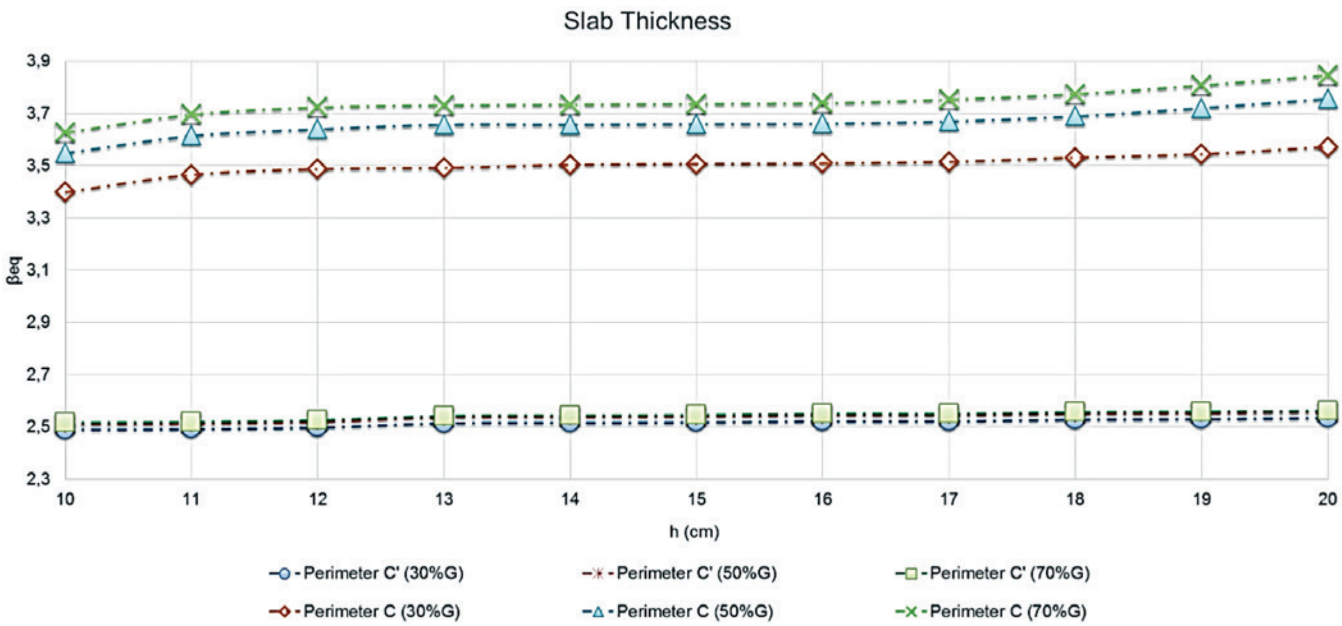

Figure 6
Results of reliability index β_{eq} for circular column considering the variation of the slab thickness

for flat slabs without shear reinforcement subjected to punching shear, supported in circular or rectangular columns, both centered, with load and bending moment transmissions. In each model, it was simulated 1 million cases of equation G to check the critical perimeters C and C', vary in each model the characteristic of the concrete compression resistance f_{ck}, the slab thickness h and the eccentricity of the normal force e.
The information about the variables involved in the problem is shown in Tables 3 and 4.
In order to determine the influence of f_{ck} variation in the probability of failure P_f for C and C' perimeters, other variables remained the same values shown in Tables 4 and 5. The same occurred with the variation of the slab thickness h and the eccentricity of loading e.
Table 5 presents the combinations of actions used to determine the probability of failure.

3. Results and discussion

Considering the reliability models described, the results obtained to the reliability index β_{eq} are shown in Figures 6 to 8 for circular columns and Figures 9 to 12 to rectangular columns.

