

# Reinforced concrete corbels strengthened with external prestressing

## Consolos de concreto armado reforçados com protensão externa



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### Abstract

Reinforced concrete structures may demand, throughout their lifetime, the increase of the capacity load due to eventual changes in the load configuration. In this context, corbels could be considered structural elements which present great challenges for installing strengthening solutions. The referred difficulty is due to the fact that the strength capacity of this kind of element is dependent on the strength of a diagonal concrete strut and a tensile steel tie, both located in a very short region also subjected to interferences. In the present paper, the behavior of reinforced concrete corbels strengthened with external prestressing is investigated. For that, nonlinear analysis of short corbels ( $a/d$  ratio between 0,5 and 1,0), strengthened or not, have been applied. Also, analytical models based on the Strut-and-Tie Model were developed and the obtained results were compared with experimental results. The obtained results have demonstrated that strengthening reinforced concrete corbels using external prestressing is a fast, efficient and safe solution.

**Keywords:** structural strengthening, short corbels, strut and tie model, external prestressing.

### Resumo

As estruturas em concreto podem exigir, ao longo de sua vida útil, o aumento da capacidade de carga devido a eventuais mudanças de utilização. Nesse contexto, os consolos podem ser considerados elementos estruturais que apresentam grandes desafios para a instalação de alternativas de reforço. Tal dificuldade é decorrente do fato de que a capacidade resistente do consolo depende da segurança conjunta de uma diagonal comprimida de concreto e de um tirante metálico em uma zona extremamente limitada e sujeita a interferências. No presente trabalho, apresenta-se o comportamento de consolos de concreto armado reforçados com barras externas protendidas. Para tanto, análises não-lineares de consolos curtos (relação  $a/d$  entre 0,5 e 1,0), dotados ou não de reforço com protensão externa, foram efetuadas. Além dos modelos numéricos, foram aplicados modelos analíticos baseados no Método das Bielas, de maneira que os resultados puderam ser comparados com resultados experimentais. Os resultados obtidos demonstram que a alternativa de reforço de consolos com protensão externa é uma solução rápida, eficiente e segura para consolos em concreto armado.

**Palavras-chave:** reforço estrutural, consolo curtos, modelo de escoras e tirantes, protensão externa.

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## 1. Introduction

Buildings are constructed with the purpose of meeting the aspirations and needs of the owners and, based on this assumption the architectural design is developed, followed by the development of the structural model.

In the course of time, the idealized initial needs of utilization may be subjected to changes in a way that some buildings need to be readequated in order to satisfy the new demands. This need of modification will be invariably related with the necessity of strengthening of the existing structures.

Other situations, such as design errors, faults in execution, unexpected loads, material deficiencies and lack of maintenance procedures can also lead to the need of strengthening. In this way, interventions may be required in order to revitalize structures and make them safe again.

Nowadays, some procedures may be highlighted as the main techniques for the strengthening of concrete structures: application of polymers (carbon, glass, synthetic, etc), use of steel plates, increase of the cross-section and prestress application. This last technique is the main topic of the present paper, where the strengthening of reinforced concrete corbels is investigated.

Corbels are structural shoulders projecting from columns, generally used to support precast/steel beams or to allow the execution of expansion joints in the structures of the buildings. Because they are extremely short and usually subject to interferences, the strengthening of corbels may be a very challenging task.

If the corbel does not present neighboring corbels in the support column, the intervention will be much easier. Figure [1] illustrates a situation of strengthening of a corbel using carbon fiber reinforced polymers (CFRP). In this problem, considering the inadequate length of the steel bearing plate and the rotational/translational movements of the existing steel beam over the corbel, the utmost face of the corbel was subjected to concrete spalling.

