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ORIGINAL ARTICLE

Considerations on the design of indirect supports in reinforced concrete beams

Considerações sobre o projeto de apoios indiretos em vigas de concreto armado

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Abstract: Indirect supports consist of beams being supported by others beams, demanding the presence of a hanger reinforcement, required by ABNT NBR 6118, that will act as a tie hanging the applied load from the supported beam to the upper chord of the supporting beam. Therefore, the present work, through the study of experimental results in the literature, aims to present recommendations for the design of indirect supports and hanger reinforcements in reinforced concrete beams. Through a parametric analysis, was obtained an equation that calculates the load to be suspended as a function of the dimensions of the beams. It was observed that in some cases the reinforcement can be dispensed. The hanger reinforcement must be added to the shear reinforcement and in the case of regions where there is a torsional moment acting on the support beam, it was noticed that only the inner legs of the support beam stirrups collaborate to hang the load.

Keywords: indirect support; hanger reinforcement; design recommendations.

Resumo: Os apoios indiretos consistem no apoio de vigas sobre vigas sendo necessária a presença de uma armadura de suspensão, exigida pela ABNT NBR 6118, que irá atuar como um tirante suspendendo a carga aplicada da viga apoiada até o banzo superior da viga de suporte. Diante disso o presente trabalho, através do estudo de resultados experimentais na literatura, tem como objetivo apresentar recomendações para o projeto dos apoios indiretos e das armaduras de suspensão em vigas de concreto armado. Obteve-se, através de uma análise paramétrica, uma equação que calcula a carga a ser suspensa em função das dimensões das vigas. Observou-se que em alguns casos a armadura pode ser dispensada. A armadura de suspensão deve ser adicionada à armadura de cisalhamento e no caso de regiões em que existe momento torsor atuando na viga de suporte percebeu-se que apenas as pernas internas dos estribos da viga de suporte colaboram para suspensão da carga.

Palavras-chave: apoio indireto; armadura de suspensão; recomendações de projeto.

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1 INTRODUCTION

In structural design, beams are usually supported by columns or other beams, with a predominance of columns. The case in which the column is the support is called direct support and indirect support is characterized by the support of one beam over another. Indirect supports are provided in the structural planning stage of the project and normally occur to meet architectural demands, for example in cases where longer spans are desired without the presence of columns.

In situations of indirect support, a beam ends up transferring its load to another beam, requiring the presence of a hanger reinforcement to ensure that this load acts on the upper face of the supporting beam, allowing it to adequately resist this force. The presence of this reinforcement allows the balance of internal forces in the structure, thus ensuring

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its safety, and if not considered, it can cause premature failure due to shear or premature yielding of the longitudinal reinforcement of the supporting beam. Yet, its design and specification are often neglected by designers [1].

Although hanger reinforcement is necessary and recommended by the standards, sufficient criteria are not presented for the design and detailing for this type of reinforcement in most standards. The Brazilian standard ABNT NBR 6118 [2], for example, deals very briefly with the subject, recommending only the need for reinforcement without further indications. Limitations on requirements are also observed in the American standard ACI 318-19 [3], in the European standard EN 1992-1-1:2004 [4] among others. The Canadian standard CSA A23.3-04 [5] among all the standards is the most complete and brings a broader approach to the design of hanger reinforcement.

Therefore, the main references on the subject consist of studies carried out by researchers that have been published in articles or academic works. Initially, Leonhardt and Mönnig [6] should be highlighted, being a theoretical basis used by other studies as a reference. In this work, considerations about the load to be hung, details of the longitudinal reinforcement present in the joint and the region of distribution of the hanger reinforcement are presented.

The experiments carried out by Mattock and Shen [7] and Mattock and Kumar [8] proved the influence of the hanger reinforcement on the behavior of beams in situations of indirect support. In the case of Mattock and Shen [7] it is worth highlighting the verification that for cases in which the supported beam has a lower height than the supporting beam, the load to be hung by the reinforcement can be reduced.

In addition, there are situations in which the supported beams transfer moment to the supporting beam that are neglected by the standards. In the study carried out by Collins and Lampert [9] it is observed that in the beam failure, despite the presence of the hanger reinforcement. The supporting beam presented inadequate behavior with the detachment of the lower zone of the connection region. In this context, the study by Mattock and Shen [7] considered the action of the torsional moment, and a cracking behavior similar to that seen in the study by Collins and Lampert [9] was observed.

The need for an adequate design of the hanger reinforcement for the proper behavior of the connection between beams, especially in situations with torsional moment acting on the supporting beam, still ignored by the standards, justify the present work that, through the analysis of experimental results present in the literature aims to provide recommendations that help in the design of these elements and serve as a basis for future studies.

2 BEHAVIOR OF INDIRECT SUPPORT

Concrete beams transmit their loads to the support preferentially through compression struts so that in the case of indirect supports, this load that arrives at the bottom is introduced into the supporting beam [6], as can be seen in Figure 1.

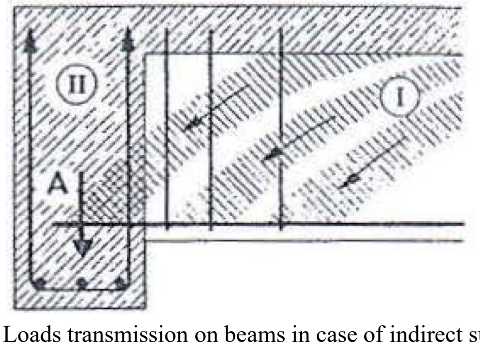


Figure 1. Loads transmission on beams in case of indirect support [6]

In order to obtain the balance of internal forces (Figure 2), is necessary at the intersection of the two beams, a reinforcement that acts as a tie to hang the load applied by the supporting beam to the upper flange of the supporting beam, and this reinforcement it is called hanger reinforcement ([6], [10]). To calculate this reinforcement, two strut ant tie models are connected, one for the supported beam and the other for the supporting beam.

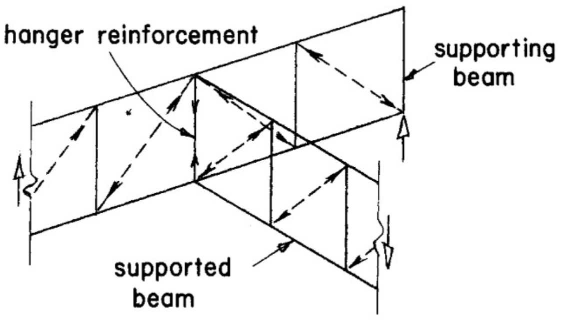


Figure 2. Strut and tie model in indirect supports [7]

Thoma [11] used a nonlinear finite element analysis (NFLE) to model numerically the static behavior of different experimentally tested beams, with one being a case of a prestressed beam with an indirect support. The beam studied is shown in Figure 3.