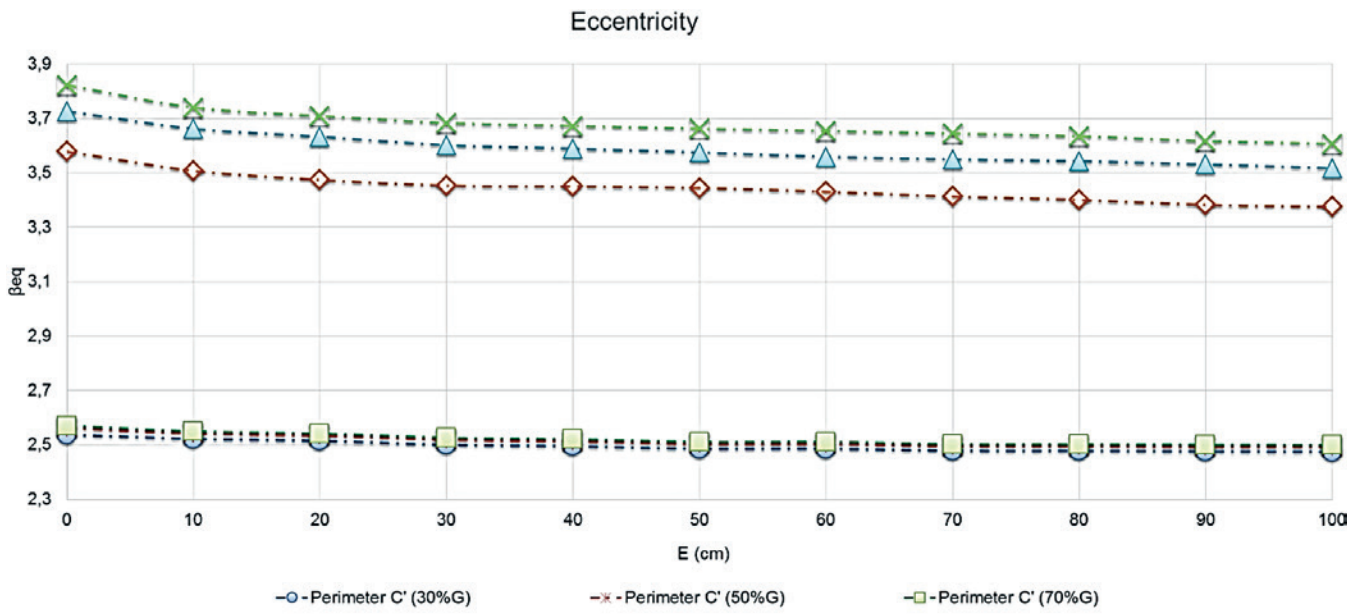

Figure 7
Results of reliability index β_{eq} for circular column considering the variation of eccentricity