In addition, after analyzing the reinforcement used, it was found that

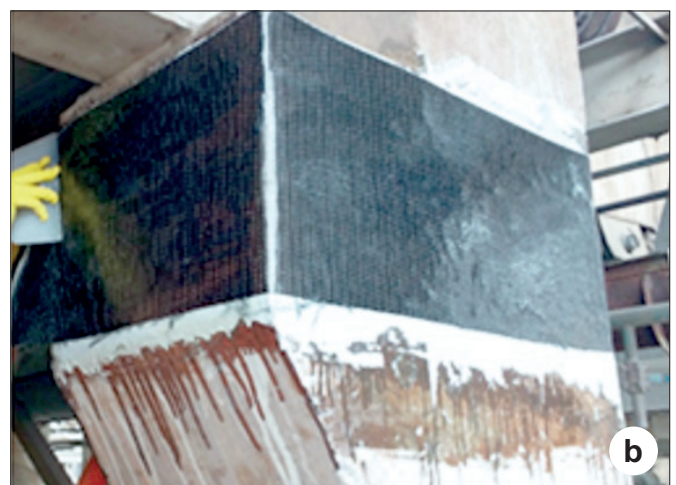
the main tie had deficiencies in relation to the ultimate load expected for the structure and could slip in the damaged zone. For this reason, an intervention using sheets of carbon fiber reinforcement polymers (CFRP) has been used. The proposed procedure simultaneously increased the main tie strength and confined the concrete that was previously subject to spalling, thus damaging the anchorage zone of the original reinforcement. However, it should be noted that if there were neighboring corbels in the support column, it would be difficult to intervene by means of a jacketing procedure.

Thus, in situations where the corbel to be strengthened presents neighboring corbels, an interesting alternative may be the use of prestressing forces, both internal or external, observing that this technique can even be used to add new corbels to the columns. Figure [2] presents an example of prestress force applied for the creation of new corbels in 57 columns. This strengthening was conducted by Voumard [1] using the Strut-and-Tie Method as tool for analysis and design.

According to Almeida [2], prestressing has the advantage of being an active solution in regard to other techniques, i.e., it is the only technique that is able to introduce active forces into the structural system, which is a great advantage.

A prestressing system is basically composed of concrete, steel, anchorage devices and corrugated ducts. Unlike the traditional reinforced concrete, that makes use of undeformed rigid bars, prestressing systems may use wires, strands or steel rod bars, all of them subjected to initial deformation in order to introduce the prestressing force.

The anchorage devices of wires and strands are usually composed of grooved wedges which prevent the prestressed steel from returning to their original position after being tensioned. Usually, the anchoring wedges in post-tensioning systems undergo an accommodation which results in a loss of approximately 6 mm in the initial elongation of the strands. Thus, the strengthening of corbels using strands would be impractical, since the losses by accommodation could be of the order of magnitude of the deformation applied, taking into account the short length of the corbel.



