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Vibrations due to walking and aerobics activities: a theoretical verification

Vibrações devidas ao caminhar e às atividades aeróbicas: uma verificação teórica

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Abstract

This paper examines the structural design aspects related to vibrations on floors of urban buildings, induced by human activities that can motivate some discomfort for the users thereof. It presents the Brazilian normative regulations concerning human comfort in structures in the presence of vibrations induced by the users in their daily activities, like walking in places such as houses or offices, and aerobic activities such as dancing, jumping and running. The paper concludes by offering relevant and eminently practical information ready to be used manually or with minimal computer resources by the professionals involved, allowing for verification in the initial design phase regarding the eventual need of careful verifications in the structural design of floors subject to vibrations due to human activities.

Keywords: vibrations on floors, human comfort, normative aspects.

Resumo

Este trabalho examina a verificação estrutural referente às vibrações em pisos de edifícios urbanos induzidas pelas atividades humanas que possam motivar algum sentimento de desconforto aos usuários. Apresenta as recomendações normativas vigentes no país relativas ao conforto humano em estruturas na presença de vibrações induzidas pelo próprio usuário em suas atividades cotidianas tais como a prática do caminhar em ambientes residenciais ou comerciais e de atividades aeróbicas em ambientes diversos, como dançar, saltar e correr. Finaliza oferecendo informações relevantes e eminentemente práticas, possíveis de serem exercitadas manualmente ou com o mínimo de recursos computacionais pelos profissionais envolvidos, permitindo verificar na fase inicial de projeto a eventual necessidade de maiores cuidados no dimensionamento estrutural de pisos sujeitos as vibrações devidas às atividades humanas.

Palavras-chave: vibrações em pavimentos, conforto humano, aspectos normativos.

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Table 2.1 – Limits for deflections

Type of effect	Reason for the limitation	Example	Displacement to consider	Threshold deflection
Sensory acceptability	Visual	Visible displacements in structural elements	Total	l/250
	Other	Vibrations felt on floor	Due to live loads	l/350

Source: ABNT NBR 6118:2007

1. Introduction

Examining the Brazilian Standards relating to the development of designs for reinforced concrete structures (ABNT NBR 6118:2007) as well as steel or composite steel/concrete structures (ABNT NBR 8800:2008), one can see that these standards do not specifically address the issue of vibrations in such structures, in reference to verification procedures for obtaining responses due to the dynamic loads induced by activities of human occupation.

Below is an excerpt of the content of each one of these standards regarding the recommended treatment for analysis of vibrations on floors due to the activities performed by the users thereof, followed by considerations compared with ISO 2631:1997/2003, with the exemplification of a numerical application, with an evaluation and conclusion as to the validity of the theoretical proposal presented herein.

2. The approach under the brazilian standards

2.1 ABNT NBR 6118-2007 – design of concrete structures – procedure

This standard, in items 3.2.8 and 11.4.2.3, defines the limit state for excessive vibrations as that which occurs when "*the structure, because of its conditions of usage, is subject to shocks or vibrations, the respective effects must be considered in determining the demands and possibility of fatigue must be considered in the design of the structural elements*," then refers to section 23 of the same standard.

Later on, in section 13.3, paragraph *a*, it again refers to vibrations and refers to section 23 as well, but showing – in this item – the data partially transcribed in Table 2.1 of this article, i.e., the effect of vibrations due to accidental loads on the sensory acceptance of users.

In section 23, specifically in item 23.3 - Limit state of excessive vibration, ABNT NBR 6118:2007 recommends that the analyses relating to vibrations in concrete structures should be done in a linear regime with the natural frequencies f_{nat} being kept distant from the structure's critical frequency f_{crit}; this is a function of the use for which the building is intended, specifying a minimum limit $f_{nat} > 1.2 f_{crit}$, as recommended in Table 2.2.

To control these vibrations, it is suggested that the behavior of the structure should be changed by modifying certain factors, including the dynamic actions of excitation or the natural frequency of the structure with the change of the structure's rigidity or mass, or the damping characteristics.

Otherwise, the analysis is redirected to international standards in cases where the dynamic analysis requires more specific precautions, at the discretion of the analyst. (ABNT NBR 6118:2007)

2.2 ABNT NBR 8800:2008 – design and execution of steel and composite steel/concrete structures for buildings – procedure

This standard, in section 11 – Vibrations, specifically in item 11.4.1, explicitly refers to vibrations by recommending that "*Floor systems subject to vibrations, such as those of large areas that have no partitions or other damping elements, must be verified in order to avoid the appearance of unacceptable transient vibrations due to people walking or other sources, according to Annex L.*"

Annex L of ABNT NBR 8800:2008, includes general introductory comments and refers to item 4.7.7.3.3: "- Frequent combinations of service," which defines that "*frequent combinations are those that repeat often during the lifetime of the structure, on the order of* 10^5 *in 50 years, or that have a total duration equal to a non-trivial part of the period, on the order of 5%.*"

And these combinations may be used for reversible limit states, i.e. those that do not cause permanent damage to the structure or other components of the building, including those related to the user comfort. It concludes by recommending that the natural frequency of the floor structure must never be less than 3 Hz."

Below, we present general procedures that should be considered for what this standard admits as an accurate assessment. These are transcribed as follows; it states that in the case of vibrations on floors, at least the criteria transcribed below shall be considered in a dynamic analysis:

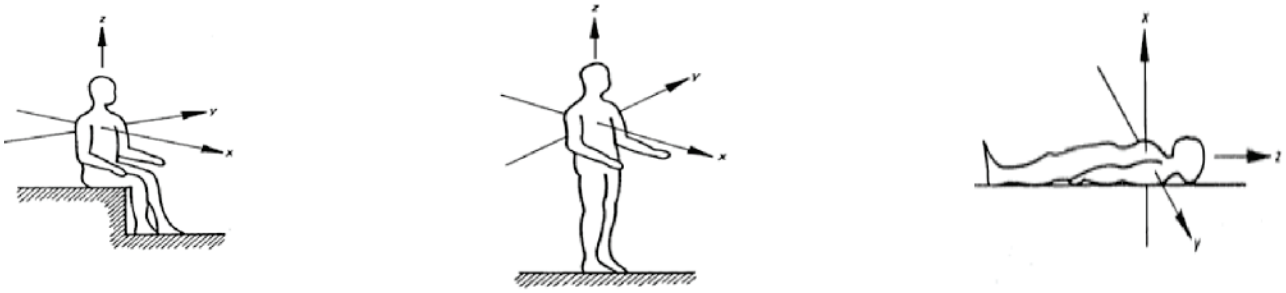
a) the characteristics and the nature of the dynamic excitations,

