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Design of compression reinforcement in reinforced concrete membrane

Dimensionamento das armaduras de compressão em chapas de concreto armado

T. F. SILVA [a]
tfreitassilva85@gmail.com

J. C. DELLA BELLA [b]
dbella@usp.br

Abstract

This paper presents a method to design membrane elements of concrete with orthogonal mesh of reinforcement which are subject to compressive stress. Design methods, in general, define how to quantify the reinforcement necessary to support the tension stress and verify if the compression in concrete is within the strength limit. In case the compression in membrane is excessive, it is possible to use reinforcements subject to compression. However, there is not much information in the literature about how to design reinforcement for these cases. For that, this paper presents a procedure which uses the model based on Baumann's [1] criteria. The strength limits used herein are those recommended by CEB [3], however, a model is proposed in which this limit varies according to the tensile strain which occur perpendicular to compression. This resistance model is based on concepts proposed by Vecchio e Collins [2].

Keywords: concrete, design, membrane, compression reinforcement.

Resumo

Este artigo apresenta métodos de dimensionamento analíticos de armaduras de compressão para chapas de concreto com malha de armadura ortogonal. Os métodos de dimensionamento, em geral, propõem formas para quantificar a armadura necessária para equilibrar os esforços de tração e verificar se a compressão no concreto atende ao limite de resistência. Para os casos em que a compressão na chapa é excessiva, uma das soluções possíveis seria a adoção de armaduras que funcionam comprimidas. Entretanto, não há muita informação na literatura para dimensionamento nestas situações. Assim, é apresentado um procedimento para determinação dessas armaduras que se fundamenta no método baseado nos critérios utilizados por Baumann [1]. Neste trabalho são utilizados como limites de resistência à compressão aqueles recomendados pelo CEB [3], porém, é proposto um modelo em que este limite varia de acordo com a deformação de tração que ocorre perpendicularmente a compressão atuante. Este modelo resistente é baseado nos conceitos propostos por Vecchio e Collins [2].

Palavras-chave: concreto armado, dimensionamento, chapas, armaduras de compressão.

[a] *Departamento de Engenharia de Estruturas e Geotécnica, Escola Politécnica da Universidade de São Paulo/EPUSP, tfreitassilva85@gmail.com, Av. Prof. Almeida Prado tv.2, nº 83, Cidade Universitária, São Paulo, Brasil;*

[b] *Departamento de Engenharia de Estruturas e Geotécnica, Escola Politécnica da Universidade de São Paulo/EPUSP, dbella@usp.br, Av. Prof. Almeida Prado tv.2, nº83, Cidade Universitária, São Paulo, Brasil.*

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1. Introduction

In any reinforced concrete structure, the best efficiency of the reinforcement is attained when it is placed in the principal tensile stress direction. However, in the case of membranes, this assumption is rarely satisfied. For each combination of loading and at each point of the structure, there is a principal tensile direction. Therefore, there are rare cases in which it is possible to determine a single position of the reinforcement which would be in its best condition.

Furthermore, structures are usually subdivided into many membranes elements where the stresses are evaluated. It is constructively inadequate that the position of reinforcement to be different for each element region. Typically, the reinforcement is placed on the structure in a pattern which makes the construction easier. This paper only discusses the cases of orthogonal positioning of the reinforcement, because it is the most common and constructively simpler. For these reasons, the direction of reinforcement, in general, does not coincide with the direction of principal tensile stress in membranes.

Due to the aspects described, the design for ULS, i.e., the quantification of the reinforcement and verification of compressive stress in concrete, is not easy. However, this problem has been studied by many researchers and there are some methods for resolution. One of the first solutions was provided by Baumann [3] in 1972. He assumes some hypotheses that make his model one of the simplest to operate.

The solutions proposed for ULS consider cases in which the reinforcement is under tension. An also important issue is how to design membranes which compressive stress in concrete does not satisfy the strength limit. Possible solutions to this problem are increasing the strength of concrete, increasing membrane thickness or adopting reinforcement which resists compressive stress.

The objective of this paper is to obtain criteria to use and to design membranes in the ULS with orthogonal grid reinforcement with at least one of the directions of reinforcement submitted to compression. For this study, the method based on Baumann's criteria [1] will be used as a basis. The formulation presented in this paper can be found with more details in Silva [18].

2. Brief history about membrane design

Researchers have long studied the problem of membrane design. Nielsen [4] proposed a model based on the cracked membrane concept, in which the reinforcement resists only axial stress and the concrete is subjected to compressive stress. Baumann [1], in 1972, was probably the first to develop equations that satisfy both the equilibrium and the compatibility of the membrane. His model is based on the premise that there is no shear stress along the cracks. The solutions reached by Baumann [1] and Nielsen [4] are the same, but deduced from different models.

Gupta [5] uses Baumann's model to obtain equations that allow ULS design. Moreover, he solved the problem of obtaining the minimum amount of reinforcement necessary and the minimum compression in concrete.

Vecchio and Collins [6] executed an experiment in which thirty reinforced concrete panels, with different amounts of reinforcement in two directions were subjected to several in-plane loadings.

Fialkow [7] adapts the proposed criteria in ACI 318-77 Building Code [8] of the American Concrete Institute (ACI) to design linear elements for membrane elements, considering not only the reinforcement axial strength and the compressive strength of concrete, but also the shear strength provided by the concrete and the reinforcement.

Based on experiments by Peter [10] and Vecchio and Collins [6], Gupta and Akbar [9] present a model in order not to only design membranes, but also to predict their response when subjected to a set of loads. Gupta and Akbar [9] divide the response of the membrane into four distinct stages. At the first, the concrete is uncracked and the reinforcement has elastic behavior. At the second, the concrete is cracked and the reinforcement in both directions has elastic behavior. At the third, the concrete still cracked and reinforcement in one direction yields. Finally, at the fourth stage, concrete is cracked and the reinforcements of both directions yield. At the first stage, the element has elastic behavior. The last stage refers to the element in the ultimate limit state, a problem for which there were already some solutions. Gupta e Akbar [9] present solutions that allow predicting the behavior of the membrane for the intermediate stages. To do so, they use some simplifying assumptions to the problem such as the non-existence of shear stress between the cracks. They mention the concept of rotation of cracks, which consists in the change of the cracks direction as the load increases.

Vecchio and Collins [2] propose the Modified Compression Field Theory (MCFT). This model considers the effect of tension-stiffening, presupposes the existence of shear stress in the crack, but only transmitted by aggregate interlock and also considers the softening of cracked concrete. Because it is more realistic, considering more variables, it is more complex, but achieves satisfactory results.

Currently, some researchers published studies on this subject. In his work, Chen [11] compiles some of the above design methods, such as the one based on the criteria proposed by Baumann [1], Nielsen [4] and elaborated by Fialkow [7].

Jazra [12] compares the MCFT with the method based on the Baumann's criteria, and presents some formulations for designing compression reinforcement for membranes.

Pereira [17] uses the equations to design membranes to calculate shells obtaining stresses from a finite element model.

3. Method based on Baumann's criteria

The design method based on the Baumann's criteria is probably the simplest to use to design membranes. For this reason, it was chosen as the basis of this study.

The method itself has no solution for the case of adopting compression reinforcement, but it will be used to propose a formulation and criteria to use this reinforcement.