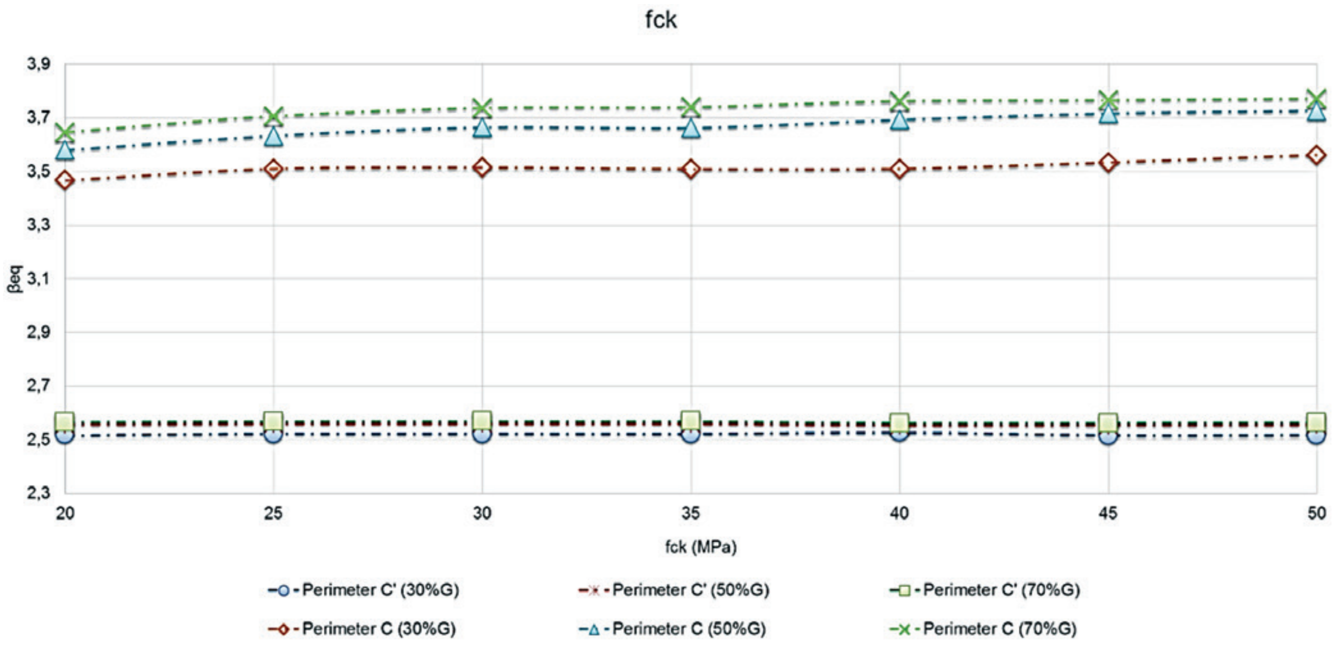

Figure 8
Results of reliability index β_{eq} for circular column considering the variation of f_{ck}