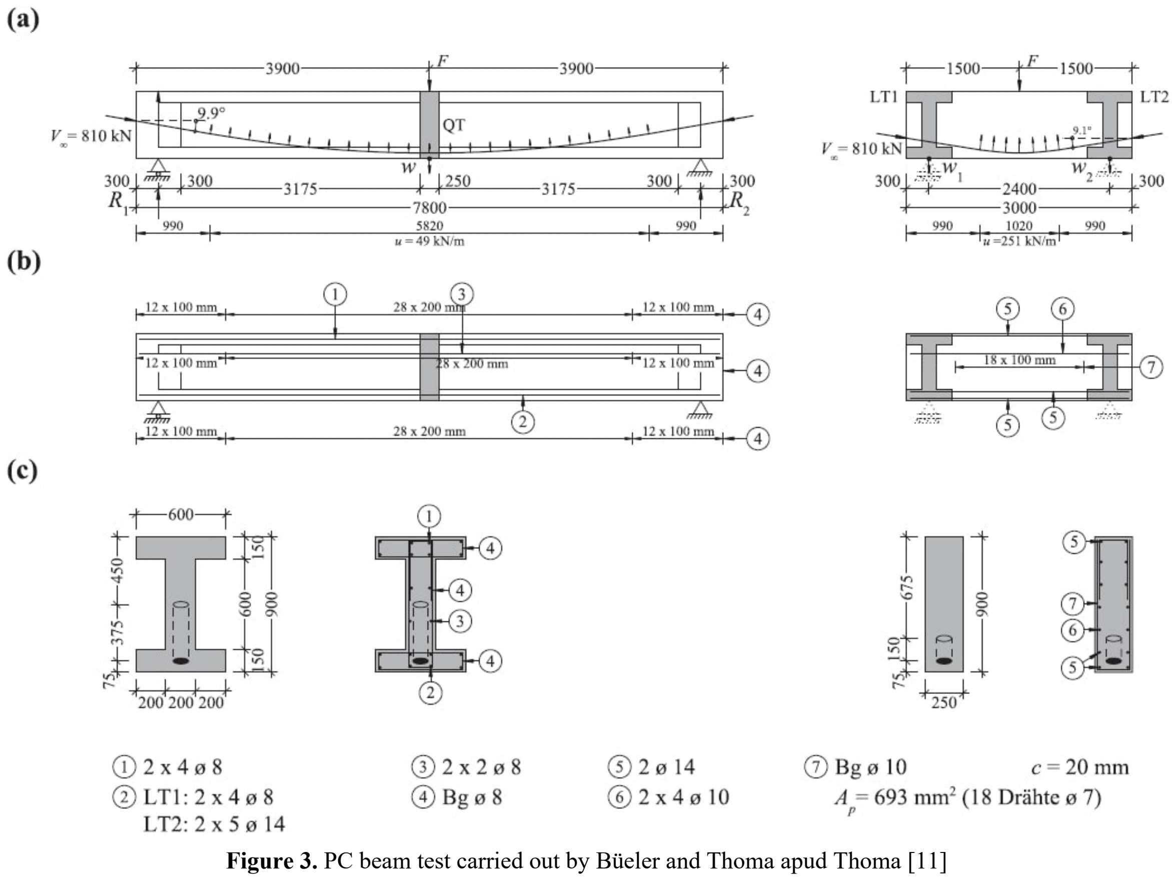


Figure 3. PC beam test carried out by Büeler and Thoma apud Thoma [11]

It was observed that both the crack orientation and the decompressed area in the longitudinal girder are determined sufficiently well by the NLFE analysis. The similar behavior can be seen in Figure 4 where the red lines represent the NFLE analysis.

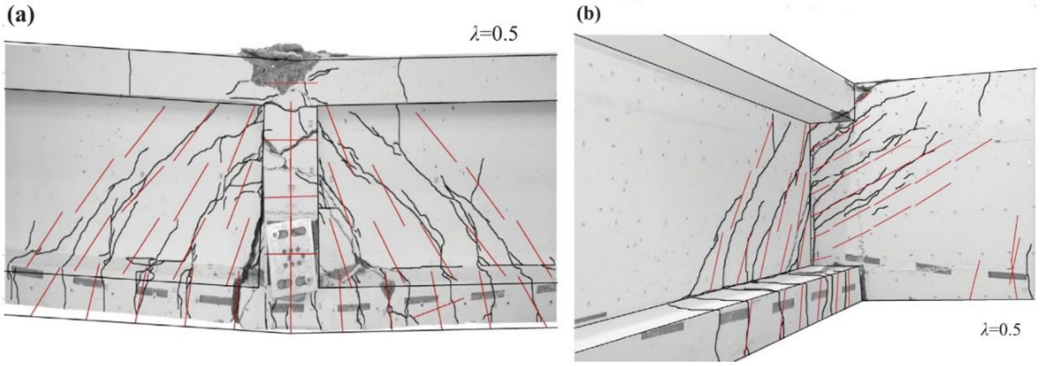


Figure 4. Crack pattern at failure: (a) view of longitudinal girder; (b) view of transverse girder–longitudinal girder. [11]

In the test, the flexural compression zone failed locally, as expected, at the intersection of the transverse and longitudinal girders. This phenomenon has not been implemented in the NFLE analysis. Despite that, the NFLE analysis showed that it could be a good method to model the indirect support with the development of new studies.

There are several types of indirect supports due to the arrangement between the supported beam and the supporting beam. In this case, the distribution of the load applied by the supported beam is different, so three cases of indirect supports will be shown in which attention must be paid when determining the load to be hung by the reinforcement.

It is initially considered the simplest case where both beams have the same height (Figure 5) and thus 100% of the load must be hung.

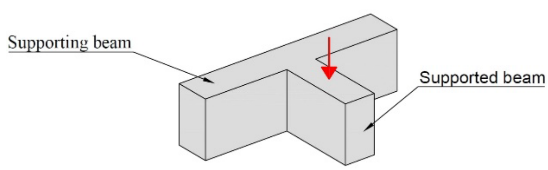


Figure 5. Indirect support where $h_1 = h_2$

The second case corresponds to the case of the supported beam (h_1) having a lower height than the supporting beam (h_2). Within this case, there are two situations in which the height of the supported beam is much smaller compared to the height of the supporting beam ($h_1 << h_2$) in which attention must be paid.