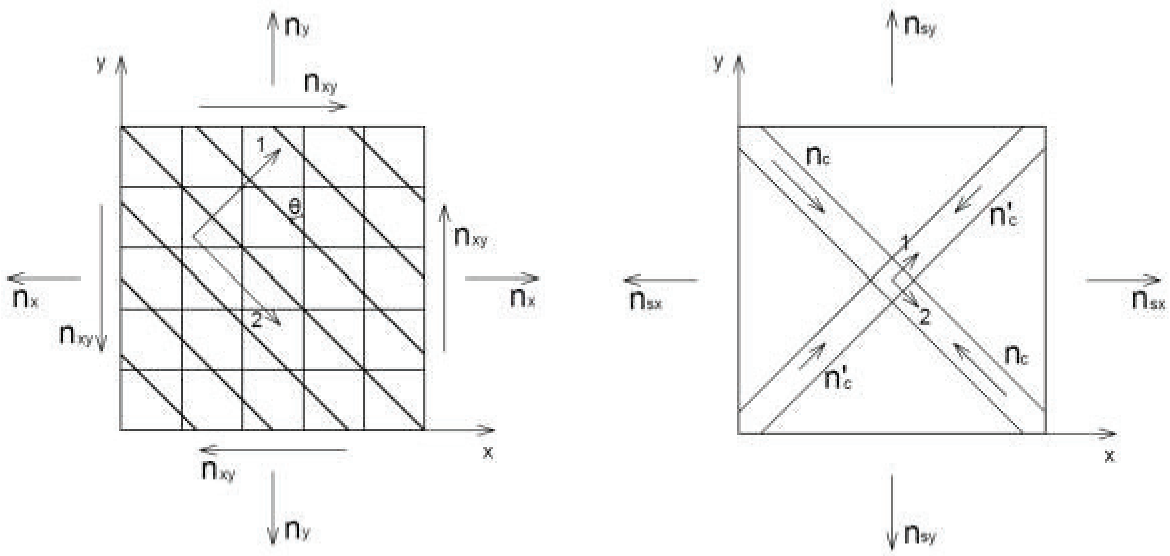
Jazra [12] compares this method with the MCFT proposed by Vecchio and Collins [2] and observes that the design obtained with Baumann`s criteria results higher stress in concrete. This conclusion was expected, because this formulation adopts some hypotheses assuming this result.

Although adequate for ULS design, it will not obtain the same efficacy in predicting characteristics to verify the SLS, such as strain and cracking.

The basic hypotheses are:

1. The cracks are approximately parallel and straight.
2. The tensile strength of concrete is null

Figure 1 – Positive membrane forces and axes considered in this paper



3. The dowel effect will not be considered
4. The effect due to aggregate interlock will not be considered
5. The bond between reinforcement and concrete is perfect
6. The tension-stiffening effect will not be considered
7. The directions of principal strains and the directions of principal stresses coincide

Considering a membrane element subjected to normal forces per unit length, n_x and n_y, and shear force also per unit length, n_{xy}, wherein the reinforcements are positioned in the direction of the x and y axes. Angle θ is formed between the principal direction of compression in concrete and the y direction. In case the element is cracked, angle θ is formed by axis y and the direction of cracks because, as hypothesized, there is no shear stress between the cracks. Thus, the axis of the principal compressive stress in concrete coincides with the axis parallel to cracks. By hypothesis, the principal directions of strain and stress in concrete are considered coincident. This configuration is shown in Figure 1.

Forces n_{sx} and n_{sy} are positive when they represent tension. When n_c and n'_c are compression, they are positive. n'_c is the minimum compression in the element, when existing, and n_c is the maximum compression. Figure 1 also shows this convention.

The problem consists in knowing forces n_x, n_y and n_{xy}, finding the necessary area of reinforcement a_{sx} and a_{sy} and verifying if compressive stress in concrete is below its strength. For this purpose, the equilibrium and compatibility equations will initially be written for the situation in which the reinforcement is subjected to tensile stresses in the x and y directions. This result in expressions 1, 2, 3 and 4, where n_{sx} and n_{sy} are the forces in the reinforcement in the x and y directions, respectively, and ε_y and ε_x are the strains in x and y and ε_c is the strain in concrete in the direction of principal compressive strain.

$$n_{sx}=n_x+n_{xy}.tg\theta \quad (1)$$

$$n_{sy}=n_y+n_{xy}.cotg\theta \quad (2)$$

$$n_c=n_{xy}.(tg\theta+cotg\theta) \quad (3)$$

$$\frac{\varepsilon_y}{\varepsilon_x}=tg^2\theta.\left[1+\frac{\varepsilon_c}{\varepsilon_x}.(1\text{-}cotg^2\theta)\right] \quad (4)$$

From these expressions, it is possible to demonstrate that the minimum reinforcement required to equilibrate the tensile stresses in the membrane occurs when angle θ is equal to 45° in case the reinforcement is subjected to tensile stresses in x and y. The demonstration of this result can be found in several works as in Leonhardt and Mönning [13], Chen [11] and Jazra [12].

3.1 Cases of design

The CEB [3] divides the design of membranes into four cases. Case I considers that the reinforcements are subject to tension in both directions, making n_{sx} and n_{sx} positive.

When one of the forces in the reinforcement takes negative values, i.e. compression, the use of reinforcement in that direction is not necessary. If there is no tensile force in the x direction, case II of design applies. If there is no tensile force in the y direction, applies case III.

When the membrane is completely compressed, case IV is characterized. In this case, reinforcement is not necessary.
This design method does not consider using of compression reinforcement in any case.

3.1.1 Case I – Reinforcement in both directions

For reinforcement to be necessary in both directions, the following conditions must be satisfied.

$$n_{sx}=n_x+|n_{xy}|>0 \quad (5)$$

$$n_{sy}=n_y+|n_{xy}|>0 \quad (6)$$

Thus, the reinforcement area is defined by 7 and 8, where f_{yd} is the yield design stress of steel.

$$a_{sx}=\frac{n_{sx}}{f_{yd}}=\frac{n_x+|n_{xy}|}{f_{yd}} \quad (7)$$

$$a_{sy}=\frac{n_{sy}}{f_{yd}}=\frac{n_y+|n_{xy}|}{f_{yd}} \quad (8)$$

For verifying concrete in case I, f_{cd2} will be used, as suggested by CEB [3] for the compressive strength because, in this case, concrete is cracked. Thus, the expression is as follows.

$$\frac{2|n_{xy}|}{h}=f_{cd2}=0,6\left(1-\frac{f_{ck}}{250}\right).f_{cd} \quad (9)$$

3.1.2 Case II – Reinforcement only in the y direction

In order to use reinforcement only in the y direction, n_{sx} must be negative while n_{sy} must be positive, satisfying the following expressions.

$$n_{sx}=n_x+|n_{xy}|\leq 0 \quad (10)$$

$$n_{sy}=n_y+|n_{xy}|>0 \quad (11)$$

Thus, eliminating the reinforcement in the x direction, angle θ will no longer be 45 ° as mentioned, but will be determined by equation 12.

$$\theta=\operatorname{arctg}\left(-\frac{n_x}{n_{xy}}\right) \quad (12)$$

Thus, in the x direction, there is no reinforcement. Expression 13 determines the reinforcement in y direction.