Table 2.2 – Critical frequency for some specific cases of structures subjected to vibrations by the action of people

Case	f_{crit} (Hz)
Sports gymnasium	8.0
Dance or concert halls without fixed seats	7.0
Offices	3.0–4.0
Concert halls with fixed seats	3.4
Walkways for pedestrian or bicycles	1.6–4.5

Source: ABNT NBR 6118:2007

Figure 3.1 – Barycentric axes of the human body



Source: ISO 2631-1:1997

such as (for example) those arising from people walking and rhythmic activities;

b) the acceptance criteria for human comfort as a function of the use and occupation of the areas of the floor;

c) the natural frequency of the floor structure;

d) the modal damping ratio;

e) the effective weights of the floor;

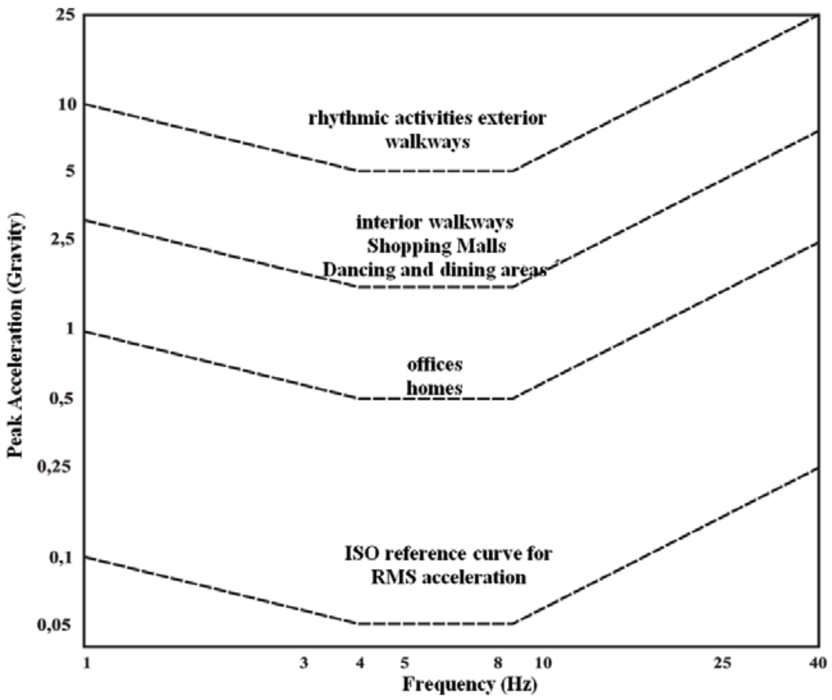
Finally, texts of international origin that may be of interest in this analysis are recommended, more precisely, in Annex S-4 thereof. (ABNT NBR 8800:2008)

3. The international approach

3.1 Standards ISO 2631-1:1997 and 2631-2:2003

International standard ISO 2631:1997/2003, consulted in order to support arguments of this technical article, defines the various methods applicable to the measurement of periodic, random and transient vibrations that may be observed in the human body in standardized positions such as standing, sitting and lying. In this

Figure 3.2 – Peak acceleration for environmental comfort due to vibrations related to human activities



Source: ASCI – Steel Design Guide Series No. 11 (1997) situation, this standard addresses the key factors that in combined form can determine the level of exposure to vibrations acceptable by man.

Figure 3.1 shows the principal axes recommended for measuring the effects of vibration, in accordance with the plan of entry in the human body according to the position of interest admitted for analysis, according to that specific standard.

This standard recommends that measurements be taken during a period of time that is sufficient and necessary to ensure reasonable statistical accuracy, without any restriction on the duration thereof. (ISO 2631-1:1997, ISO 2631-2:2003)

Possibly, in occurrence of measurements in different periods with clear differences in characteristics there between, separate analyses should be made for each period and these facts must be reported; likewise it is essential that other factors be recorded, such as age, gender, size, physical capability, etc., of the users.

Moreover, the AISC ("American Institute of Steel Construction") considers the abacus of Figure 3.2 with peak acceleration rates related to gravitational acceleration, whereby the various types of possible use for the floors are categorized with regard to vibrations due to human activities. (MURRAY et alii, 2003)

The areas of the abacus shown in Figure 3.2 located below the dashed lines correspond to the maximum acceptable limits for peak accelerations corresponding to the respective existing descriptions thereof, for the respective natural frequencies.

4. Aspects of the analysis

Brazilian standards ABNT NBR 6118:2007 and ABNT NBR 8800:2008 do not address the merits of theoretical formulations to develop dynamic analysis of problems involving structural vibrations, whatever their origin, leaving the suggestion that the design engineer research the topic in the relevant literature.

It is known that in order to find the response in the field of the dynamic analysis of structure, the analyses can be performed both in the time domain and in the frequency domain. Dynamic analysis in the time domain is more suitable in structural projects, considering that all work is carried out only with the resources of mathematics of real numbers, while analysis in the frequency domain makes use of complex numbers, with no practical meaning for the engineer. (FERREIRA, W.G, 2002)

5. The theoretical sequence of calculation

5.1 Initial considerations

Manual verification of the dynamic conditions of a particular floor slab begins by examining the physical characteristics thereof in order to better adapt it to a representative structural model regarding the aspect of the structural analysis.

The methodology shown below is based on the recommendations of the AISC ("American Institute of Steel Construction") compiled from several published articles, and on Brazilian standards ABNT NBR 6118:2007 and ABNT NBR 8800:2008.