The first situation (called support from the top) is characterized by being a situation of indirect support where the upper surface of the supported and supporting beams coincide (Figure 6), in this condition the load to be hung can be reduced because in elements of reinforced concrete that are subject to shear forces, in addition to the truss model, there are other alternative resistant mechanisms (such as the arch effect and the aggregate interlock) that transfer the internal stresses from one cross-section to another [12].

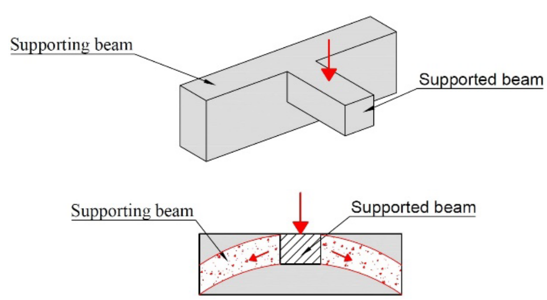


Figure 6. Indirect support with support from the top (where $h_1 << h_2$)

The second situation (called support from below) it is characterized by being an indirect support in which the lower surface of the supported and supporting beams coincide (Figure 7), in this condition is necessary to hang the entire load.

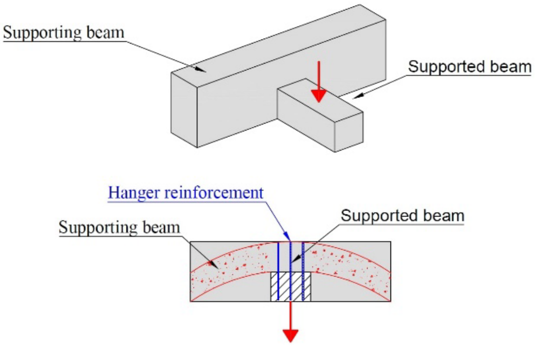


Figure 7. Indirect support with support from below (where $h_1 << h_2$)

Finally, the third case corresponds to the case where the supported beam is higher than the support beam (Figure 8). In this situation, 100% of the load must be hung.

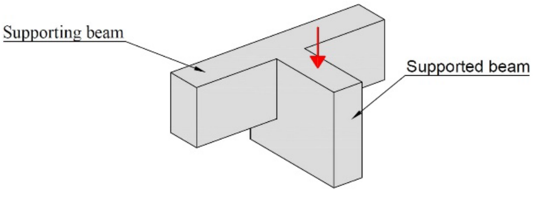


Figure 8. Indirect support where $h_1 > h_2$

With this, it is observed that attention is needed to the load to be hung in the design of indirect supports so that you can optimize your project by reducing this load and maintaining structural safety, in situations where it is possible.

In addition, an important situation that modifies the behavior of this region is the case where there is an action of a torsional moment in the supporting beam. The superposition of the forces resulting from the shear force and the torsional moment ends up altering the distribution of forces in the support beam and results in a weakening of this beam, where the stirrup legs adjacent to the back face of the support beam do not effectively contribute to the hanging of the beam load.

To understand this alteration, it is initially considered a beam subjected to torsional moment (T), in which shear stresses are developed at the top and on the side surfaces, as can be seen in Figure 9a. The principal stresses are shown in Figure 9b. These principal stresses eventually cause cracking around the body as seen by the line A-B-C-D-E in Figure 9c.

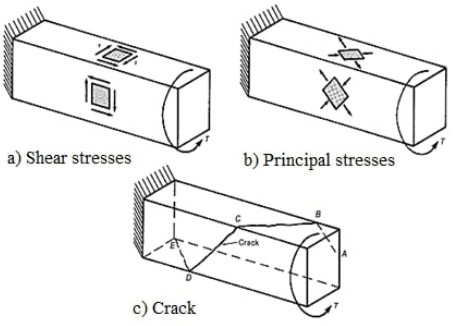


Figure 9. Principal stresses and crack pattern due to pure torsion [13]

However, if the beam is subjected to combined torsion and shear, the two shear components are added on one face (front face of Figure 10) and act in the opposite direction to each other on the other face. As a result, the inclined crack (Figure 10c) starts at the face where the stresses add up (crack AB) and extends along the top face of the beam (crack BC). If the bending moment acting on the part is large enough, the (bending) cracks extend almost vertically along the back face (crack CD). The compression zone due to bending near the bottom of the beam prevents cracks from extending over the entire height of the faces.

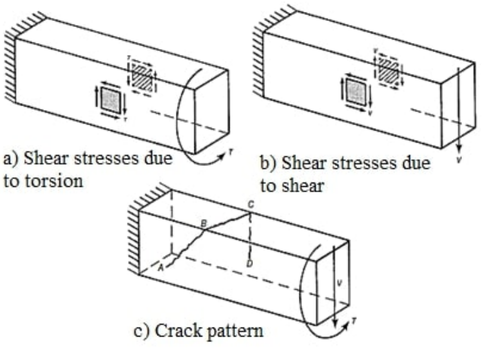


Figure 10. Shear stresses and crack pattern due to the combination of torsion and shear[13]

From the behavior of the structure when subjected to both shearing force and torsional moment, it is observed that since the posterior face presents only flexural cracking, the stirrup leg in this region does not act as a hanger reinforcement, only the inner leg of the stirrup does. Based on this consideration, in the case of a torsional moment, a different analysis is required regarding the hanger reinforcement.

3 INDIRECT SUPPORT DESIGN

With this, experimental results and design recommendations will be presented regarding four important topics in the design of indirect supports: load to be hung by the reinforcement, addition of the hanger reinforcement to the shear reinforcement, limit stress for which the reinforcement is not necessary and cases with rotation restraint.

3.1 Load to be hung

To determine the load for the design of hanger reinforcement, it is preferable to opt for the adoption of an analysis by a strut and tie model, but to simplify the design, Leonhardt and Mönnig [6] and Wight [13] propose equations that allow the calculation of the load to be hung, these expressions being indicated in Equations 1 and 2 respectively.

$$A_{sus} = \frac{h_1}{h_2} \cdot \frac{V_{u1}}{f_{yd}} \tag{1}$$

where A_{sus} = cross-sectional area of hanger reinforcement (m^2); V_{u1} = factored shear at the end of the supported beam (kN); f_{yd} = design yield strength of transverse reinforcement (kN/m^2); h_1= supported beam height (m); h_2 = supporting beam height (m).

$$A_{sus} = \left(1 - \frac{h_b}{h_2}\right) \cdot \frac{V_{u1}}{f_{yd}} \tag{2}$$

where h_b = vertical distance between the bottom of the two beams (m).