$$a_{sy}=\frac{n_{sy}}{f_{yd}}=\frac{n_y-\frac{n_{xy}^2}{n_x}}{f_{yd}} \quad (13)$$

To verify the concrete stress, f_{cd2} will be used for the same reason given for case I, as shown by Equation 14:

$$\frac{-n_x-\frac{n_{xy}^2}{n_x}}{h}\leq f_{cd2}=0,6\left(1-\frac{f_{ck}}{250}\right).f_{cd} \quad (14)$$

3.1.3 Case III – Reinforcement only in the x direction

Case III is similar to case II; the only difference is the reinforcement direction. Hence, for reinforcement to be necessary only in the x direction, the following inequations must be satisfied.

$$n_{sx}=n_x+|n_{xy}|>0 \quad (15)$$

$$n_{sy}=n_y+|n_{xy}|\leq 0 \quad (16)$$

Similarly, also in this case, angle θ will not be 45 °, but is determined by equation 17.

$$\theta=\operatorname{arctg}\left(-\frac{n_{xy}}{n_y}\right) \quad (17)$$

In this case, a_{sx} is null and a_{sy} is given by equation 18.

$$a_{sx}=\frac{n_{sx}}{f_{yd}}=\frac{n_x-\frac{n_{xy}^2}{n_y}}{f_{yd}} \quad (18)$$

The verification of concrete stress is given by expression 19.

$$\frac{-n_y-\frac{n_{xy}^2}{n_y}}{h}\leq f_{cd2}=0,6\left(1-\frac{f_{ck}}{250}\right).f_{cd} \tag{19}$$

3.1.4 Case IV – Membrane without reinforcement

In order not to use reinforcement, there can be no tensile stresses in membrane, thus loading conditions shall satisfy the following inequations.

$$n_{sx}=n_x+|n_{xy}|\leq 0 \tag{20}$$

$$n_{sy}=n_y+|n_{xy}|\leq 0 \tag{21}$$

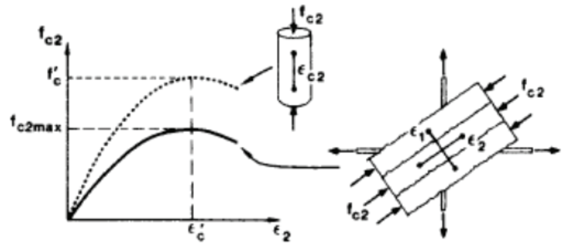
Therefore, in this case, it must only check if the compressive stress in the concrete is less than the limit strength. For this verification, differently from the other cases, it is used as a reference value f_{cd1} of CEB [3] for the strength of concrete, because there is no cracking in this case. From membrane equilibrium, this verification can be written as:

$$\left(-\frac{n_x+n_y}{2}+\sqrt{\frac{(n_x-n_y)^2}{4}+n_{xy}^2}\right).\frac{1}{h}\leq f_{cd1}=0,85.\left(1-\frac{f_{ck}}{250}\right).f_{cd} \tag{22}$$

3.2 Considerations about the study of compression reinforcement

Keeping the design cases proposed by CEB [3] in mind, it will be examined which of them is consistent for studying compression reinforcement.
Firstly, in case I, compression reinforcement could only be effectively used if it were arranged in the direction of the cracks, which would help reduce compressive stress in the concrete. However, in this case, there is already an orthogonal grid of reinforcement, and if another layer of bars is placed in the other direction, the solution would be constructively bad, only recommended in special cases.
Another possibility would be to use a larger reinforcement area than necessary to limit compression strain and to reduce stress in the principal compressive direction. However, this solution would lead to a brittle rupture, because concrete would collapse before reinforcements yield.
For cases II and III, it is reasonable to think that placing reinforcement in the direction that it was not reinforced, it will affect the stress field in the membrane and it will reduce the compressive stress in the concrete.
For case IV, for which there is no reinforcement, it is evident that if reinforcement is placed appropriately in this membrane, it will help to reduce the compressive stresses.
Therefore, this paper will only study design cases II, III and IV.

Figure 2 – Stress-strain diagram for cracked concrete in compression (Vecchio e Collins, 1986)



4. Strength model and verification of compressive stress in concrete

In the method based on Baumann`s criteria presented, the strength values of concrete follow those recommended by the CEB [3]. However, this imposes a discontinuity of concrete strength between the case in which the concrete is cracked and that in which it is intact. Therefore, this work will be adopted a resistance model for the concrete that optimizes the use of the material. To do so, the objective is to find strength values for the concrete that are between f_{cd1} and $f_{cd2,}$ using the tensile strain that occurs perpendicular to the compressive strain as a parameter.
Vecchio and Collins [2] propose a formulation which includes concrete softening due to cracking differently from CEB [3]. For them, the loss of strength of cracked concrete is related to principal tensile strain ε_1 imposed on the membrane. Equations 23 and 24 define the stress-strain diagram for the compression in the concrete proposed by Vecchio and Collins [2]. Considering the strain value that leads to stress peak in concrete ε'_c=2‰, equation 24 is obtained. Figure 2 shows this model.

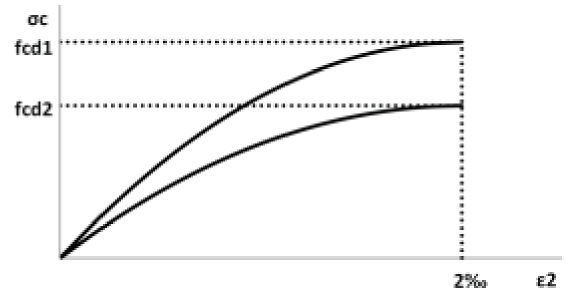
$$f_{c2}=f_{c2max}.\left[2.\left(\frac{\varepsilon_2}{\varepsilon`_c}\right)-\left(\frac{\varepsilon_2}{\varepsilon`_c}\right)^2\right] \tag{23}$$

$$f_{c2max}=\frac{f`_c}{0,8+170.\,\varepsilon_1}\leq f`_c \tag{24}$$

Expression 23 is similar to that suggested by CEB [3] for the stress-strain diagram of concrete, only changing the strength limit. In this paper, it will use the limits proposed by CEB [3], but interpolated by equation 24. Thus, it follows that:

$$\sigma_c=f_{c2max}.\left[2.\left(\frac{\varepsilon_2}{\varepsilon`_c}\right)-\left(\frac{\varepsilon_2}{\varepsilon`_c}\right)^2\right] \tag{25}$$

Figure 3 – Stress-strain diagram adopted



$$f_{cd2} \leq f_{c2max} = \frac{f_{cd1}}{0{,}8 + 170.\,\varepsilon_1} \leq f_{cd1} \quad (26)$$

The maximum strain used in this work is the same as that suggested by CEB [3] for zones subjected to axial compression, and this limit is 2 ‰. Thus, Figure 3 shows the stress-strain diagram used in this work.