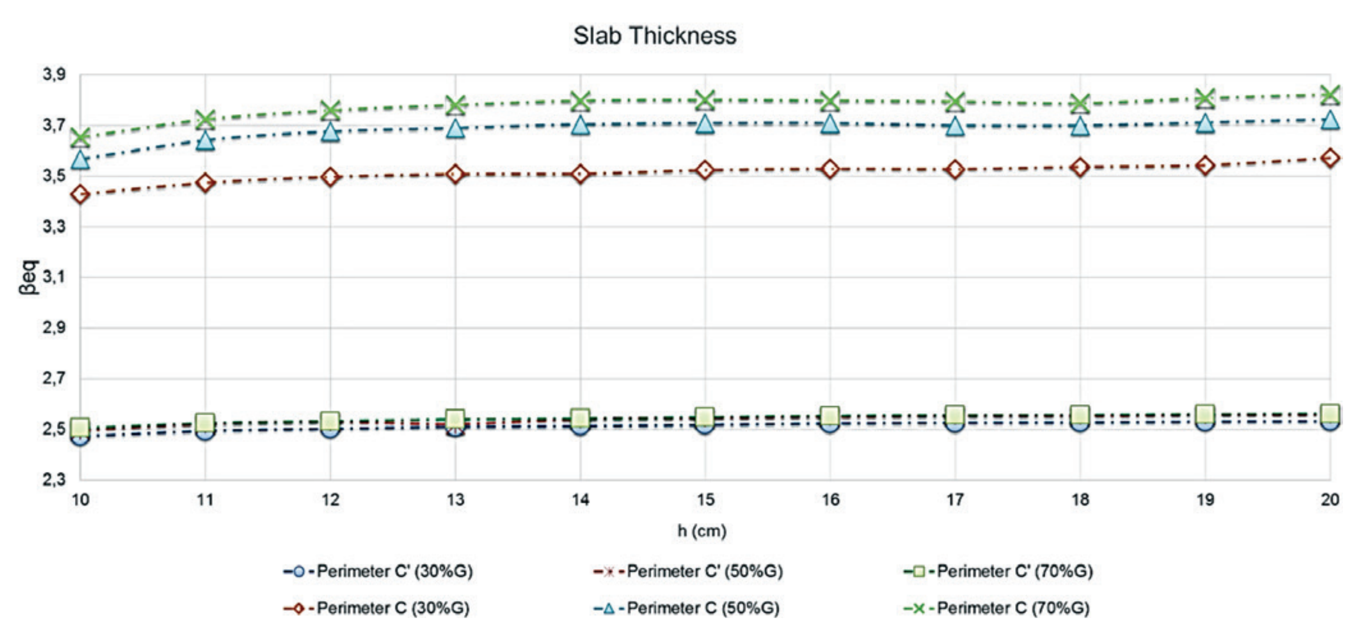

Figure 9
Results of reliability index β_{eq} for rectangular column considering the variation of the slab thickness

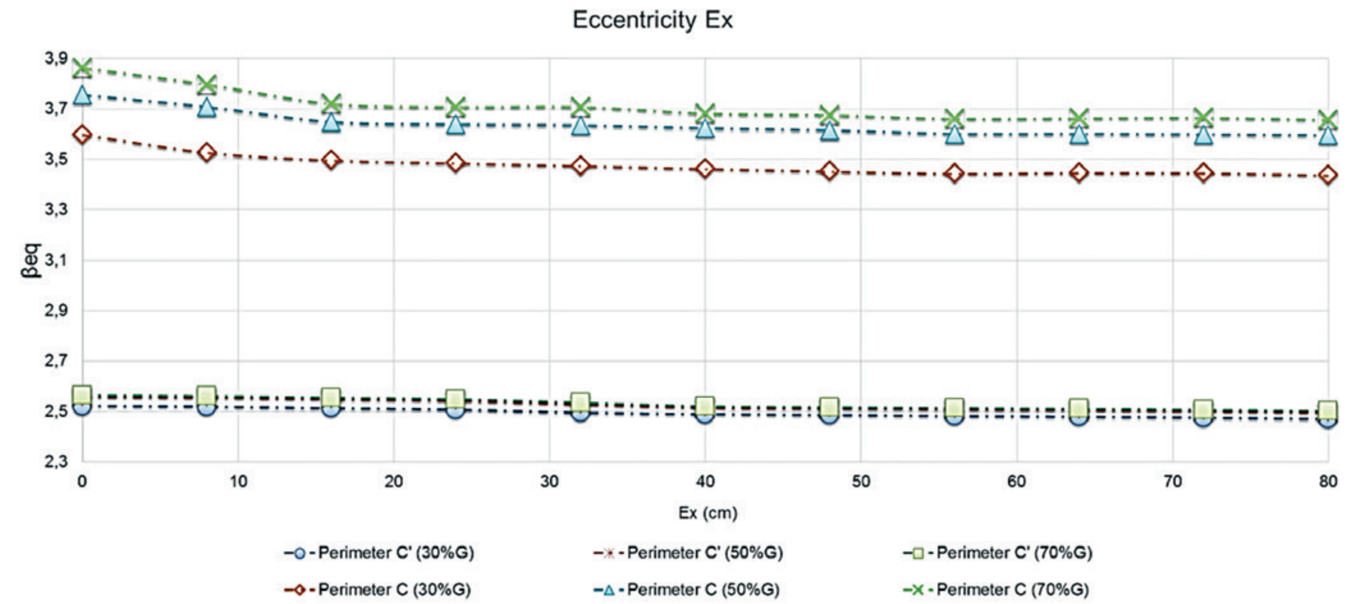

Figure 10
Results of reliability index β_{eq} for rectangular column considering the variation of eccentricity e_x