5.1.1 Unidirectional floor slab

In this case, the vibrations occur in a single direction of the floor, and the structural model can be conceived as a system supported in just one direction with secondary beams in steel hot rolled sections or joists. This case applies most frequently to walkway designs intended for pedestrian traffic. They are designed with a floor slabs normally supported on longitudinal girders supported at the ends, as shown in Figure 5-1b.

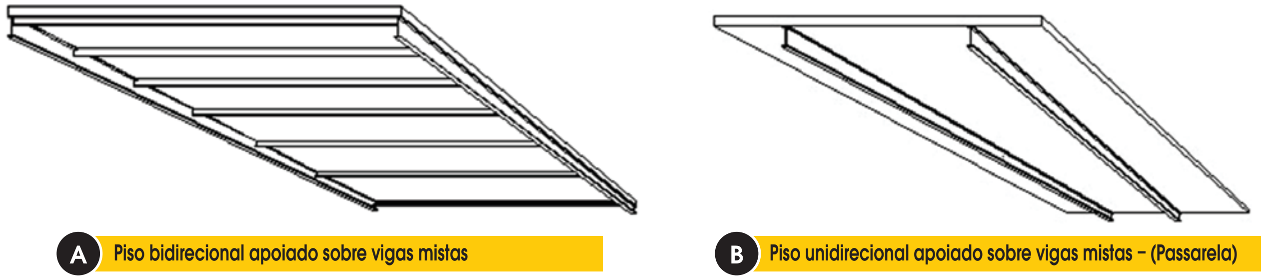
5.1.2 Bi-directional floor slab

In this case, the vibrations occur in two directions of the floor, and the structural model is designed as a floor system supported in both directions, as shown in Figure 5.1a and supported by one of the following alternatives:

- On secondary beams
- On secondary joists
- On primary girders
- On primary joists
- On rigid walls in the primary direction

This solution is regularly found in floor designs in multi-story buildings intended for residential and/or business activity subject to movement of people, such as walking, jumping, and dancing. Depending on the purpose of their use, they are designed with a slab normally supported on transversal secondary girders, and these in turn are supported at their ends on the primary girders or longitudinal rigid walls. Figure 5.1 shows two basic models for analysis.

Figura 5.1 – Modelos típicos para o projeto de pisos de edificações

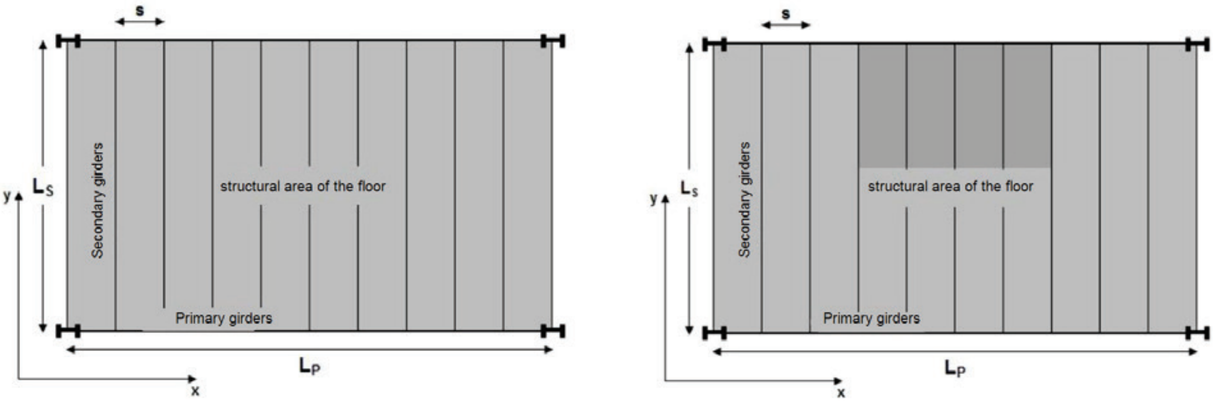


A Piso bidirecional apoiado sobre vigas mistas

B Piso unidirecional apoiado sobre vigas mistas – (Passarela)

Fonte: PRETTI (2012)

Figure 5.2 – Structural model of floors subjected to dynamic actions due to human activities



Floor subject only to walking

Floor subject to walking and dancing

Source: PRETTI and FERREIRA (2012)

5.2 Structural model

The floor slab examined in this article is specified in the building plan as shown in Figure 5.2, where one can identify the various parts that comprise the model. In the model are listed the Cartesian axes according to the directions of the secondary and primary support girders and the spacing between the secondary girders. The dimensions to be considered in this floor slab structural model are as follows:
Direction x – Main dimension L_P [m]
Spacing between girders s [m]
Direction y – Secondary dimension L_S [m]

5.3 Preliminary calculation and specifications

5.3.1 Effective height of floor slab

Solid slab
Slab thickness d_e [m]
Ribbed slab
Slab thickness d_m [m]
Rib height d_n [m]
Effective height $d_l = d_m + d_n$ [m]

Average thickness $d_e = d_m + {}^{d_n}\!/_2$ [m]

Center of gravity of the slab y_{CG} [mm]

5.3.2 Steel beams

Primary direction (x)
Area of cross-section A_x [mm²]
Inertia in relation to the x-x axis I_x [mm⁴]
Nominal height of the girder d_x [mm]
Secondary direction (y)
Area of cross-section A_y [mm²]
Inertia in relation to the x-x axis I_y [mm⁴]
Nominal height of beams or joists d_y [mm]

5.3.3 Modulus of elasticity of the materials

Modulus of elasticity of the concrete E_c [kN/m²]
Modulus of elasticity of the steel E_a [kN/m²]

5.3.4 Dynamic modular ratio for obtaining the secondary section

The modular ratio is a proportion established between the modulus of elasticity of the concrete in relation to the modulus of elasticity of the steel, for the purposes of homogenization of the materials and obtainment of the equivalent composite section transformed into a single material, used in the various calculations. In this study, the sections are homogenized in relation to the steel and modular ratio is given as:

$$n = \frac{E_a}{1{,}35\,E_c} \tag{5.1}$$

Where $1{,}35\ E_c$ refers to the dynamic modulus of elasticity of the concrete. (MURRAY et alii, 2003)