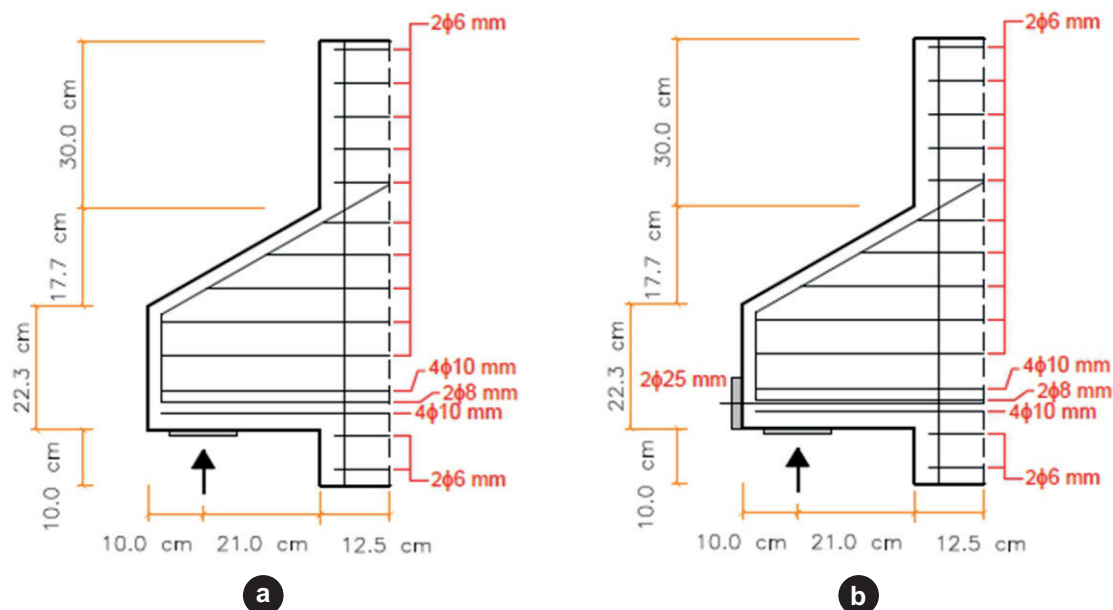
**Figure 1**

Corbel strengthened with carbon fiber reinforcement polymer (CFRP):

a) initial situation, b) strengthening finished







**Figure 5**  
Geometry and reinforcement of the corbels: a) WII-4, b) WII-2

[5.b]). After the strengthening, the corbel was then loaded until the failure. For the corbels of each group that received external prestressing bars, the suffix 2 was inserted to the identification name. For the corbels that did not receive external prestressing bars, the suffix 4 was inserted to the identification name, according to notation assumed by Godycka's [3].

For the numerical analysis of the corbels, the package software ATENA2D was select and more details of which can be found in Cervenka et al [15]. ATENA2D is a program focused on the study

of reinforced concrete structures, capable of performing nonlinear analysis by means of the Finite Element Method. The selected package software when used correctly is able to predict the failure mode, the cracking and yielding loads.

In the nonlinear analysis, ATENA2D uses the constitutive model SBETA for concrete, which has 20 parameters (Table [1]) that can be specified by the user or automatically defined from international codes. The passive reinforcement was modeled as embedded, while the external reinforcement (prestressing bars) were simu-

**Table 1**  
Sbeta parameters

Parameter	Formula
Cylinder strength	$f'_c = -0,85 * f'_{cu}$
Tensile strength	$f'_t = 0,24 * f'_{cu}^{\frac{2}{3}}$
Initial elastic modulus	$E_c = (6000 - 15,5 * f'_{cu}) * \sqrt{f'_{cu}}$
Poisson's ratio	$\nu = 0,2$
Softening compression	$w_d = -0,0005 \text{ mm}$
Type of tension softening	1 - exponential, based on $G_F$
Compressive strength in cracked concrete	$c = 0,8$
Tension stiffening stress	$\sigma_{st} = 0$
Shear retention factor	Variável (item 2.1.7)
Tension-compression function type	Linear
Fracture energy $G_f$ according to VOS 1983	$G_F = 0,000025 f_t^{ef} \text{ [MN/m]}$
Orientation factor for strain localization	$\gamma_{max} = 1,5 \text{ (item 2.1.3)}$

Source: CERVENKA, JENDELE E CERVENKA [15]

**Table 2**  
Yielding strength for the used bars

Reinforcement bar	Yielding strength (MPa)	Hardening
φ 32 mm	390	No
φ 25 mm	396	No
φ 10 mm	493	Yes
φ 8 mm	483	No
φ 6 mm	291	No

lated without adhesion to the concrete, being considered only anchored in the lateral steel plates. Table [2] presents the characteristics of the bars used, taking into account that a bilinear model was selected to simulate the steel behavior.

The Newton-Raphson Method was applied to the solution scheme of the nonlinear system and the boundary conditions and properties of the materials were defined in order to appropriately simulate the experimental setup developed by Godycka [3].