Analyzing β_{eq} obtained results, both in circular and rectangular columns, it is noted that the C perimeter formulation has a lower probability of failure P_f than the corresponding C' perimeter formulation. The lowest $\beta_{eq} \cong 3,4$ result to C perimeter agrees with Table 1 (considered the high cost and high safety as a consequence of the failure occurrence) and it is similar to the index provided for concrete beams calibration as proposed by (Nowak [13]) to the ACI calibration.
The cases submitted to lower proportions of permanent actions resulted in lower indexes β_{eq}, as expected, due to the increase of the variable actions proportion.
The index β_{eq} of C' perimeter equation remained constant when subjected to the modification of eccentricity variables of load e, concrete strength f_{ck} and slab thickness h. This behavior emphasizes the proper calibration of this equation. The C perimeter equation was more sensitive to the variation of these parameters.
It is important to emphasize that $\beta_{eq} = 2,5$ result, evident in all simulated cases for the formulation of C' perimeter, indicates a higher probability of failure than the desired one. Comparing it with Table 1, this value remains below the expected $\beta = 3,0$ and with the same magnitude as provided for bending slabs (proposed by Nowak [13]). Nevertheless, remember that punching shear phenomenon is associated with brittle fracture of the structure and it would be ideal to obtain lower probabilities of failure for punching

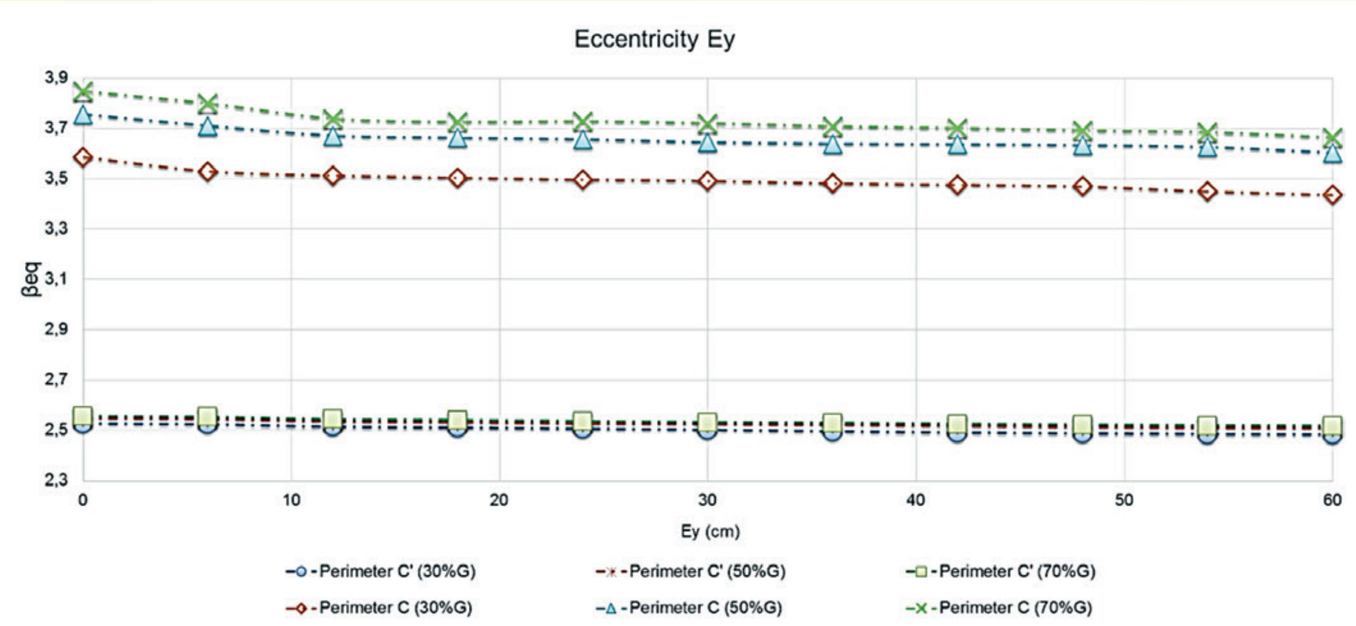

Figure 11
Results of reliability index β_{eq} for rectangular column considering the variation of eccentricity y_x