Equation 1 suggested by Leonhardt and Mönnig [6] has limitations, underestimating the loads for the case of support from below and overestimating for supported beams with a height greater than the supporting beam. The underestimation behavior for the case of support from below can be observed in Figure 11, where the green line corresponds to the expected behavior and the blue line corresponds to the expression of Equation 1. In this case, the values obtained by the equation are equal to the value expected until the h_1/h_2 ratio is equal to 1, where after that point it starts to present higher values.

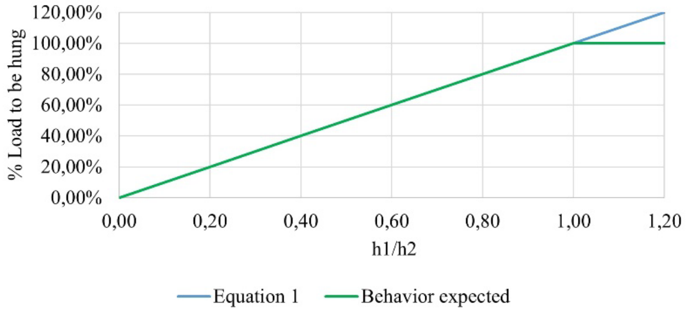


Figure 11. Relation between h1/h2 and load to be hung – Support from below

Regarding the support from the top, it can be noted that the values obtained from Equation 1 are below the expected result until the h1/h2 ratio is equal to 1. For a ratio higher than 1, Equation 1 overestimates the load to be hung (Figure 12), which despite being a more difficult situation to occur in the practice of projects, causes oversizing of the reinforcement.

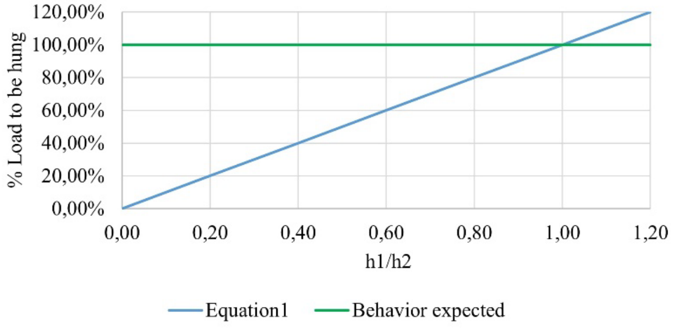


Figure 12. Relation between h1/h2 and load to be hung – Support from the top

Equation 2 suggested by Wight [13], which is the same as indicated by the Canadian standard CSA A23.3-04 [5], presents a more adequate behavior for situations with support from below, with the values obtained by the equation equal to the ones that are expected, despite having the same limitations as Equation 1 for cases with support from the top.

In view of the observed, it is necessary to adopt the following restriction (Equation 4) to use the expression indicated by Wight [13], and the load is calculated using the Equations 3 and 4:

$$A_{sus} = \left(1 - \frac{h_b}{h_2}\right) \cdot \frac{V_{u1}}{f_{yd}} \text{ to } h_1 \leq h_2 \tag{3}$$

$$A_{sus} = \frac{V_{u1}}{f_{yd}} \text{ to } h_1 > h_2 \tag{4}$$

The study carried out by Baek [1] aimed to study the influence of the depth of lateral load application on the behavior of the connection region between the beams. The experiment was carried out on two specimens, where the dimensioning and detailing of the reinforcements followed the determinations of the Canadian standard CSA A23.3-04 [5].

Two specimens were made where the beams were intentionally designed so that shear failure occurs in the supporting beam, with all other failure mechanisms avoided. The first specimen (B1) has two crucifix forms, where the supported beams are H4 and H5, and the supporting beam is called B1. Beams H4 and H5 correspond to 4/6 and 5/6 of the 600mm height of the supporting beam. The beam layout as well as the dimensions (in mm) of the beams are shown in Figure 13.

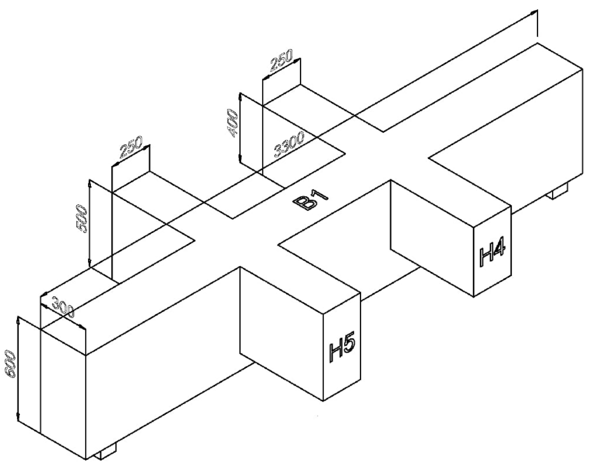


Figure 13. Description of specimen B1 (dimensions in mm) [1]

The specimen (B2) has only a crucifix shape, consisting of a supporting beam (B2) with the same height as the beam B1 of the first specimen and the beam H6 supported at the same height as the beam B2. In addition, beam B2 is loaded by direct loading at the top indicated by H0. The beam layout as well as the dimensions (in mm) of the beams are shown in Figure 14.

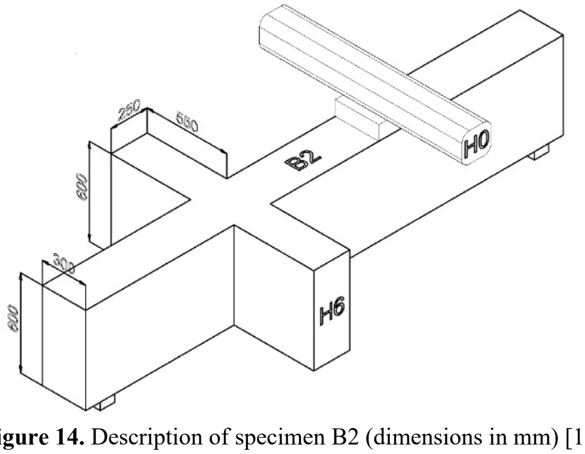


Figure 14. Description of specimen B2 (dimensions in mm) [1]

The hanger reinforcements were designed according to the simplified equation of CSA A23.3-04 [5] presented in Equation 2. If the load to be carried by the hanger reinforcement is taken by F then the reinforcements need to hang 0.67·F, 0.83·F, 1.0·F and 0 for the supported beams H4, H5, H6 and H0, respectively.