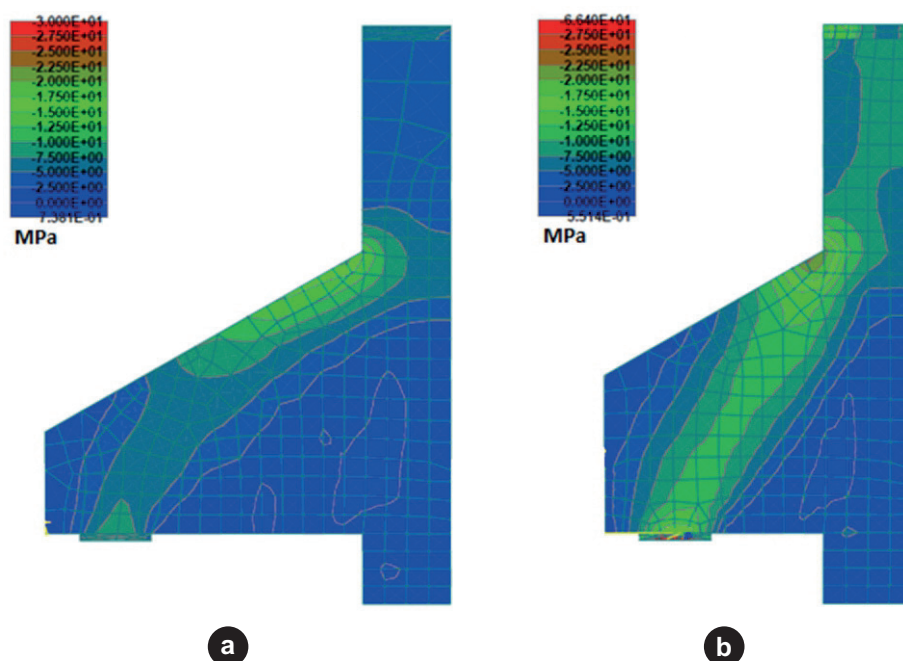
The finite element meshes used for the corbels was generated based on the definition of 6 macro-elements, all of them using a quadrilateral type element with average size of 3 cm. The conditions of symmetry were considered in order to drastically reduce the number of elements and allow faster convergence. The final mesh used for the corbel WI consisted of 289 finite elements and 596 nodes, while the corbel WII had 284 elements and 564 nodes. Figure 6 shows the stress contour for corbel WI-4 (150 kN) and WII-4 (275 kN) in regarding to the experimental load level where the first diagonal cracks appeared. Since both corbels have the same reinforcement ratio, one may observe that corbel WII-4 (275 kN) has cracked for higher stresses when compared to

corbel WI-4 (150 kN), taking into account the differences in their geometries. As the ratio  $a/d$  decreases the stress required to form the first cracks will be probably higher.

Figure 7 shows a comparison between the cracking layout obtained for the WI-4 and WI-2 corbels, for a load level of 150 kN. As can be observed, the application of external prestressing resulted in a decrease in the number of cracks and in the crack openings for the corbel WI-2. The width of the crack openings was about 10 times smaller than those verified for the corbel WI-4. For the corbel WII-2, the application of the external prestressing also reduced the number of cracks, but the size of the main openings was 2.5 times smaller than those of the corbel WII-4.

Both corbels, WI-4 and WII-4, presented the main cracks at the intersection between the columns and the corbels, in the upper face of the corbels. By another hand, with the application of the external prestressing in corbels WI-2 and WII-2, the main cracks did not occur in the region mentioned before. For the prestressed corbels the main cracks occurred in the diagonal strut. This behavior would not be interesting, once the concrete failure by diagonal splitting occurs before the steel yielding (fragile failure). However, one must observe that Godycka [3] had as main objective obtain the corbel failure by the concrete and not by the external prestressing bars.