4.1 Verification of compressive strength of concrete

The strength limit for concrete is herein calculated considering the concepts presented in item 4. Thus, the way to check if the compressive force respects this limit is different from that presented in the method based on Baumann`s criteria, because the concrete capacity now depends on the tensile strain to which the membrane is subjected in ULS. In case IV, verification of concrete is the same of that in item 3, because in this case there is no tension in the membrane and the compressive strength of concrete is always given by f_{cd1}.
For the cases II and III, it should be first checked if:

$$\sigma_c = \frac{n_c}{h} \geq f_{cd1} \quad (27)$$

This study admits that f_{cd1} is the maximum limit for the compressive strength of concrete in any case. If inequation 27 is satisfied, the compressive stress in the concrete is above the limit and, it should thus evaluate the possibility of using compression reinforcement. The way to do this evaluation will be presented in item 5.
For the case in which inequation 27 is not satisfied, it should be verified if:

$$\sigma_c = \frac{n_c}{h} \leq f_{cd2} \quad (28)$$

As f_{cd2} is the lowest limit for the compressive strength of concrete, if expression 28 is satisfied, the compressive stress in the concrete respects the strength limit imposed and it will not therefore be necessary to use compression reinforcement. If inequations 27 and 28 are not satisfied, it consequently follows that:

$$f_{cd2} \leq \frac{n_c}{h} \leq f_{cd1} \quad (29)$$

In this case, the strains in the membrane must be considered to determine the strength limits to be used, because it will depend on ε_1.

4.1.1 Calculation to determine the limit of compressive strength of the concrete

The objective of this item is to find the value of f_{cd2max}. However, it depends on the strain of the membrane. A calculation method based on that presented by Jazra [12] will be presented. This calculation is valid for cases II and III. Due to their being analog, changing just the reinforcement position (y axis to case II and x axis to case III), only case III will be described. For case II, equation 30 must be replaced by the equivalent equation to ε_y and the same process must be repeated. Thus, from Mohr circle, it follows that:

$$\varepsilon_x = \frac{\varepsilon_1 + \varepsilon_2}{2} + \left(\frac{\varepsilon_1 - \varepsilon_2}{2}\right).\cos 2\theta \quad (30)$$

So:

$$\varepsilon_1 = \frac{2.\varepsilon_x - \varepsilon_2.(1 - \cos 2\theta)}{(1 + \cos 2\theta)} \quad (31)$$

Therefore:

$$f_{c2max} = \frac{f_{cd1}}{0{,}8 + 170.\left[\frac{2.\varepsilon_x - \varepsilon_2.(1 - \cos 2\theta)}{(1 + \cos 2\theta)}\right]} \quad (32)$$

From equation 25 it is possible to express the compressive strain as a function of strength.

$$\varepsilon_2 = \varepsilon`_c.\left(1 - \sqrt{1 - \frac{f}{f_{c2max}}}\right) \quad (33)$$

Considering by hypothesis that ε_x is equal to the yield strain of steel, it follows that:

$$\varepsilon_2 = \varepsilon`_c.\left(1 - \sqrt{1 - \frac{\frac{\sigma_c}{f_{cd1}}}{0{,}8 + 170.\left[\frac{2.\varepsilon_{yd} - \varepsilon_2.(1 - \cos 2\theta)}{(1 + \cos 2\theta)}\right]}}\right) \quad (34)$$

For case II, equation 34 modifies and results in expression 35:

$$\varepsilon_2=\varepsilon'_c.\left(1-\sqrt{1-\dfrac{\dfrac{\sigma_c}{f_{cd1}}}{0{,}8+170.\left[\dfrac{2.\varepsilon_{yd}-\varepsilon_2.(1+\cos 2\theta)}{(1-\cos 2\theta)}\right]}}\right) \quad (35)$$

Equations 34 and 35 can be solved by iterative methods. Assuming an initial value to ε_2 for which the function exists (i.e., the radicand will not be negative), it will converge to the solution. If the radicand assumes a negative value in any iteration, then the problem has no solution, and therefore the stress in concrete is higher than the maximum limit.

5. Design of compression reinforcement for cases II and III

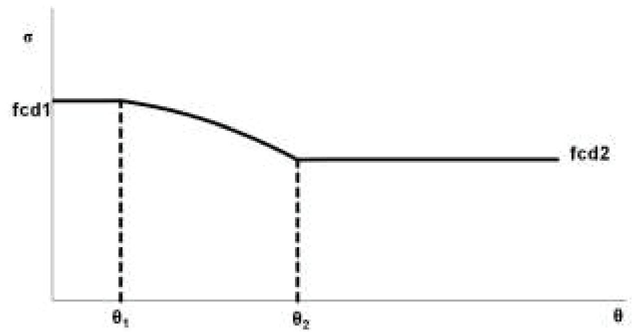
All the demonstrations in this item will be made only for design case III. Case II is analogous and only its final formulation will be presented. For those cases in which the compressive stress in concrete is higher than the strength calculated as shown in item 4, it should be checked if adopting compressed reinforcement in the direction in which there was no reinforcement previously, it will be effective to decreases the stress in concrete so that it will be lower or equal to the strength. First, for the purposes of this problem the same assumptions given by the method based on Baumann`s criteria, presented in item 3, will be used.
Moreover, some considerations about strain must be made. First, it will be admitted that the strain in the x direction is equal to the yield stress in steel. This assumption limits the strain in the membrane, optimizing the compressive strength of the concrete, besides resulting in a reinforcement area in which ductile rupture in ULS occurs. In other words, even if to solve the problem it would be necessary to over-reinforce the membrane, this result will be discarded because the membrane would collapse in a brittle way.
Furthermore, it is assumed that strain ε_2 is always equal to ε'_c, leading the concrete to the strength limit and, consequently, reducing the consumption of reinforcement. Summarizing the hypotheses, it follows that:

1. The cracks are approximately parallel and straight.
2. The tensile strength of concrete is null
3. The dowel effect will not be considered
4. The effect due to aggregate interlock will not be considered
5. The bond between reinforcement and concrete is perfect
6. The tension-stiffening effect will not be considered
7. The directions of principal strains and the directions of principal stresses coincide
8. The strain in x direction is equal to the yield strain of steel ($\varepsilon_x = \varepsilon_{yd}$).
9. The principal compressive strain is equal to the strain resulting in the peak stress in concrete ($\varepsilon_2 = \varepsilon'_c$).

5.1 Design limits

With these hypotheses, it is intended to determine the cases in which it is possible to design compression reinforcement. Thus, firstly, a membrane subjected to stresses such that tensile reinforcement in the y direction is not necessary, therefore, it lies in design case III, and the compressive stress in the concrete is higher than the strength f_{c2max}, as shown in item 4 is assumed. As hypothetically $\varepsilon_x = \varepsilon_{yd}$ and $\varepsilon_2 = \varepsilon'_c$, the compressive strength of concrete is given by equation 36.