5.3.5 Service loads per unit of floor area

The loads normally acting on floors can be classified as follows. In the event that other loads occur, they must also be considered.
Dead loads
Weight of the floor slab g_1 [kN/m²]
Weight of the incorporated form g_2 [kN/m²]
Weight of finishing material below the floor slab g_3 [kN/m²]
Weight of finishing material above the floor slab g_4 kN/m²]

Weight walls on the floor slab g_5 [kN/m²]
Weight of secondary beam per linear meter g_6 [kN/m]
Weight of primary girder per linear meter g_7 [kN/m]
Weight of people on the floor area g_8 [kN/m²]
Live loads
Weight of people on the floor area q_1 [kN/m²]
Weight of furniture on the floor area q_2[kN/m²]
Weight of partitions on the floor area q_3 [kN/m²]

5.4 Analysis of the floor regarding secondary direction

Under this analysis criterion, the calculations will be developed first by favoring the verification with regard to the secondary direction of the floor, then by focusing on the primary direction, and finally consolidating them into a single step called consolidated calculations.

5.4.1 Effective beam panel width of the secondary section

$$L_{colab} = s \tag{5.2}$$

5.4.2 Transformed moment of inertia of the secondary section per unit of width of the floor slab

$$D_{L_s} = \frac{d_e^3}{12n} \tag{5.3}$$

In this expression the term d_e when the slab is solid, equals the effective height of the slab d_l and when it is ribbed, equals the average thickness thereof, as defined in item 5.4.1.

5.4.3 Mode of vibration of the panel in the direction of the secondary girders

For the purpose of manual verification on a slab floor regarding the dynamic load due to the action of people moving about on the floor, as characterized in item 5.3, in situations due to walking, only the lowest modes are of interest. Similarly in other situations such as those due to dynamic loads of the same origin, but with other characteristics, such as those due to aerobic activities like jumping, dancing, and physical exercise it is also sufficient to check only the occurrences corresponding to the first, second and third modes of vibration. (MURRAY et alii, 2003)

5.4.4 Position of the neutral axis in the secondary direction

In this script for manually calculating the position of the neutral axis of the secondary section obtained is referenced to the underside of the slab when the floor is ribbed and the underside face of the slab when it is solid. In both cases, the slab is supported on the secondary steel beams. For cases in which the floor is a ribbed slab or steel–concrete composite, the expression to be used is the following:

$$\overline{y_{\sec}} = \frac{A_y(d_n + y_{CG.}) - \left(L_{Colab}/n\right) d_m \left(d_m/2\right)}{A_y + \left(L_{Colab}/0\right) d_k} \tag{5.4}$$

For cases in which the floor slab is solid, the expression to be used is the following:

$$\overline{y_{\sec}} = \frac{A_y\, y_{CG.} - \left(L_{Colab}/n\right) d_m \left(d_m/2\right)}{A_y + \left(L_{Colab}/2\right) d_m} \tag{5.5}$$

Expression 5.5 is a particular case of the previous expression 5.4, when the ribbing is nullified. Note that the calculated value is positive when the position of the neutral axis is above the adopted reference, in this case, the underside of the slab, if ribbed or steel–concrete composite. The underside is also used when the slab is solid. Otherwise, it will be negative, and the signal should be considered algebraically in the calculations.

5.4.5 Transformed moment of inertia I_s of the secondary section

This transformed moment of inertia of the secondary section of the floor, or compound moment of inertia, is calculated by expression 5.6, where all of the terms have been defined previously. This refers to the moment of inertia of a section comprised of a beam and the effective part of the floor slab transformed into a single material, here steel is adopted.

$$I_{TS} = I_y + A_y\left(d_n + y_{CG} + h_a - \overline{y_{\sec}}\right)^2 + \left[\frac{\left(L_{colab}/n\right) d_m{}^3}{12}\right] + \left(L_{colab}/n\right) d_m \left(\overline{y_{\sec}} + d_m/2\right)^2 \tag{5.6}$$

At this stage of the calculation script, it is necessary to consider the type of beams that are specified for the secondary direction of the floor. Two cases can occur:

- Case 1 – Steel secondary joists
- Case 2 – Steel secondary beams

In case 1, where these elements are joists simply supported on the upper flanges of the border girders in the primary direction, additional considerations must be admitted in order to determine the actual transformed moment of inertia in the secondary direction. In this case, the actual moment of inertia is given by the following expression:

$$I_{Effet} = \frac{1}{\dfrac{\gamma}{I_{chords}} + \dfrac{1}{I_{TS}}} \tag{5.7}$$

$$\gamma = \frac{1}{C_T} - 1 \tag{5.8}$$

In order to determine the term C_T it is necessary to evaluate the ratio L_S/d where L_S is the span of the secondary direction and d is the height of the supporting joists of the floor slab in the same direction, as a function of the sections of the materials of the flanges of the top chords of joists.

Hypothesis 1:

In joist formed by chords comprised of single or double angles, the term C_T is calculated by the expression;

$$C_T = 0{,}90\left(1 - e^{-0{,}28\left(L_S/d\right)}\right)^{2{,}8} \quad \text{for} \quad 6 \le L_S/d \le 24 \tag{5.9}$$

Hypothesis 2:

The joist being formed by the chords comprised of single round steel bars, the term C_T is calculated by the expression;

$$C_T = 0{,}721 + 0{,}00725\left(L_S/d\right) \quad \text{for} \quad 10 \le L_S/d \le 24 \tag{5.10}$$

If these steel sections have a solid web, welded web or web bolted directly to the webs of the primary girders of the floor, this transformed moment of inertia is considered to be the actual transformed moment of inertia with no need for additional calculations as in expression 5.7. In this case, term $C_T = 1$.