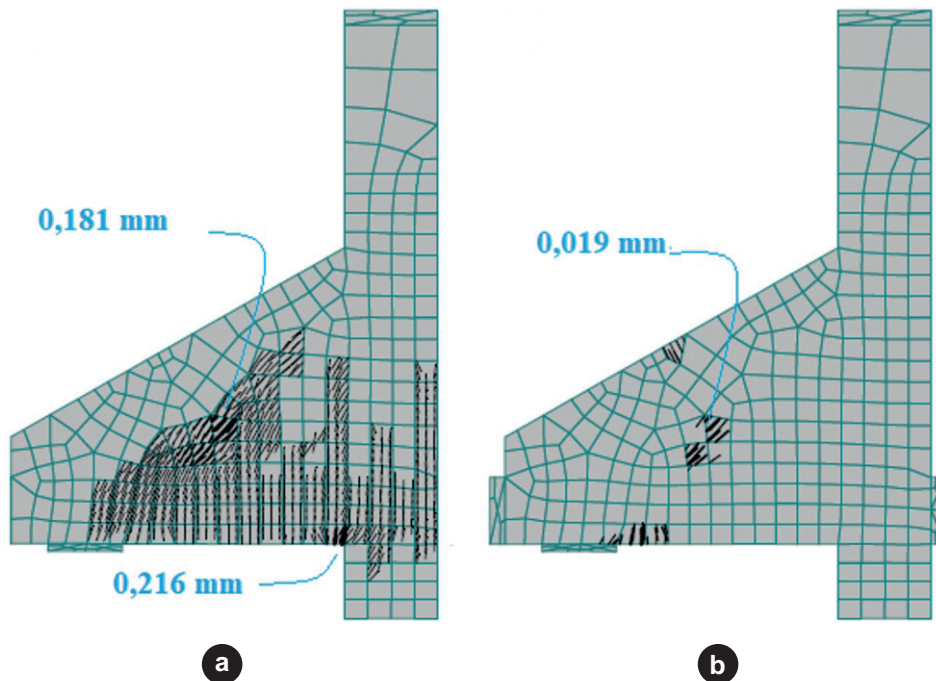
Figure 8 shows the comparison between the numerical cracks obtained using the nonlinear analysis and the experimental cracks obtained by Godycka [3] for the corbel WII-4. Comparing both cracking maps, one can observe that the nonlinear analysis was able to adequately predict the cracking behavior of the corbel WII-4, even estimating the crack openings with good precision. The same good performance was obtained for corbel WI-4. However, it has not been possible to compare the cracking maps



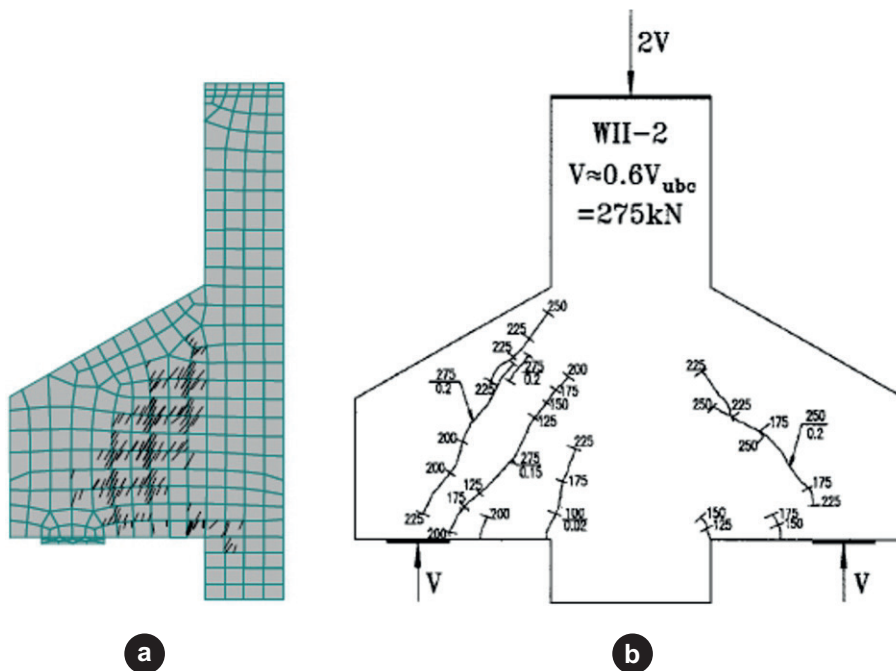
**Figure 6**  
Stress contour for the corbels: a) WI-4 (150 kN), b) WII-4 (275 kN)

for corbels WI-2 and WII-2, once the cracking scenarios for this specimens were not provided by Godycka's. Figure [9] shows the development of stresses in the main tie

reinforcement and in the external bars of the corbel WI-2 according to the applied load. The determination of the stresses in external reinforcement using the nonlinear analysis was