Figure 4 – Compressive strength of concrete as a function of θ to case III



$$f_{c2max}=\frac{f_{cd1}}{0{,}8+170.\left[\dfrac{2.\varepsilon_{yd}-\varepsilon'_c.(1-\cos 2\theta)}{(1+\cos 2\theta)}\right]} \quad (36)$$

In which:

$$f_{cd2}\leq f_{c2max}\leq f_{cd1}$$

The graph that describes the strength as a function of θ is shown in Figure 4. For case III, all the functions of θ have domain 0 ≤ θ ≤ |45º|. For case II, the domain is |45º| ≤ θ ≤ |90º|.
It can determine the values of θ_1 and θ_2 shown in Figure 4. Angle θ_1 is the one which equates f_{c2max} at f_{cd1}. Thus, it can be demonstrated that:

$$\theta_1=\frac{\arccos\left(\dfrac{0{,}00118-2.\varepsilon_{yd}+\varepsilon'_c}{(\varepsilon'_c-0{,}00118)}\right)}{2} \quad (37)$$

As the cosine function produces the same result, no matter the angle signal, both positive and negative θ_1 are solutions. Similarly, θ_2 is the value that equates f_{c2max} at f_{cd2}. Thus, it follows that:

$$\theta_2=\frac{\arccos\left(\dfrac{0{,}003627-2.\varepsilon_{yd}+\varepsilon'_c}{(\varepsilon'_c-0{,}003627)}\right)}{2} \quad (38)$$

Also for θ_2, both positive and negative solutions satisfy equation 38. However, if θ exceeds a certain limit, strain ε_y assumes positive values. Thus, the area of reinforcement in y results in negative values, which is not physically possible. As by hypothesis $\varepsilon_x = \varepsilon_{yd}$ and $\varepsilon_2 = \varepsilon'_c$, it can calculate to what values of θ, ε_y is lower than 0. The objective is to find θ* for which $\varepsilon_y = 0$. From Mohr circle, it follows that:

$$\varepsilon_1+\varepsilon_2=\varepsilon_x+\varepsilon_y \quad (39)$$

Then:

$$\varepsilon_1=\varepsilon_{yd}-\varepsilon'_c \quad (40)$$

Also through the Mohr circle, by replacing θ with θ*, it follows that:

$$\varepsilon_x=\frac{\varepsilon_1+\varepsilon_2}{2}+\left(\frac{\varepsilon_1-\varepsilon_2}{2}\right).\cos 2\theta^* \quad (41)$$

Replacing 40 in 41, θ* is given by equation 42:

$$\theta^*=\frac{\arccos\left(\dfrac{\varepsilon_{yd}}{\varepsilon_{yd}-2.\varepsilon'_c}\right)}{2} \quad (42)$$

Table 1 shows the values of θ_1, θ_2 and θ* to steels determined by NBR 6118 [14]. It can be observed that for steels CA-50 and CA-60 there are no values of θ_1. This is because for strain values assumed by hypothesis for this problem, the strength of the concrete never reaches the value of f_{cd1} for these steels. Thus, the strength of concrete reaches its maximum when θ = 0°. Thus, it equalizing strength with stress in concrete, if θ1 exists and θ = 0, then nxy = 0 and:

$$f_{cd1}=\frac{n_c}{h} \quad (43)$$

If θ_1 exists and 0 < θ ≤ $|\theta_1|$, then:

$$f_{cd1}=\frac{2.n_{xy}}{h.\text{sen}(2\theta)} \quad (44)$$

Table 1 – Values of θ1, θ2 and θ* for steels prescribed by NBR 6118 to case III

	ε_{yd} (‰)	$\lvert\theta_1\rvert$ (°)	$\lvert\theta_2\rvert$ (°)	$\lvert\theta^*\rvert$ (°)
CA-25	1,04	12,17	42,74	39,07
CA-50	2,07	DOES NOT EXIST	31,74	35,03
CA-60	2,48	DOES NOT EXIST	26,79	33,74

If θ_1 does not exist and θ = 0, so n_{xy} = 0 and:

$$\frac{f_{cd1}}{0{,}8+170.\left[\dfrac{2.\varepsilon_{yd}-\varepsilon'_c.(1-\cos 2\theta)}{(1+\cos 2\theta)}\right]}=\frac{n_c}{h} \quad (45)$$

If θ_1 exists and $|\theta_1|$< θ < $|\theta_2|$ or if θ1 does not exist and 0 < θ < $|\theta_2|$, then:

$$\frac{f_{cd1}}{0{,}8+170.\left[\dfrac{2.\varepsilon_{yd}-\varepsilon'_c.(1-\cos 2\theta)}{(1+\cos 2\theta)}\right]}=\frac{2.n_{xy}}{h.\text{sen}(2\theta)} \quad (46)$$

If $|\theta_2|$ ≤ θ < |θ*|, then:

$$f_{cd2}=\frac{2.n_{xy}}{h.\text{sen}(2\theta)} \quad (47)$$

Considering this situation, the objective is to find for which values of forces it is possible to design compression reinforcement. For normal forces, there is no mathematical limit, there is only constructive limit to the reinforcement ratio prescribed by NBR 6118 [14]. For shear force, there is a limit, but the formulation of which varies with the kind of steel adopted. This is because limits θ_1, θ_2 and θ* are different for each steel. For CA-25, as |θ*| <$|\theta_2|$, equation 47 will never be valid. Thus, it should be known which maximum value of n_{xy} can be assumed for this steel. Then, taking back equation 46, it is possible to demonstrate that:

$$|n_{xy}|\leq\frac{f_{cd1}.h}{2}.\left(\frac{\text{sen}(2.|\theta_{xy}|)}{0{,}8+170.\left[\dfrac{2.\varepsilon_{yd}-\varepsilon'_c.(1-\cos(2.|\theta_{xy}|))}{(1+\cos?\,(2.|\theta_{xy}|))}\right]}\right) \quad (48)$$

In which:

$$|\theta_{xy}|=33{,}76° \quad (49)$$

For CA-50 and CA-60, as |θ*| > $|\theta_2|$, equation 47 is valid. From it, it is possible to demonstrate that:

$$|n_{xy}|\leq\frac{f_{cd2}.h.\text{sen}(2|\theta^*|)}{2} \quad (50)$$

Therefore, if n_{xy} respects the condition imposed by 48 or 50, the problem has a solution, in other words, there is a reinforcement which will decreases stress in concrete until its maximum strength. Table 2 shows the maximum values of θ for each kind of steel.

Table 2 – Maximums values for θ_{max} in case III

	ε_{yd} (‰)	θ_{max} (°)
CA-25	1,04	33,76
CA-50	2,07	35,03
CA-60	2,48	33,74

5.2 Design reinforcement in case III

Assuming a membrane that is subjected to forces which respects the conditions imposed by equations 48 and 50, the intention is to calculate the amount of reinforcement necessary to be positioned in the y direction in order to compressive stress in concrete is equal to the maximum strength f_{c2max}. This calculation method was based that presented by Jazra [12]. Therefore, with equations 43, 44, 45, 46 and 47 the intention is to find the value of θ which is the solution to the problem. First, if nxy = 0, then θ = 0°. If θ_1 exists and $0 < \theta \leq |\theta_1|$, so:

$$\theta=\frac{\arcsen\left(\frac{2.n_{xy}}{f_{cd1}.h}\right)}{2} \quad (51)$$

If θ_1 exists and $|\theta_1| < \theta < |\theta_2|$ or if θ_1 does not exist and $0^o < \theta < |\theta_2|$, then:

$$\theta=\frac{\arcsen\left(\frac{\frac{2.n_{xy}}{f_{cd1}.h}}{0{,}8+170.\left[\frac{2.\varepsilon_{yd}-\varepsilon'_c.(1-\cos 2\theta)}{(1+\cos 2\theta)}\right]}\right)}{2} \quad (52)$$

Finaly, if $|\theta_2| \leq \theta < |\theta^*|$, then:

$$\theta=\frac{\arcsen\left(\frac{2.n_{xy}}{f_{cd2}.h}\right)}{2} \quad (53)$$

Therefore, to find θ, it should follow the steps.