5.4.6 Loads acting on the compound secondary sections

Based on Brazilian standard ABNT NBR 6118:2007, the combination of active loads to be considered in the calculations is obtained with the direct application of expression 5.11, where it is admitted that such loads have the character of frequent actuation. Loads with this characteristic are those who behave according to the normative definition stated in section 2.2 above. The primary variable action F_{q1} is taken with its frequent value $\Psi_1 F_{qk,1}$ and all other variables actions are taken with their quasi-permanent values $\Psi_2 F_{qk}$. Under these conditions, the combination of loads for verifying the limit state regarding excessive vibrations recommended by ABNT NBR 6118:2007 and ABNT NBR 8800:2008 is as follows:

$$F_{d,serv} = \sum_{i=1}^{m} F_{gk,i} + \psi_1 F_{qk,1} + \sum_{j=2}^{n}\left(\psi_{2,j} F_{qk,j}\right) \tag{5.11}$$

5.4.7 Deflection due to static load in the middle of the secondary span

The deformation due to the static load in the secondary direction corresponding to the homogenized section in the direction of the Cartesian axis, y, in Figure 5.2, is calculated for the condition in which the supports are given by the following expression:

$$\Delta_S = \frac{5.F_{s,serv} L_S}{384\, E_a I_{Effet}} \tag{5.12}$$

5.4.8 Natural frequency in the secondary direction

It is appropriate that the natural frequency in the secondary direction is calculated and specified, given that it may assist in the event that it is necessary to make some ad-hoc changes to the parts of the structure in order to attain a favorable end result regarding the aspect of verification of vibration. The calculation is also recommended because the verification as to vibrations in the manner structured in this study is linear with the assumption of superposition of the applied actions.

This intermediate verification allows the proposed structural system to be verified at this moment with regard to the provisions of items 23.1 of standard ABNT NBR 6118:2007, L.1.2 and L.3.3 of standard ABNT NBR 8800:2008. Standard ABNT NBR 6118:2007 states that $f_{nat} > 1{,}2 f_{crit}$, where the critical or excitation frequency is obtained in Table 5.1, and one should also consider that ABNT NBR 8800:2008 recommends that the natural frequency should always be above 3 Hz. It is appropriate that these regulatory limits be met simultaneously.

Thus, the natural frequency in the secondary direction is calculated by:

$$f_{natSec} = 0{,}18\sqrt{\frac{g}{\Delta_S}} \tag{5.13}$$

In the event that this natural frequency does not meet the regulatory recommendation at this stage, the modeled structural system can be revised, whether in the section of the girders, or in the section of floor slab, or both, according to the interest and possibility allowed by design specifications, thereby adapting it.

Table 5.1 – Critical frequency for some specific cases of structures subjected to vibrations by the action of people

Case	f_{crit} (Hz)
Sports gymnasium	8.0
Dance or concert halls without fixed seats	7.0
Offices	3.0–4.0
Concert halls with fixed seats	3.4
Walkways for pedestrian or bicycles	1.6–4.5

Source: ABNT NBR 6118:2007

5.4.9 Transformed moment of inertia of the secondary section per girder

This is given by:

$$D_S = \frac{I_{Effet}}{s} \tag{5.14}$$

Where s refers to the distances between the secondary floor girders, as defined in Figure 5.2.

5.4.10 Load acting on the secondary section

The calculation of the applied load on the secondary direction (direction y) of the structural model of Figure 5.2 is done by the expression:

$$W_S = k\left(\frac{F_{d,serv}}{s}\right)B_S L_S \tag{5.15}$$

In this expression, the constant k takes on different values according to the border condition of the floor slab, specifying $k = 1{,}0$ when the slab is simply supported in the secondary direction (in the case of secondary joists supported on the upper table) and $k = 1{,}5$ when dealing with a continuous slab, in the direction examined (in the case of steel beams welded or bolted onto the webs of the primary girders of the floor).
In expression 5.15, the term B_s corresponding to the effective width calculation of the floor in secondary direction is given by:

$$B_S = C_S\left(\frac{D_{LS}}{D_{VS}}\right)^{1/4} L_S \qquad B_S \leq \frac{2}{3}L_S \tag{5.16}$$

The value of B_S adopted will be the lower of the values obtained in expressions 5.16. The first expression only the term C_S is still unknown, being arbitrated as a function of the position of the floor on the story, as follows:
$C_S = 1$, when dealing with a floor at the edge of the story.
$C_S = 2$, for all other cases.

5.5 *Analysis of the floor in the primary direction*

In this section, the calculations relating to the dynamic verification of the floor will be shown, according to the primary direction thereof.

5.5.1 Effective beam panel width of the primary section

The effective beam panel width of the slab L_{Colab} in the direction of the primary girders is determined as follows:

$$\begin{aligned} L_{Colab} &= 0{,}4L_S \quad \text{if} \quad 0{,}4L_P < L_S \quad \text{or} \\ L_{Colab} &= \frac{L_S}{2} \quad \text{if} \quad 0{,}4L_P > L_S \end{aligned} \tag{5.17}$$

5.5.2 Position of the neutral axis of the primary section

The position of the neutral axis of the composite section formed by the girder and supported slab is given by the following expression:

$$\overline{y_P} = \frac{A_P\left(\frac{d_n}{2} + h_{apoio} + y_{CG\,prin}\right) - \left(\frac{L_{ColabP}}{n}\right)\left(d_m + \frac{d_n}{2}\right)}{A_P + \left(\frac{L_{ColabP}}{n}\right)\left(d_m + \frac{d_n}{2}\right)} \tag{5.18}$$

The calculated value is positive when the position of the neutral axis is below the reference adopted, in this case, the underside of the slab of the ribbed slab and concrete–steel composite and the underside of the solid slab. Otherwise it will be negative.