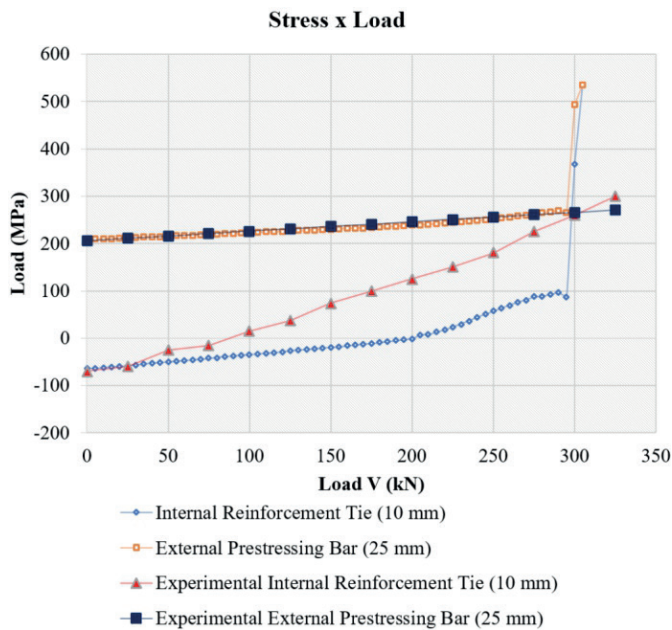


**Figure 7**  
Cracking maps for the corbels: a) WI-4 (150 kN), b) WI-2 (150 kN)



**Figure 8**  
Cracking map of the corbel WII-4: a) nonlinear analysis, b) experimental analysis

Source: Godycka [3]



**Figure 9**  
Stresses in the internal and external reinforcement of the corbel WI-2

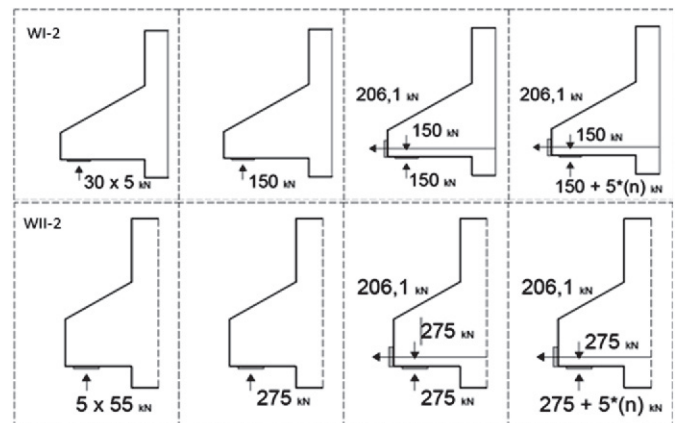
Source: Adapted from Godycka [3]

very close to that one obtained in the experimental results. By another hand, for the internal tie reinforcement, the stress difference was of the order of 100 MPa depending on the applied load.

The loading protocol for corbels WI-4 and WII-4 (corbels not strengthened) consisted in the application of loading steps of 5 kN until the attainment of the failures. For corbel WI-2 the following loading protocol was used: application of 30 loading steps of 5 kN (loading level where the first experimental diagonal cracks occurred); application of a load of 150 kN at the same point and in a direction opposite to that of loading; application of a prestressing force of 206,1 kN in the 25 mm bars and application of loading steps until the attainment of the failure (Figure [10]).

For corbel WII-2 the following loading protocol was used: application of 55 load steps of 5 kN (load where the first diagonal experimental cracks occurred); application of a load of 275 kN at the same point and in the opposite direction to that of loading; application of a prestressing force of 206,1 kN in the 25 mm bars and application of loading steps until failure (Figure [10]).

The predicted failure load using the nonlinear analysis for the corbels WI-4, WI-2, WII-4 and WII-2 was 245 kN, 305 kN, 360 kN and 460 kN respectively, as can be seen in Table [3]. Comparing these values with the experimental ones obtained by Godycka [3], differences between 2% and 24% below the experimental values are observed. These differences may be assumed as satisfactory, since all determinations were in favor of safety. From the nonlinear analysis conducted previously, proposals of strut-and-tie models for the conventional and externally strengthened corbels may be proposed, as follows.