The test consisted of two phases for each specimen. Initially, the load was applied to the beams until one of the shear spans of the supporting beam failed. In the second phase, the span that failed was stiffened and loading was resumed until the failure of the other span (Figure 15). In this way the four spans could be tested.

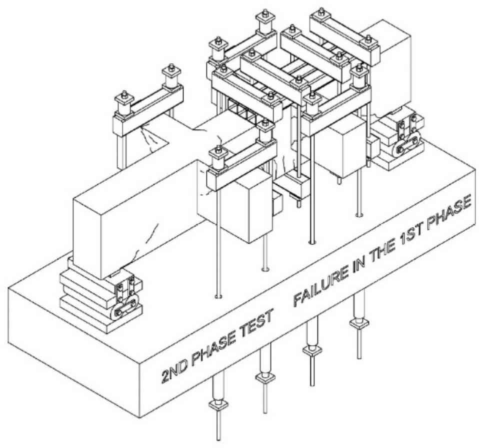
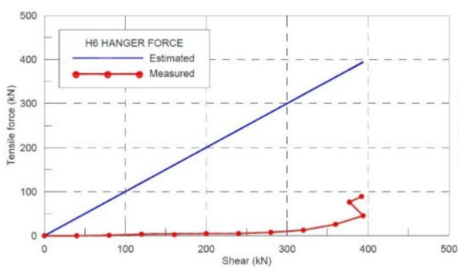
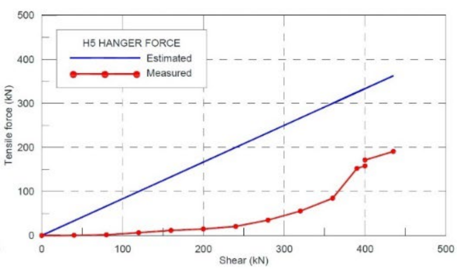


Figure 15. Test scheme [1]

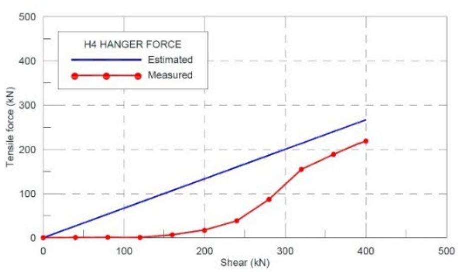
With the results for the two specimens, the effect of the beam depth on the forces acting on the hanger reinforcements can be evaluated, comparing the design force calculated according to CSA A23.3-04 [5] with the forces corresponding to the deformations measured in the hanger reinforcement. Figure 16 shows the comparison between the estimated force and the force observed in the tests for beams H6, H5 and H4.



(a) Joint region of H6



(b) Joint region of H5



(c) Joint region of H4

Figure 16. Tensile force on hanger reinforcement [1]

Thus, it is observed that the load on beam H6 in the test was only 20% of the design value estimated by CSA A23.3-04 [5]. For beam H5, the load on the hanger reinforcement was 46% lower than expected in the design and finally, for beam H4 the load was only 12% lower than the design force. The reference values are the highest observed in the tests and were observed for the maximum loads supported by the beams.

In order to better visualize this comparison, the author makes the relationship between the ratio hb⁄h2 used in the design equation and the values obtained in the tests. This analysis for the maximum load is shown in Figure 17.

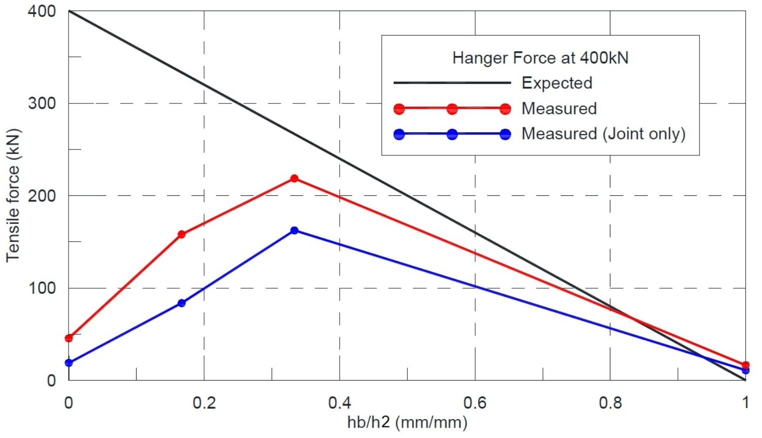


Figure 17. Development of tensile force as a function of the ratio hb/h2 [1]

It was expected that the maximum load on the hanger reinforcements would occur for the case of the deepest supported beam (H6), but this was not achieved, observing that the maximum load to be hung was obtained in the beam H4, which has a height of 4/6 of the height of the supporting beam and compared to beams H5 and H6 should be, by the design equation, the smallest load.

According to the author, the presence of diagonal tensile cracks in the concrete was observed, indicating that compressive stresses were flowing from the supported beam to the supporting beam throughout the height of the supporting beam. This means that there was force transfer over the height of the supported beam and not just at the bottom, as assumed by the Canadian code. This flow of compressive stresses was likely present for H6, H5 and H4, as evidenced by diagonal cracks formed in the supported beam and flowing into the supporting beam, as shown in the Figure 18.