5.5.3 Transformed moment of inertia I_{TP} of the primary section

The transformed moment of inertia of the section in the primary direction of the structural system, therefore, is determined through the following expression:

$$\begin{aligned} I_{TP} = I_P + A_P\left(\frac{d_n}{2} + h_{apoio} + \overline{y}_{CG.P} + \overline{y}_{LN.P}\right) + \\ \left\{\frac{\left[\frac{L_{Colab}}{n}\right](d_e)^3}{12}\right\} + \left(\frac{L_{Colab}}{n}\right)(d_e)\left[y_{LN\,\mathrm{Pr}\,in} + \frac{(d_e)}{2}\right]^2 \end{aligned} \tag{5.19}$$

When the secondary girders are joists and simply supported on the primary girders, the reduced rigidity on the primary girders must be considered due to the flexibility that occurs in these supports. Hence, the transformed moment of inertia is calculated through the following expression:

$$I_{TP\,\mathrm{Re}\,d} = I_P + \frac{(I_{TP} - I_P)}{4} \tag{5.20}$$

5.5.4 Loads acting on the compound primary sections

The loads acting on the slab and secondary girders are transferred to the primary girders through the following expression:

$$F_{d,P} = L_S \cdot \left(\frac{F_{d.Serv}}{L_{Colab}}\right) + g_4 \tag{5.21}$$

5.5.5 Deflection due to the static load in the middle of the primary span

The deflection due to the static load in the primary direction, corresponding to the homogenized section in the direction of the Cartesian axis, x, in Figure 5.2, is calculated for the condition in which the supports are girders and admitted as being free from the action of the bending moments. This deflection is given by the following expression:

$$\Delta_P = \frac{5 F_{d,P} L_P}{384\, E_a I_{TP,Red}} \tag{5.22}$$

Note that the load used $F_{d,P}$ in expression 5.22 for obtaining the deflection at the center of the primary girders is the nominal load, i.e., is not increased by weighting coefficients, as recommended by Brazilian standards. It is also important to note that the term $I_{TP.Red}$ corresponding to the reduced moment of inertia will only occur when the secondary girders are simply supported on the primary girders as occurs with joists. If this type of bond does not occur, this term is replaced by the transformed moment of inertia I_{TP} calculated by expression 5.19.

5.5.6 Natural frequency in the primary direction

Like the calculation of the natural frequency in the secondary direction, at this instant it is also appropriate that the natural frequency in the primary direction of the system being analyzed should be calculated and specified, although not of interest to the final verification of the floor regarding the aspect of the vibrations. Moreover, this specification, at this stage of the analysis, is recommended because the verification regarding vibrations in the manner structured in this study is linear, as previously stated.

This intermediate verification allows for the determination in this phase of analysis as to whether the structural system in the primary direction meets item 23 of ABNT NBR 6118:2003, in which the ratio $f_{nat} > 1{,}2 f_{crit}$ must be observed, according to the purpose of usage specified in Table 5.1 shown previously.

$$f_{nat\,\mathrm{Prin}} = 0{,}18 \sqrt{\frac{g}{\Delta_P}} \tag{5.23}$$

In the event that this natural frequency does not meet the provisions of the standards, the structural system modeled may be revised by changing the section of the primary girders or section of the floor slab, or both, according to interest and design possibility, adapting it to the regulatory recommendation.

5.5.7 Transformed moment of inertia per unit of length in the primary direction

The transformed moment of inertia of the slab in the primary direction D_P is calculated as follows:

$$D_P = \frac{I_{TP}}{L_S} \tag{5.24}$$

5.5.8 Effective width of the floor in the primary direction

The calculation width of the floor slab in the primary direction to be adopted B_P is calculated as follows:

$$B_P = C_P \left(\frac{D_S}{D_P}\right)^{1/4} L_S \tag{5.25}$$

In this expression, the term C_P is a function of the conditions for binding of the secondary girders on the primary girders, assuming the value 1.60 when the secondary girders are supported on the flanges of the primary girders as, for example, when there is the specification of the joists in the secondary direction. When the secondary girders are connected directly to the webs of the primary girders as occurs, for example, when the specification of solid-web steel girders, the term C_P takes the value 1.80.

5.5.9 Total load per unit of length of the primary section

The portion of the total load per unit of length W_p in the primary direction is given by the expression:

$$W_P = \frac{F_{d,P}}{L_S} B_P L_P \tag{5.26}$$

All terms of expression 5.26 were previously defined and are known from the specifications in items 5.3 and 5.5.4 except B_P, corresponding to the calculation width of the floor, to be considered according to the following conditions:

$$\begin{aligned} B_P &= C_P \left(\frac{D_S}{D_P}\right)^{1/4} L_P \quad \text{when} \quad C_P \left(\frac{D_S}{D_P}\right)^{1/4} L_P \le \frac{2}{3} L_S \\ B_P &= \frac{2}{3} L_S \quad \text{if} \quad C_P \left(\frac{D_S}{D_P}\right)^{1/4} L_P > \frac{2}{3} L_S \end{aligned} \tag{5.27}$$

5.6 Combined analysis of the floor in both directions

With the analysis completely developed in both directions of the proposed floor system as shown in Figure 5.2, it is then possible to perform the final calculations assuming the superposition of effects and proceeding according to the procedures shown below. The actions and effects will now be defined as combined and refer to the system as a structural assembly supported in both directions.

5.6.1 Combined natural frequency

The combined natural frequency for the proposed floor system is calculated by the expression:

$$f_{nat.Comb} = 0{,}18 \sqrt{\frac{g}{\Delta_S + \Delta_P}} \tag{5.28}$$

5.6.2 Calculation of the combined load acting on the system

The total combined acting load for the proposed system is calculated by the expression:

$$W_{Comb} = \left(\frac{\Delta_S}{\Delta_S + \Delta_P}\right) W_S + \left(\frac{\Delta_P}{\Delta_S + \Delta_P}\right) . W_P \qquad (5.29)$$

5.6.3 Response due to the action of people walking

With the sequence of calculations developed here, one is able to obtain the dynamic response of the structural system of the floor due to walking loads. Accordingly, the response will be obtained in terms of peak acceleration relative to gravitational acceleration by applying expression 5.30, only valid for dynamic actions due to walking loads. (MURRAY et alii, 2003)

In expression 5.30, the load P_0 and the damping factor β, according to this criterion for calculating the dynamic response due to walking loads, are obtained in Table 5.2, according to the type of use of the floor under analysis.

The dynamic response of the floor due to walking loads is obtained by expression 5.30 below:

$$\frac{a_{peak}}{g} = \frac{P_0 e^{-0,35\, f_{nat.Comb}}}{\beta W_{Comb}} \qquad (5.30)$$

This result calculated thusly should be compared with the values recommended in the last column of Table 5.2 for completion of the analysis.

The percentage values obtained for the ratio a_{peak}/g in expression 5.30 can also be compared with the thresholds for these percentages of peak accelerations acceptable and defined for each case of usage of floors in function of human activity, considered adjusted in relation to the base curve in ISO 2631-1:1997, as shown in Figure 3.2.