**Figure 10**  
Loading protocol of the corbels WI-2 and WII-2

#### 4. Analytical model for the strengthened corbels

The determination of the load capacity of the short corbel without strengthening (WI-4 and WII-4) was performed according to the strut and tie model proposed by ABNT NBR 6118 [7] (Figure [3]). The strength parameters used for the struts and nodes are the normative parameters described in the item 3 and recommended by ABNT NBR 6118 [7]. In order to compare the results with the available experimental data, a unit security coefficient ( $\gamma_c = 1$ ) was used. The following procedure was used to determine the load capacity of the corbels WI-4 and WII-4: a) Definition of the width of the strut considering the load opening under the support plate; b) Determination of the maximum force acting on the strut taking into account the strength parameters; c) Balance of forces at the top node; d) Checking the stress in the main tie; e) Checking the stresses on the faces of the upper and lower nodes; f) Definition of the load capacity. As one can see it was an iterative procedure and if any verification was not attended the procedure was repeated again, i.e., the balance of forces was redone and the checks of the struts, nodes and tie verified again, until the convergence.

For the corbels with external prestressing, a model formed by 2 diagonal struts and one tie is proposed, as shown in Figure [11]. Node 1 can be considered as being a CCT node type. Although the external prestressing bars rest on plates behind the nodal zone, it should be remembered that the original internal reinforcement tends to anchor in the nodal zone, causing tensile stress in this area. By another hand, Node 2 will be a CCC node type and will be formed by the confluence of 3 struts.

The definition of the width of the strut was made from the edges of the anchor plates and the loading plate, using the entire available depth of the corbel. The prestressing force applied to the external bars followed the Equation [04], used by Godycka [3].

$$P_s = 0,53 \cdot A_{ps} \cdot f_y \quad (4)$$

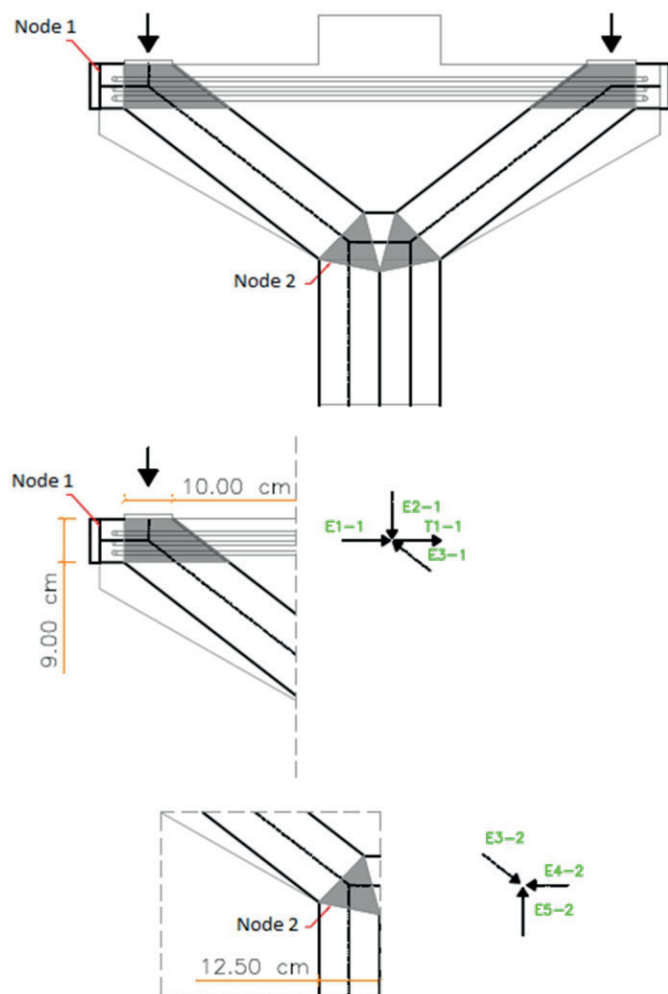
Where:

$P_s$  – Prestressing force;

$f_y$  – Yielding strength of the prestressing steel bar;

$A_{ps}$  – Area of the prestressing steel bar.