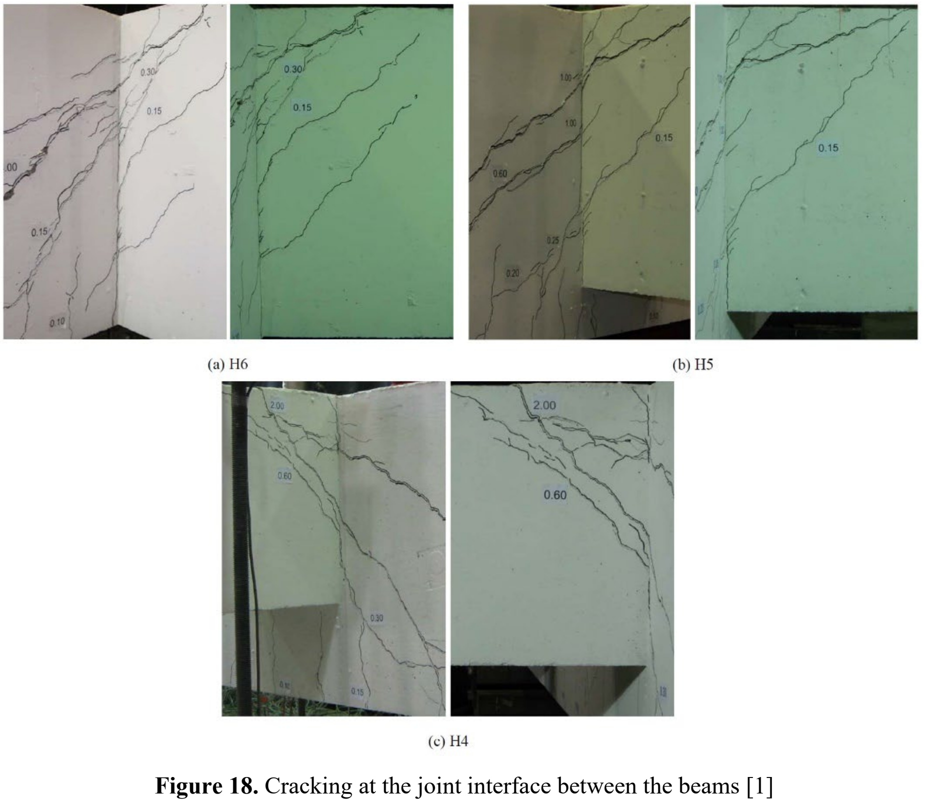


Figure 18. Cracking at the joint interface between the beams [1]

Given the data of the hanging loads, the remainder of the load was transferred directly by diagonal compression stresses from the supported beam to the supporting beam. The value of this portion referring to direct transmission was determined by subtracting the hanging force from the total load applied to the interface. The comparison between the hanging load and direct transmission values is shown in Figure 19.

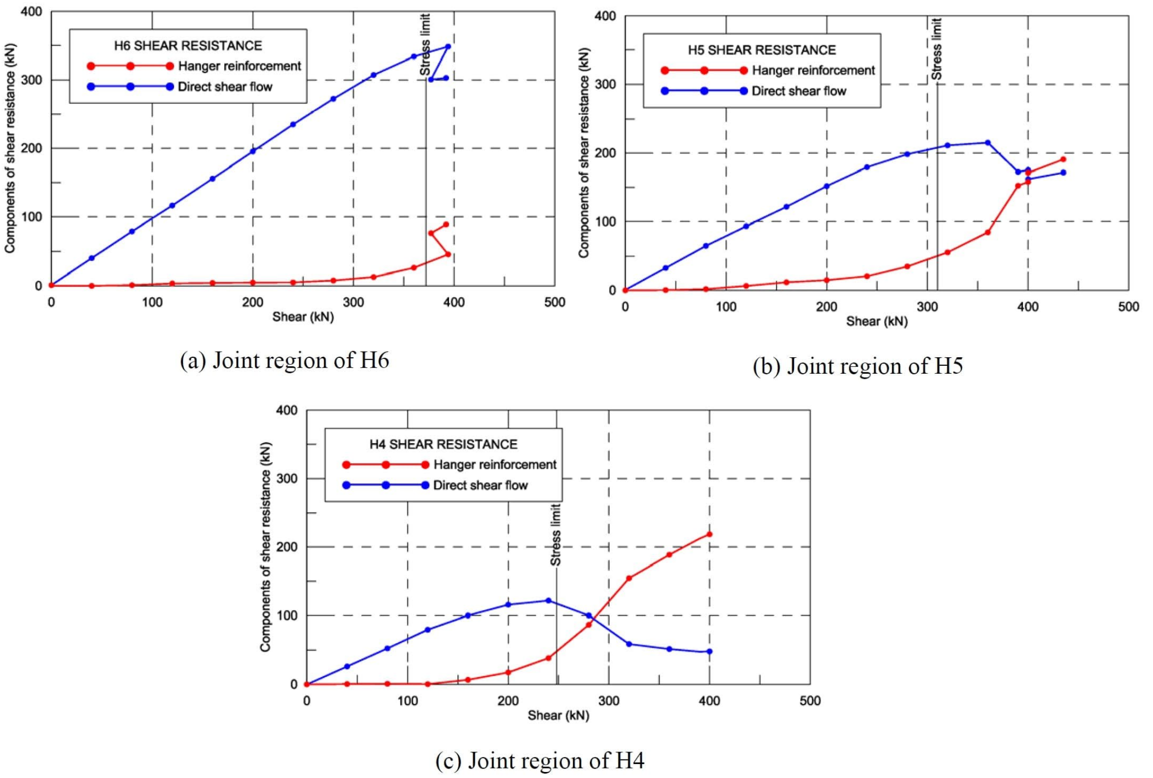


Figure 19. Comparison between the forces transmitted by the hanger reinforcement and the direct shear flow [1]

It is observed that the direct transmission by diagonal compression was much higher than that transferred by the hanger reinforcement in beam H6 (Figure 19a). For beam H4 the two mechanisms contributed similarly (Figure 19b), while the shear transferred by the hanger reinforcement is predominant in beam H4 (Figure 19c). Thus, as the depth of the supported beam is greater than 2/3 of the height of the supporting beam, there is a reduction in the demand for hanger reinforcement due to direct transfer so for these cases the equation for the design of the CSA A23.3-04 [5] proved to be conservative.

Due to the low sampling in which this behavior was observed, more studies are needed to be focused on this consideration for an adequate use of this greater reduction and future application in design standards for concrete structures. At the current moment of knowledge on the subject, it is not understood properly which portion is transferred directly by compression through the height, so due to the safety of the structure, it is recommended to use Equation 3, which is based on the Canadian standard.

3.2 Addition of the hanger reinforcement to shear reinforcement

This is one of the main points of attention because, in the case of superposition case of superposition of reinforcements, congestion can occur in the intersection region, and if this superposition is necessary and not considered, the section weakens. Leonhardt and Monnig [6] defend that the highest of the values between the hanger reinforcement and the necessary shear reinforcement should be adopted, but the CSA A23.3-04 [5], ACI 318-19 [3] and EN 1992- 1-1:2004 [4] consider that the reinforcement overlap is necessary.

Using the strut and tie model formulated by Mattock and Shen [7] it is observed that as the load is applied to the lower part, the presence of a tie is necessary to hang the load transferred by the supported beam. In order to observe this behavior, a truss can be taken with a load applied to the center of its span in two situations: the first is considered the load applied to the upper face and the second to the load applied to the lower face.

It was observed that for the first situation (direct loading) the vertical tie in the region of application of the load did not present axial force and could therefore be disregarded for the balance of the region, having the schematic of its structure shown in Figure 20.