1. If $n_{xy} = 0$, θ = 0.
2. If $n_{xy} \neq 0$, use the iterative method to find θ through equation 52.
3. If converges, for CA-25, two solutions can be found, but it is only valid that one which it respects θ <θmax.
4. If converges, for CA-25, check if θ ≤ θ1. Because equation 52 is not valid for this domain, θ must be found using equation 51.
5. If converges, for CA-50 and CA-60, θ found is solution.
6. If does not converge, find the solution using equation 53.

With values of $\varepsilon_x = \varepsilon_{yd}$, $\varepsilon_2 = \varepsilon'_c$ and θ, it is possible to obtain the value of ε_1 and ε_y. Taking back equation 31 and 39, then:

$$\varepsilon_y=\frac{2.\varepsilon_{yd}-\varepsilon'_c.(1-\cos 2\theta)}{(1+\cos 2\theta)}+\varepsilon'_c-\varepsilon_{yd} \quad (54)$$

It is possible to calculate the forces in the reinforcement using equations 1 and 2. The reinforcements are given by:

$$a_{sx}=\frac{n_{sx}}{\sigma_x}=\frac{n_{sx}}{E_{cs}.\varepsilon_x}=\frac{n_{sx}}{E.\varepsilon_{yd}}=\frac{n_{sx}}{f_{yd}} \quad (55)$$

$$a_{sy}=\frac{n_{sy}}{\sigma_y}=\frac{n_{sy}}{E_{cs}.\varepsilon_y} \quad (56)$$

5.3 Design reinforcement in case II

In this item, the formulation for designing compression reinforcement in case II will be presented. As already exposed, the demonstration of equations is the same for case III and only the final equations will be presented here. Thus, for case II, the design limits presented in item 5.1 also must be considered, but the domain of functions of θ is |45°| ≤ θ ≤ |90°|.

In this case, $\varepsilon_y = \varepsilon_{yd}$. By adapting expression 36, f_{c2max} is represented by equation 57.

$$f_{c2max}=\frac{f_{cd1}}{0{,}8+170.\left[\frac{2.\varepsilon_{yd}-\varepsilon'_c.(1+\cos 2\theta)}{(1-\cos 2\theta)}\right]} \quad (57)$$

Thus, for case II, the limits of θ assume the values shown in Table 3. For CA-25, it follows that:

$$|n_{xy}|\leq\frac{f_{cd1}.h}{2}.\left(\frac{\mathrm{sen}(2.|\theta_{max}|)}{0{,}8+170.\left[\frac{2.\varepsilon_{yd}-\varepsilon'_c.(1+\cos(2.|\theta_{max}|))}{(1-\cos(2.|\theta_{max}|))}\right]}\right) \quad (58)$$

For CA-50 e CA-60, it follows that:

$$|n_{xy}|\leq\frac{f_{cd2}.h.\mathrm{sen}(2|\theta_{max}|)}{2} \quad (59)$$

If the shear force to which the membrane is subjected respects condition 58 or 59, then the value of θ must be found by using the

Table 3 – Values of θ_1, θ_2 e θ^* and θ_{max} for steels prescribed by NBR 6118 in case II

	ε_{yd} (‰)	$\|\theta_1\|$ (°)	$\|\theta_2\|$ (°)	$\|\theta^*\|$ (°)	θ_{max} (°)
CA-25	1,04	77,83	47,26	50,93	56.24
CA-50	2,07	DOES NOT EXIST	58,26	54,97	54,97
CA-60	2,48	DOES NOT EXIST	63,21	56,26	56,26

same steps used in case III, but using the formulation found herein. First, if n_{xy} = 0, then θ = 90 °. If θ_1 exists and $|\theta_1| \leq \theta < 90$ °, then:

$$\theta=\frac{\arcsen\left(\frac{2.n_{xy}}{f_{cd1}.h}\right)}{2} \quad (60)$$

If θ_1 exists and $|\theta_2| < \theta < |\theta_1|$ or if θ_1 does not exist and $|\theta_2| < \theta < 90°$, then:

$$\theta=\frac{\arcsen\left(\frac{\frac{2.n_{xy}}{f_{cd1}.h}}{0{,}8+170.\left[\frac{2.\varepsilon_{yd}-\varepsilon'_c.(1+\cos 2\theta)}{(1-\cos 2\theta)}\right]}\right)}{2} \quad (61)$$

Finally, if $|\theta^*| < \theta \leq |\theta_2|$, then:

$$\theta=\frac{\arcsen\left(\frac{2.n_{xy}}{f_{cd2}.h}\right)}{2} \quad (62)$$

For case II, $\varepsilon_{yd} = \varepsilon_y$. So, with $\varepsilon_2 = \varepsilon'_c$ e θ, it is possible to find the value of ε_1 and then ε_x. Thus, it follows that:

$$\varepsilon_x=\frac{2.\varepsilon_{yd}-\varepsilon'_c.(1+\cos 2\theta)}{(1-\cos 2\theta)}+\varepsilon'_c-\varepsilon_{yd} \quad (63)$$

It is possible to calculate the forces in the reinforcement using equations 64 and 65. The reinforcements are given by:

$$a_{sx}=\frac{n_{sx}}{\sigma_x}=\frac{n_{sx}}{E.\varepsilon_x} \quad (64)$$

$$a_{sy}=\frac{n_{sy}}{\sigma_y}=\frac{n_{sy}}{E.\varepsilon_y}=\frac{n_{sy}}{E.\varepsilon_{yd}}=\frac{n_{sy}}{f_{yd}} \quad (65)$$

6. Design of compression reinforcement to case IV

Case IV is different from cases II and III because the concrete strength conditioning is not the same. As in this case the membrane is in biaxial compression state, the concrete strength could be even higher than the value of f_{cd1}, as recommended by the CEB [3]. However, in this study, concrete strength will be considered equal to f_{cd1}. The objective of the formulation that will be presented is to design the reinforcement in x and y directions for membranes in which stress in concrete is higher than its strength.