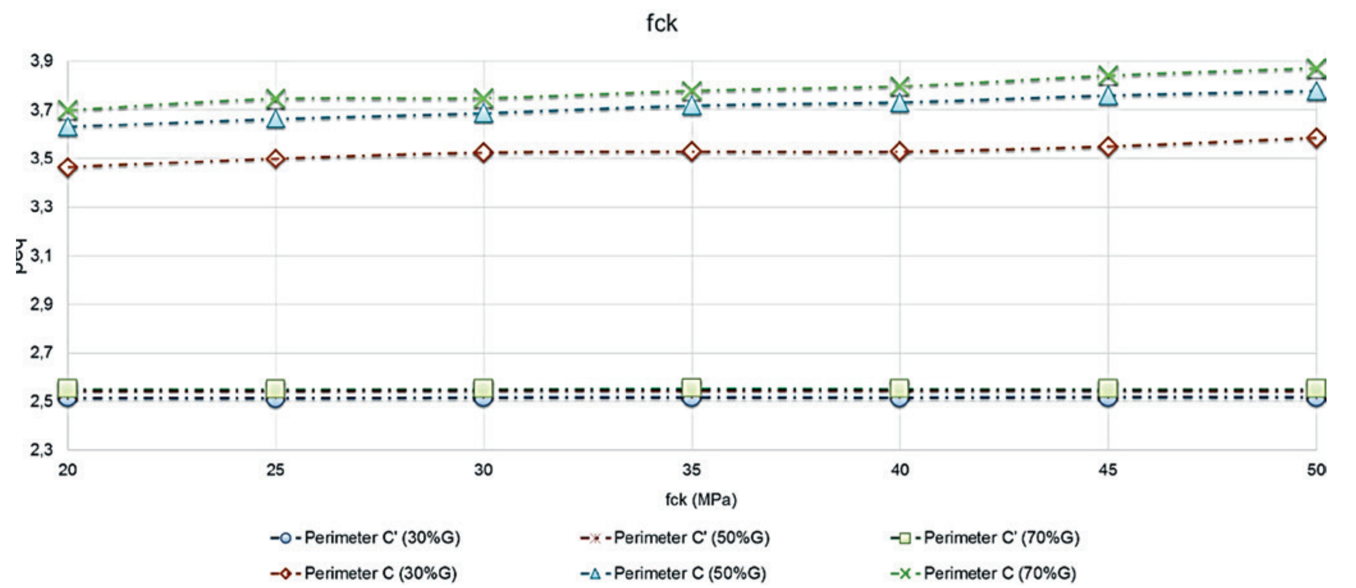

Figure 12
Results of reliability index β_{eq} for rectangular column considering the variation of f_{ck}

shear formulations when compared to the probabilities from bending cases.
Therefore, it is interesting to highlight the need to required punching shear reinforcement in cases where the structural global stability depends on punching shear resistance of slabs (prescribed in Section 19.5.3.5 of NBR6118: 2014), situation that promotes, primarily, the reduction of the probability of failure.

4. Conclusions

This paper presented a comparative study of the safe safety analysis of flat slabs without shear reinforcement and subjected to punching, according to the formulation of the Brazilian standard for structural design.
The results indicate that the reliability of the formulation proposed to the C perimeter obtains reasonable safety in the evaluated situations, containing the index β_{eq} sensitive to slab thickness variation, the concrete strength and load eccentricity, whose minimum value is approximately 3,4 and acceptable according to the patterns of the Brazilian standard.
On the other hand, the C' perimeter presented lower reliability index β_{eq} in all simulated cases, and therefore, significant probability of failure. This situation may suggest the need to revise the standard, either by adopting a minimum reinforcement for punching resistance, either by reducing the resistant stress τ_{Rd1} in the Ultimate Limit State. These results are still considered premature and indicate the need to a deeper study in this subject. We suggest for future studies to increase the verified reliability cases of C' formulation and

include flat slabs containing punching shear reinforcement because the presence of reinforcement can (in principle) reduce the probability of failure to acceptable levels.

5. References

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