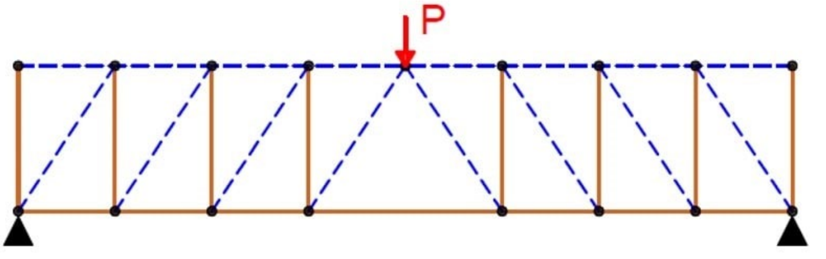


Figure 20. Strut and tie model for a bi-supported beam with centered force applied to the upper face

When taking the second situation (indirect loading) it was noticed that the vertical tie is loaded exactly with the value of the applied load, which indicates that it acts by hanging 100% of the applied load and consequently its presence is necessary to balance the region, being observed the schematic of the structure in Figure 21.

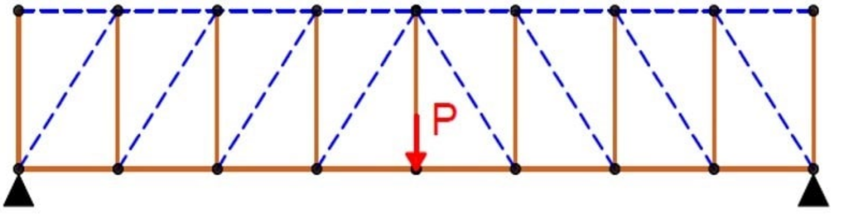


Figure 21. Strut and tie model for a bi-supported beam with centered force applied to the lower face

Given this and associated with the current knowledge regarding the behavior of the intersection zones, the superposition of hanger and shear reinforcements is recommended, and attention should be doubled on the congestion of reinforcements, observing the limitations imposed by the codes for shear reinforcement rate as well as limitations of spacing so that an adequate vibration of the concrete of the structure can be ensured.

3.3 Limit stress for which the hanger reinforcement is not required

In reinforced concrete elements that are subject to shear forces, in addition to the truss model, there are other alternative resistant mechanisms that transfer the internal stresses from one cross section to another [12]. The arc effect and the aggregate interlock are two examples of these mechanisms, where they are considered in ABNT NBR 6118 [2] by an additional portion defined in the standard as V_c. Thus, the hanger reinforcement in certain circumstances can be dispensed because for low stresses other mechanisms may be sufficient to adequately transmit the load.

Fusco [10] comments that only in cases of very thin web beams ($b_w/b_f = 1/6$, represented in Figure 22) the truss scheme is mobilized at the beginning of cracking, so that on the other beams, the alternative resistant schemes act preliminarily, whose contribution progressively decreases as the part cracks.

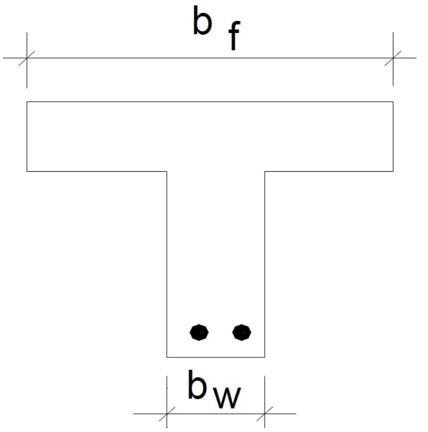


Figure 22. T-beam cross section [10]

One of the studies that evaluated the limiting stress was carried out by Mattock and Shen [7]. In this study, the authors aimed to evaluate the efficiency of the hanger reinforcement in the case where the supported beam rests on one side of the supporting beam of same height. The specimens were E-shaped grids (Figure 23), where the supporting beam presents a restriction to rotate in order to create a negative moment in the supported beam in the intersection region.

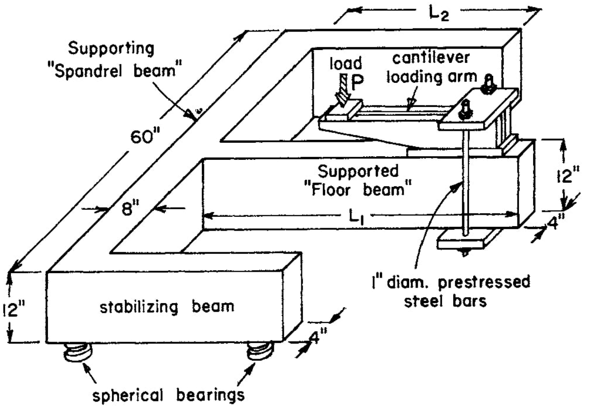


Figure 23. Typical structure of the tested specimens [7]

The specimens were dimensioned by varying the arrangement of the hanger reinforcement, the shear forces and the maximum resistant moment of the section. Specimens 1 and 4 were designed so that the shear force on the supported beam was close to $0.21b_w \cdot d \cdot \sqrt{f_c'}$ (in MPa). The same flexural strength was designed for specimen 2, but due to a lower a/d ratio, the shear for this case was close to the maximum allowed by the American standard in its 1989 version $0.83 \cdot b_w \cdot d \cdot \sqrt{f_c'}$ (in MPa). To maintain the shear value at the failure of specimen 2, specimen 3 was dimensioned so that its flexural strength was twice that of specimen 2. Specimen 5 was designed so that the shear was $0.66 \cdot b_w \cdot d \cdot \sqrt{f_c'}$ (in MPa) and the flexural strength corresponding to this shear value.

With regard to the hanger reinforcement, in all cases, sufficient reinforcement was provided to satisfy the requirements of the American standard, wherein specimens 1 and 4 the minimum reinforcement requirement ruled. In specimens 1, 2 and 3, the hanger reinforcement was arranged adjacent to the interface between the two beams, being designed with a yield strength equal to the shear at failure by bending of the supported beam.

In the case of specimens 4 and 5, their design was carried out considering that the stirrups to support all the shear transferred from the supported beam to the supporting beam must be positioned in a region of 0.5 times the height of the beam for each side of the beams, which may be waived if the stress is less than $0.25 \cdot \sqrt{f_c'}$ (in MPa), if $h_b > h_1/2$, or in case the supported beam rests on the supporting beam by its lower face. Therefore, for specimen 4, hanger reinforcement was not available since the maximum stress in the supported beam was less than $0.25 \cdot \sqrt{f_c'}$ (in Mpa). In specimen 5, all stirrups on the supporting beam within a distance of d/2 from the axis to each side were considered as hanger reinforcement. The data from the tests are presented in Table 1.