First, it is assumed that stress in concrete is equal to its limit in ULS. Another problem hypothesis is that the membrane is always in biaxial compression state; in other words, the inclusion of reinforcement which leads to tensile stress in membrane will not be contemplated by this study. Thus, the hypotheses are:

1. The tensile strength of concrete is null
2. The bond between reinforcement and concrete is perfect
3. The membrane is always in biaxial compression state
4. The directions of principal strains and the directions of principal stresses coincide
5. The concrete strength is given by f_{cd1}
6. The principal compressive strain is equal to the strain resulting in the peak stress in concrete ($\varepsilon_2 = \varepsilon'_c$). Thus, the principal compressive stress is equal to the compressive strength ($\sigma_c = f_{cd1}$).
7. The effect due to aggregate interlock will not be considered
8. The tension-stiffening effect will not be considered

By equilibrium of membrane, the following expressions are obtained where n'_c is the force in the direction of minimum compression.

$$n_c=-n_x+n_{xy}.\mathrm{cotg}\theta+n_{sx} \quad (66)$$

$$n_c=n_{sy}-n_y+n_{xy}.\mathrm{tg}\theta \quad (67)$$

$$n_c=n'_c+n_{xy}.(\mathrm{tg}\theta+\mathrm{cotg}\theta) \quad (68)$$

$$n_c=-\frac{(n_x-n_{sx})+(n_y-n_{sy})}{2}+\sqrt{\frac{((n_x-n_{sx})-(n_y-n_{sy}))^2}{4}+n_{xy}^2} \quad (69)$$

$$n'_c=-\frac{(n_x-n_{sx})+(n_y-n_{sy})}{2}-\sqrt{\frac{((n_x-n_{sx})-(n_y-n_{sy}))^2}{4}+n_{xy}^2} \quad (70)$$

6.1 Design limits

The objective in this item is to define the cases for which it is possible to design compression reinforcement. By hypothesis, the membrane is always in biaxial compression state, then $n'_c \geq 0$. Thus, from equation 68, it follows that:

$$\mathrm{sen}2\theta \geq \frac{2.n_{xy}}{n_c} \quad (71)$$

Then:

$$|n_{xy}| \leq \frac{f_{cd1}.h}{2} \quad (72)$$

Equation 72 means an absolute limit to n_{xy}
As the assumptions define only one fixed strain, there are infinite solutions within a range. The parameter that defines this interval is θ. Thus, it is interesting to delimit the angles θ which are possible to be assigned to the problem. Thus, using equation 71, it can be demonstrated that:

$$\theta_{c1} \leq \theta \leq \theta_{c2} \quad (73)$$

In which:

$$\theta_{c1} = \frac{\operatorname{arcsen}\left(\frac{2.n_{xy}}{f_{cd1}.h}\right)}{2} \text{ to } 0^{\circ} \leq \theta \leq |45^{\circ}| \quad (74)$$

$$\theta_{c2} = \frac{\operatorname{arcsen}\left(\frac{2.n_{xy}}{f_{cd1}.h}\right)}{2} \text{ to } |45^{\circ}| \leq \theta \leq |90^{\circ}| \quad (75)$$

Besides this criterion, by hypothesis the membrane is always in biaxial compression state, the strains in any direction are always negative. Thus, in order to not obtain reinforcement area with a negative sign, which is an incongruity, the reinforcement forces should also be negative. Hence, in order to $n_{sx} \leq 0$, θ must respect the following premise.

$$\theta \leq \theta_x = \operatorname{arctg}\left(\frac{n_{xy}}{n_x + n_c}\right) \quad (76)$$

Similarly, in order to $n_{sx} \leq 0$, the following criteria must be followed.

$$\theta \geq \theta_y = \operatorname{arctg}\left(\frac{n_c + n_y}{n_{xy}}\right) \quad (77)$$

6.2 Reinforcement design

The method that will be presented was based on Jazra [12]. First, the forces to which the membrane is submitted must respect the equation 72. Once this criterion is verified, it must arbitrate a value of θ such that it respects the limits imposed by inequations 73, 76 and 77. There will be infinite values possible to θ, but just one will lead to a minimal reinforcement area. Thus, it follows that:

$$n_{sx} = n_c + n_x - n_{xy}.\operatorname{cotg}\theta \quad (78)$$

$$n_{sy} = n_c + n_y - n_{xy}.\operatorname{tg}\theta \quad (79)$$

With n_{sx} and n_{sy}, it is possible to calculate n'_c through expression 70. As the direction of the principal stress is the same of the direction of the principal strain by hypothesis, force n'_c is related to strain ε_1. These terms are related by the constitutive model for the concrete. Therefore:

$$\varepsilon_1 = \varepsilon`_c.\left(1 - \sqrt{1 - \frac{n'_c}{f_{cd1}.h}}\right) \quad (80)$$

Obtaining the value of ε_1 and as the value of $\varepsilon_2 = \varepsilon'_c$ and θ is known, it is possible to calculate ε_x and ε_y through expressions 81 and 82, obtained by the Mohr circle.

$$\varepsilon_x = \frac{\varepsilon_1 + \varepsilon_2}{2} + \left(\frac{\varepsilon_1 - \varepsilon_2}{2}\right).\cos 2\theta \quad (81)$$

$$\varepsilon_y = \frac{\varepsilon_1 + \varepsilon_2}{2} - \left(\frac{\varepsilon_1 - \varepsilon_2}{2}\right).\cos 2\theta \quad (82)$$

Thus, reinforcement is designed by:

$$a_{sx} = \frac{n_{sx}}{\sigma_x} = \frac{n_{sx}}{E_{cs}.\varepsilon_x}$$

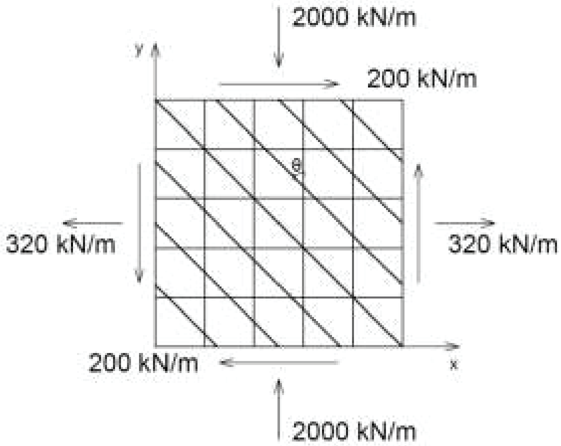
$$a_{sy} = \frac{n_{sy}}{\sigma_y} = \frac{n_{sy}}{E_{cs}.\varepsilon_y}$$

The reinforcement areas obtained are not necessarily the minimum. Therefore, attempts must be made to find this minimum.

7. Example of design

Assuming a membrane with 12cm thickness, f_{ck} equal to 25 MPa, CA-50 and subjected to forces per unit length as shown in Figure 5. First, it must be verified which design case this problem belongs to. In order to do that, it must calculate forces n_{sx} and n_{sy} from equations 83 and 84.

Figure 5 – Example of design



$$n_{sx}=n_x+|n_{xy}| \quad (83)$$

$$n_{sy}=n_y+|n_{xy}| \quad (84)$$

Then:

$$n_{sx}=n_x+|n_{xy}|=320+|200|=500\ kN/m$$
$$n_{sy}=n_y+|n_{xy}|=-2000+|200|=-1800\ kN/m$$

From inequations 15 and 16, as $n_{sx}>0$ and $n_{sy}\leq 0$, it is not necessary to use tensile reinforcement in y direction, which characterizes design case III. Then, first, concrete stress must be verified.

$$\theta=arctg\left(-\frac{n_{xy}}{n_y}\right)=arctg\left(-\frac{200}{-2000}\right)=5{,}71°$$

$$n_c=n_{xy}.(tg\theta+cotg\theta)$$

$$n_c=200.(tg5{,}71°+cotg5{,}71°)=2020\ kN/m$$

$$\sigma_c=\frac{n_c}{h}=\frac{2020}{0{,}12}=16833{,}33\ \frac{kN}{m^2}=16{,}83\ MPa$$

$$f_{cd1}=0{,}85.\left(1-\frac{f_{ck}}{250}\right).f_{cd}=0{,}85.\left(1-\frac{25}{250}\right).\frac{25}{1{,}4}=13{,}66\ MPa$$

$$f_{cd2}=0{,}60.\left(1-\frac{f_{ck}}{250}\right).f_{cd}=0{,}60.\left(1-\frac{f_{ck}}{250}\right).\frac{25}{1{,}4}=9{,}64\ MPa$$