5.6.4 Response due to aerobic actions of people

The analysis regarding aerobic actions utilizes the following calculations shown previously to where the combined natural frequency in sub-section 5.6.1 is calculated. With this natural frequency calculated it is possible to perform a first approximation as to the acceptability of the floor for aerobic activities using Table 5.3, according to the aerobic activity listed in the first column.

The natural frequency calculated according to section 5.6.1 should be compared with the minimum natural frequency required in Table 5.3, interpolating, when necessary, for the obtainment thereof according to the characteristics of the floor under analysis in relation to those specified in table 5.3. This calculated natural frequency, being less than the required tabulated minimum natural frequency, indicates that the floor cannot be accepted, thus proceeding to a second analysis, now using the following expression:

$$f_{nat\,.Comb} \geq (f_{nat})_{req} = f\sqrt{1 + \frac{k}{a_{peak}/g}\,\frac{\alpha_i . w_p}{w_t}} \qquad (5.31)$$

In this expression, the terms are defined as follows:

f_{nat} – Natural frequency of the floor system under analysis

$(f_{nat})_{req}$ – Minimum natural frequency required at each forced frequency

f – Excitation frequency as shown in Table 5.4, $f = if_{step}$

f_{step} – Pitch frequency of the activity

i – Harmonic under analysis as shown in Table 5.4

k – Constant depending on the type of use of the floor
    Dancing room = 1.30
    Lively concert = 1.70
    Sporting events = 1.70
    Aerobic activities = 2.00

α – Dynamic coefficient according to Table 5.4

Table 5.2 – Recommended values for the parameters of equation 5.30 and limits (a_{peak}/g) due to walking

Type of floor usage	Constant force (P_0) (kN)	Damping (β)	Peak/Gravitational acceleration (a_{peak}/g)x100%
Offices, homes and churches without walls, ceilings or platforms (decks)	0.29	0.02	0.50%
Offices, homes and churches with removable dividers and non-structural components	0.29	0.03	0.50%
Offices with partitions, walls between floors	0.29	0.05	0.50%
Malls and shopping centers	0.29	0.02	1.50%
Footbridges – indoor	0.41	0.01	1.50%
Footbridges – outdoor	0.41	0.01	5.00%

Source: MURRAY, ALLEN & UNGAR (2003) – Adaptation

Table 5.3 – Application of equation 5.31 for rhythmic activities

Activity Threshold peak acceleration Type of construction	Excitation frequency[1] (Hz) (f)	Weight of users (kN/m²) (w_p)	Total weight (kN/m²) (w_t)	Minimum required natural frequency [3] (H_z)
Dancing and dining				
Rate of acceleration – $a_0/g = 0{,}02$				
Heavy floor – 5.0 kN/m²	3.00	0.6	5.6	6.4
Light floor – 2.5 kN/m²	3.00	0.6	3.1	8.1
Lively concerts and sports events				
Rate of acceleration – $a_0/g = 0{,}05$				
Heavy floor – 5.0 kN/m²	5.00	1.5	6.5	5.9 [2]
Light floor – 2.5 kN/m²	5.00	1.5	4.0	6.4 [2]
Aerobic exercises only				
Rate of acceleration – $a_0/g = 0{,}06$				
Heavy floor – 5.0 kN/m²	8.25	4.2	5.2	8.8 [2]
Light floor – 2.5 kN/m²	8.25	4.2	2.7	9.2 [2]
Jumping exercises and weight lifting				
Rate of acceleration – $a_0/g = 0{,}02$				
Heavy floor – 5.0 kN/m²	8.25	2.5	5.12	9.2 [2]
Light floor – 2.5 kN/m²	5.00	2.5	2.62	10.6 [2]

Notes: (1) - Equation 5.31 is provided for all harmonics listed in Table 5.4 where the excitation frequency that governs the movement is shown
(2) - May be reduced if, according to equation 5.33, the product of the damping and the mass is sufficient to reduce the resonance of the 2nd and 3rd harmonics to an acceptable level.
(3) - Values calculated based on equation 5.31.

Source: MURRAY, ALLEN & UNGAR (2003)

Table 5.4 – Estimated load for rhythmic events

Activity	Excitation frequency (Hz) (f)	Weight of users (*) (kN/m²) (w_p)	Dynamic coefficient (α_i)	Dynamic load (kN/m²) ($\alpha_i w_p$)
Dancing				
1st harmonic	1.50–3.00	0.60	0.50	0.300
Lively concert				
1st harmonic	1.50–3.00	1.50	0.25	0.400
2nd harmonic	3.00–5.00	1.50	0.05	0.075
Sports events				
1st harmonic	1.50–3.00	1.50	0.25	0.400
2nd harmonic	3.00–5.00	1.50	0.05	0.075
Jumping exercises				
1st harmonic	2.00–2.75	0.20	1.50	0.300
2nd harmonic	4.00–5.50	0.20	0.60	0.120
3rd harmonic	6.00–8.25	0.20	1.00	0.020

Note: (*) – The values above are based on maximum occupancy density for areas commonly encountered. Under special conditions, these densities may be greater and must be calculated in such cases.

Source: MURRAY, ALLEN & UNGAR (2003)

a_{peak}/g – Percentage of peak acceleration/ gravitational acceleration

w_p – Weight per unit of area of the participants distributed on the floor

w_t – Total weight per unit of area of the participants and of the floor

Important note:

The portions relating to weights w_p and w_t, should be considered in the entire area of the structural system of the floor under analysis.

Expression 5.31 should be used for all harmonics listed in Table 5.4, substituting the terms thereof according to the aerobic activity to which the floor is subjected.

According to the foregoing criterion, the percent ratio of peak acceleration in relation to gravitational acceleration can be calculated by the expressions presented below, where appropriate. Similarly, as stated in item 5.6.4, the maximum acceptable levels for the ratio between peak acceleration in relation to gravitational acceleration are shown in the abacus in Figure 3.2.