**Figure 11**  
Proposed strut and tie model for the corbel with the external prestressing

The load capacity of the corbels WI-4 and WII-4 was calculated at 182,56 kN and 271,79 kN, respectively. Yielding was not observed for the bars and both limit loads were determined by the strength of the diagonal struts. It should be highlighted that from a theoretical point of view, it is always recommended to verify the yielding of the reinforcement before the failure of the struts or nodal regions, in order to have ductile behavior in reinforced/prestressed concrete structures.

For the corbel WI-2 the calculated maximum load was 267,51 kN, which depended on the strength of the diagonal strut. The increase of the effective width of the strut, caused by the anchorage plate of the external bars, led to a higher load in relation to the corbel WI-4, but did not change the failure mode. The presence of external prestressing force in the corbel WI-2 reduced the forces in the internal conventional reinforcement from 345,11 kN to 139,01 kN. The calculated load capacity for the corbel WII-2 was 339,85 kN. As verified for corbel WI-2, the increasing in the width of the diagonal strut did not prevent the failure of the corbel initiating in the concrete. The verification of node 1 demonstrated stresses very

**Table 3**

Failure loads: experimental, nonlinear analysis and strut-and-tie model

Corbel	Experimental [3]	Nonlinear analysis	Strut and tie model
WI-4	250 kN	245 kN (-2%)	183 kN (-27)
WI-2	350 kN	305 kN (-13%)	268 kN (-23%)
WII-4	475 kN	360 kN (-24%)	272 kN (-43%)
WII-2	525 kN	460 kN (-12%)	340 kN (-35%)

close to the code limit strength. By another hand, for node 2, the stresses were kept below the code limit strength. The presence of external prestressing reduced the tensile forces in the internal conventional reinforcement from 284,12 kN to 78,02 kN, accentuating the failure of the concrete strut by diagonal tension. Table 3 presents a comparison of the load capacities of the corbels determined using non-linear analysis, strut and tie model and the load capacities obtained experimentally by Godycka [3].

The values obtained using the truss model based on the parameters recommended by ABNT NBR 6118 [7], were 23% and 35% below the obtained experimental values, demonstrating that the determination of the load capacity by the Strut and Tie Method is adequate, once all the values were in favor of safety.

## 5. Conclusions

The strengthening of reinforced concrete corbels using the installation of post-tensioning external bars is a viable alternative and an effective technique. The proposed solution is able to increase the load capacity of the corbels and also reduce the cracking of this type of structure.

The conducted nonlinear analysis has shown that the development of stresses in the short corbels may be represented by diagonal struts and a main tie, even for the corbels that received post-tensioned external bars. However, it should be noted that in the design situation a ductile mode of failure should always be sought. The ductile mode of failure is characterized by the yielding of the reinforcement prior to the failure of the struts or nodal regions. For the experimental results obtained by Godycka [3], there was an intention of having failure initiating by the struts, and both nonlinear analysis and the proposed strut and tie model were able to detect this fragile behavior.

The cracking of the corbels was influenced by the external prestressing, which by its turn altered the location of occurrence of the larger cracks. The width of the crack was also affected, being about 10 times smaller, depending on the  $a/d$  ratio, when compared to the cracks width of the corbels without strengthening.

The determination of the load capacity of the corbels using nonlinear analysis was very effective, leading to values between 2 and 24% below the values obtained experimentally by Godycka [3]. This fact demonstrates that the nonlinear analysis can lead to values in favor of safety for the limit loads (cracking, yielding and failure), serving as virtual laboratory for the verification of complex structures.

The determination of the load capacity by the proposed strut and tie model based on the parameters of ABNT NBR 6118 [7] led to lower values than those obtained experimentally by Godycka [3], even with the use of  $\gamma_c = 1$ . If the safety factors are applied, the safety level would be even higher, and the service load condition could be assumed satisfied, a general doubt when using the Strut and Tie Method for design.

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