In this case $\sigma_c > f_{cd1}$, the compressive stress in the concrete is higher than the maximum limit of strength. Then, it will check if it is possible design compression reinforcement in the y direction to decrease the compressive stress in concrete.
First, because steel CA-50 is being used, it must verify if n_{xy} respects inequation 85.

$$|n_{xy}|\leq\frac{f_{cd2}.h.sen(2|\theta^*|)}{2} \quad (85)$$

Then, θ* will be calculated through equation 86.

$$\theta^*=\frac{arccos\left(\frac{\varepsilon_{yd}}{\varepsilon_{yd}+2.2}\right)}{2}$$
$$\theta^*=\frac{arccos\left(\frac{2{,}07}{2{,}07+2.2}\right)}{2}=\frac{arccos\left(\frac{2{,}07}{6{,}07}\right)}{2}=35{,}03° \quad (86)$$

Now, it is possible to find the limit to n_{xy}:

$$|n_{xy}|\leq\frac{9642{,}9.0{,}12.|sen(2.35{,}03)|}{2}=543{,}7\ kN/m$$

Therefore, because the n_{xy} is lower than the limit, it is possible to calculate the compression reinforcement for this case. Thus, using equation 87, it follows that:

$$\theta=\frac{arcsen\left(\frac{\frac{2.n_{xy}}{f_{cd1}.h}}{0{,}8+170.\left[\frac{2.\varepsilon_{yd}+2‰.(1-cos2\theta)}{(1+cos2\theta)}\right]}\right)}{2}$$
$$\theta=\frac{arcsen\left(\frac{\frac{2.200}{13660{,}7.0{,}12}}{0{,}8+170.\left[\frac{2.2{,}07‰+2‰.(1-cos2\theta)}{(1+cos2\theta)}\right]}\right)}{2} \quad (87)$$

Iteratively, it is possible to obtain the value of θ. It can be seen from Table 4 that, in this case, value θ converges. In this example, the steel is assumed to be CA-50, it is not necessary to check if $\theta \leq \theta_1$, because θ_1 does not exist.
Going forward, it is now possible to calculate ε_y.

$$\varepsilon_y=\frac{2.\varepsilon_{yd}-\varepsilon'_c.(1-cos2\theta)}{(1+cos2\theta)}+\varepsilon'_c-\varepsilon_{yd}$$
$$\varepsilon_y=\frac{2.2{,}07‰+2‰.(1-cos(2.8{,}27°))}{(1+cos(2.8{,}27°))}-2‰-2{,}07‰=-1{,}91‰ \quad (88)$$

Table 4 – Iterative calculation of θ

i	θ_i (°)	ε_1 (‰)	f_{c2max} (MPa)	σ_c (Mpa)	θ_i (°)
1	1,000	2,072	11,856	95,512	8,164
2	8,164	2,154	11,714	11,856	8,266
3	8,266	2,156	11,710	11,714	8,269
4	8,269	2,156	11,710	11,710	8,269
5	8,269	2,156	11,710	11,710	8,269
6	8,269	2,156	11,710	11,710	8,269

Thus, the forces in reinforcements are calculated

$$n_{sx}=n_x+n_{xy}.tg\theta \qquad (89)$$
$$n_{sx}=320+200.tg8,27°=329,07\ kN/m$$

$$n_{sy}=n_y+n_{xy}.cotg\theta \qquad (90)$$
$$n_{sy}=-2000+200.cotg8,27°=-623,78\ kN/m$$

Finally, the reinforcement areas are given by:

$$a_{sx}=\frac{n_{sx}}{f_{yd}}=\frac{329,07}{43,49}=7,57cm$$

$$a_{sy}=\frac{n_{sy}}{\sigma_y}=\frac{n_{sy}}{E.\varepsilon_y}=\frac{-623,78}{21000.(-1,91‰)}=15,52\ cm^2$$

8. Conclusions

Methods are presented herein to determine compression reinforcement in design case II, III and IV provided by CEB [3]. The limits for this design were also presented; in other words, cases in which it is possible to adopt compression reinforcement so that compressive stress in concrete is reduced to its strength are delimited. In all the cases, these limits are only related to the shear force to which the membrane is subjected.

Furthermore, a model for concrete strength was used that interpolates the values of strength between f_{cd1} and f_{cd2} according to the curve obtained by Vecchio and Collins [2], so that there is no discontinuity of strength values between cases II and IV and cases III and IV. Also, we presented how to evaluate whether the compressive stress in the concrete is lower than the limit to this strength model.

Due to this model adopted for concrete, for cases II and III, the reinforcement design became more complex and iterative methods were necessary for resolution. However, it leads to fewer amounts of reinforcement than those found when using only the values suggested by CEB [3] for strength.

About case IV, it was found that there are infinite solutions for reinforcement design, although just one leads to the minimum reinforcement required. This occurs due to the smaller number of fixed variables as compared to cases II and III. It is possible to find the most economic solution design through the trial and error method.

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10. Symbols

a_{sx} reinforcement area in x-direction
a_{sy} reinforcement area in the y-direction
E_{CS} modulus of elasticity of concrete
$f_{c'}$ characteristic compressive strength of concrete according to ACI
f_{c2} principal compressive concrete stress in concrete
f_{cd} design compressive strength of concrete
f_{ck} characteristic compressive strength of concrete
f_{cd1} design compressive strength of uncracked concrete
f_{cd2} design compressive strength of cracked concrete
f_{c2max} maximum compressive strength of concrete
f_{yd} design yield stress of reinforcement
f_{yk} characteristic yield stress of reinforcement
h thickness of membrane
n_c maximum compressive force in concrete
n'_c minimum compressive force in concrete
n_x normal force in the x-direction
n_y normal force in the y-direction
n_{xy} shear force in membrane
n_{sx} reinforcement force in the x-direction
n_{sy} reinforcement force in the y-direction
ε_1 strain in direction 1
ε_2 strain in direction 2
ε_x strain in direction x
ε_y strain in direction y
ε_{sx} reinforcement strain in the x-direction
ε_{sy} reinforcement strain in the y-direction
ε_c maximum compressive strain in concrete
$\varepsilon_{c'}$ minimum compressive strain in concrete
ε_{yd} design yield strain of reinforcement
θ angle between y-axis and direction of principal compression in concrete
θ_1 limit angle between curves fcd1 and fc2max
θ_2 limit angle between curves fc2max e fcd2
θ^* limit angle that define εy sign
θ_{max} limit angle to design of compression reinforcement in case II and III
θ_{xy} angle that corresponds to maximum shear stress
θ_{c1} limit angle to membrane remains in biaxial compressive state in case IV
θ_{c2} limit angle to membrane remains in biaxial compressive state in case IV
θ_x limit angle to reinforcement force in y-direction to be of compression in case IV
θ_y limit angle to reinforcement force in y-direction to be of compression in case IV
σ_c maximum compressive stress
$\sigma_{c'}$ minimum compressive stress
σ_{cd} design compressive stress