Case 1:

In resonance, situation whereby there occurs the sum of the system's natural vibration energies with those of the forced vibration at the time when they are equal $f_{nat.Comb} = f$ expression 5.31 can be rearranged to the following format:

$$\frac{a_{peak}}{g} = \frac{1{,}30\,\alpha_i w_p}{2\beta \quad w_t} \tag{5.32}$$

In this expression β represents the damping rate of the structural system. The result of expression 5.32 should be less than the percentage ratio of peak acceleration in relation to gravitational acceleration specified in the first column of Table 5.3, just below the intended use of the floor.

Case 2:

Out of resonance, in which $f_{nat.Comb} \geq 1{,}2f$, and expression 5.30, are transformed into:

$$\frac{a_{peak}}{g} = \frac{1{,}30}{\left(\frac{f_{nat}}{f_{for}}\right)^2 - 1}\,\frac{\alpha_i w_p}{w_t} \tag{5.33}$$

Finally, when the harmonic excitation frequency $f = if_{step}$, where i represents the harmonic of interest, is equal to or shown to be very close to the natural frequency f_{nat} of the structural floor system, its percentage acceleration of peak acceleration in relation to gravitational acceleration should be calculated by expression 5.31. In cases where the lowest harmonics exhibit levels of excitation frequencies far from the natural frequency, the ratio of peak acceleration to gravitational acceleration should be calculated by expression 5.33 for the harmonic of interest.

6. Numerical application

Exemplifying this information numerically, it is assumed a floor slab originally designed for offices, for which one first wishes to verify the acceptability thereof when subjected to users walking, and secondly, due to the change of use, when subjected to aerobic loads due to the addition of a dance floor. This floor slab, due to its characteristics of initial use and after modification, falls under the first row of Table 5.2 for the purpose of obtaining the initial information necessary for the calculations and in Table 5.3 in the final analysis. Table 5.5 shows the input data of the numerical example for verification of floor as shown in Figure 5.2.

So, by making the substitutions of these initial values appropriately in the respective expressions shown previously, according to the script shown, one arrives at the percentage value of peak acceleration in relation to gravitational acceleration a_{Peak}/g of calculation.

Once these values are obtained, a comparison is made between the calculated values and the thresholds a_{Peak}/g recommended in Table 5.2. If a_{Peak}/g calculated is less than the equivalent tabulated value, one can conclude that for the initial proposed data, the exemplified floor slab satisfies the minimum conditions capable of avoiding the harmful effects of vibrations. Otherwise, it does not meet this condition.

In this case, the floor – as initially specified – would satisfy the verification with regard to walking, and would not satisfy if a dance floor were to be added.

In the dynamic verification of the floor when walking, the rate of peak acceleration relative to gravitational acceleration would reach 0.31%, a value below the maximum acceptable limit of 0.50% (Table 5.2). In the verification as to dancing on the dance floor added thereto, the rate of peak acceleration in relation to gravitational acceleration would reach 4.13%, lower than the acceptable limit of 5.00% (Table 5.3), but the minimum acceptable natural frequency would be below the required threshold of 10.84 Hz calculated by expression 5.31, reaching only the value of 10.46 Hz.

Therefore the floor, in the case of adding a dance floor, would require technical intervention to adjust the natural frequency calculated by expression 5.28 to the minimum acceptable natural frequency required, obtained through expression 5.31.

7. Conclusion

According to standard ABNT NBR 6118:2007, the dynamic analysis for the structural element examined in this article would only be met with the calculation of the natural frequencies thereof, and subsequent comparison of these results with the respective critical frequencies shown in Table 2.2, satisfying the inequality $f_{nat}>1{,}2f_{crit}$ and the acceptable maximum displacements recommended in Table 2.1; the two conditions must be met simultaneously.

Standard ABNT NBR 8800:2008 lists several minimal criteria to be considered, however does not detail them numerically, suggesting texts of international origin for the purposes of consultation and categorization of the dynamic analysis under examination.

This article presents the development of calculations relating to a dynamic analysis of a floor, allowing for an analysis in two cases and obtaining the dynamic response when subjected to actions of human activities due to walking and aerobic activities such as dancing. For the other aerobic activities listed in Tables 5.3 and 5.4, the procedure is the same.

The theoretical development is systematized for immediate application, allowing for manual application, or application with the aid

Table 5.5 – Input data of the numerical example

Description	Unit	Slab	Beams in directions	
			Secondary	Primary
Type	–	Mixed	W460x60	W460x60
Dimension in secondary direction	cm	800	800	800
Dimension in primary direction	cm	800	800	800
Floor slab table height	mm	70	–	–
Floor slab ribbing height	mm	50	–	–
Characteristic strength of concrete	MPa	25	–	–
Modulus of elasticity of the concrete	GPa	28	–	–
Modulus of elasticity of steel beams	GPa	–	200	200
Variable load on the floor	kN/m^2	0.70	–	–
Permanent load on the floor	kN/m^2	4.36	–	–
Distance between the axes of the beams	mm	–	3000	8000
Mass per linear meter of steel beams	kg	–	60	60
Cross-sectional area of steel beams	mm^2	–	7,620	7,620
Moment of inertia of the steel beams	mm^4	–	256.52x106	256.52x106

Source: PRETTI – Autor

of simple computer systems, thereby securely meeting the most immediate and expeditious needs of the professionals involved in this type of analysis.

Ends up suggesting numerical values that can be replaced in the sequence of formulas and theoretical considerations presented, thereby enabling a final examination of the various aspects of interest involved in the dynamic analysis of floors, such as the type of occupancy, the human excitation load, the damping ratio, and the acceptable percentage of threshold acceleration, according to the international literature, in addition to meeting Brazilian regulatory recommendations.

The calculated values were compared with the tabulated values in the final analysis on the acceptance of the floor in relation to the vibrations caused by walking. It is checked whether the calculated values are below the limits specified in Tables 5.2 and 5.3, according to the respective specified use of the flooring.

It is observed with this relatively simple sequence of calculations that one can quickly and efficiently conduct the analysis of floors with regard to vibrations due to human activities to which they are submitted, as a function of the diverse and possible uses thereof, making it possible to make speedy technical decisions able to guide the need for more elaborate analyses, guiding any possible preliminary and immediate physical interferences in the structural elements examined for the purpose of mitigating user-induced vibrations.

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9. References

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