Table 1. Test results [7]

Specimen	1	2	3	4	5
P_{max} (kips)	8.18	35.00	35.00	8.90	25.80
V_{test} (kips)	8.64	35.37	35.41	9.36	26.20
$\frac{V_{test}}{b_w \cdot d/\sqrt{f_c'}}$	2.86	12.14	12.93	3.09	8.73
M_{test} (kips·in)	181.1	180.0	317.8	168.3	84.6
M_{calc} (kips·in)	165.0	165.7	296.4	166.4	128.4
$\frac{M_{test}}{M_{calc}}$	1.10	1.09	1.07	1.01	0.66

After the tests, it was observed that specimens 1 and 4, with the ratio $V_{test}/\,(b_w \cdot d/\sqrt{f_c'})$, presented in Table 1, close to 3) behaved similarly, even though the hanger reinforcement was only arranged in specimen 1. Thus, this similar behavior shows that the criterion for waiving the use of hanger reinforcement is suitable for loads less than $0.25 b_w \cdot d \cdot \sqrt{f_c'}$ (in MPa). In the case of specimen 1, the hanger reinforcement did not yield and only carried 35% of the shear at failure. Diagonal tensile cracking did not develop in these specimens and, therefore, the truss behavior was not significant.

In specimens 2 and 3, the desired behavior was achieved, being observed the appearance of a pattern of diagonal cracks by tension (Figure 24) that define the diagonal compression struts in the truss behavior of the supported beam. In addition, the hanger reinforcement yielded before failure and carried a maximum load close to shear in yielding the longitudinal flexural reinforcement. In these specimens, the stress was above the limit stress so the presence of hanger reinforcement is necessary for the structure to behave properly.

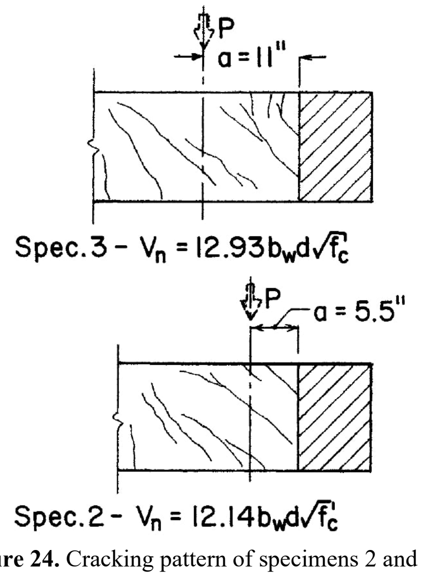


Figure 24. Cracking pattern of specimens 2 and 3 [7]

Therefore, the Canadian standard CSA A23.3-04 [5] in Equation 5 as well as Mattock and Shen [7] and Wight [13] in Equation 6 indicate the possibility of waiving the hanger reinforcement by assigning a limit stress (τ_{lim}) at the interface between the beams.

$$\tau_{lim} = 0.23 \cdot \lambda \cdot \phi_c \cdot \sqrt{f'_c} \tag{5}$$

$$\tau_{lim} = 0.25 \cdot \sqrt{f'_c} \tag{6}$$

where $\hat{o}_{lim}$ = limit stress for reinforcement waiver (MPa); λ = factor to consider low density concrete; f'_c = specified compressive strenght of concrete (MPa); ϕ_c = resistance factor for concrete [5].

Pereira et al. [14] present cases of indirect support to evaluate the limiting stresses indicated by Mattock and Shen [7] and the Canadian standard [5]. The case diagrams are presented in Figure 25, where the hatched beam corresponds to the supported beam, and the results of the cases are presented in Table 2.

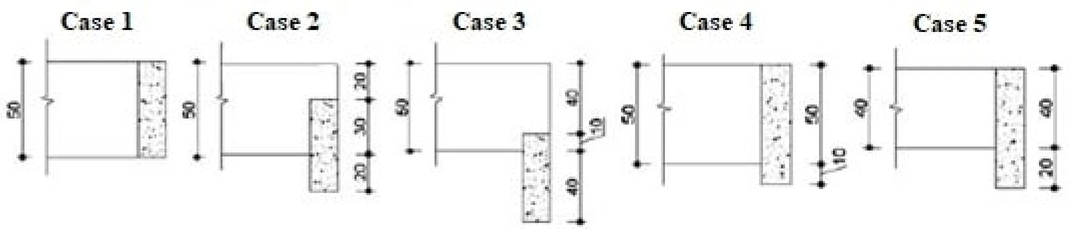


Figure 25. Support situations analyzed by Pereira et al. [14]

Table 2. Shear stress values at the supported beam interface and limits [14]

Case	b (cm)	h (cm)	τ_d (MPa)	CSA A.23.3-04 [5] (MPa)	Mattock e Shen [7] (MPa)
1	15.00	50.00	1.15	0.85	1.43
2	15.00	50.00	1.15	0.85	1.43
3	15.00	50.00	1.15	0.85	1.43
4	15.00	50.00	1.16	0.85	1.43
5	15.00	40.00	1.38	0.85	1.43

It is important to highlight that the analysis of this criterion by the Canadian standard is only allowed for situations in which the upper surfaces of the beams coincide, so that, by this consideration, only cases 1 and 5 would allow this analysis. Even so, it can be observed that the limit of the Canadian standard was exceeded in all cases, but for the limit of Mattock and Shen [7] none of the cases would require hanger reinforcement.

It appears that the expression indicated by Mattock and Shen [7] allows higher stresses at the interface in which it is possible to dispense the hanger reinforcement, but it has been observed in only a specimen, so for the structure safety, the adoption of a more conservative criterion as presented by the Canadian standard is the most recommended.

To use Equation 5 according to the Brazilian standard ABNT NBR 6118 [2] it is necessary to convert the resistance f'_c, present in the Canadian standard, to the resistance f_{ck} present in the Brazilian standard. Considering a standard deviation of the compressive strength of 3 MPa, Souza and Bittencourt [15] present an expression that relates the two factors and is expressed in Equation 7.

$$f'_c = f_{ck} - 2.04 \tag{7}$$

where f_{ck}= characteristic compressive resistance of concrete (MPa).

Substituting Equation 7 in Equation 5, the limit stress for application of the Brazilian standard is presented in Equation 8:

$$\tau_{\text{lim}} = 0.23 \cdot \lambda \cdot \phi_c \cdot \sqrt{f_{ck} - 2.04} \tag{8}$$

The λ factor considers the possibility of using low-density concrete, having 1 as the value for normal-density concrete and the ϕ_c factor assuming the value indicated by the Canadian standard of 0.65 since it does not have a corresponding value in the Brazilian standard. Substituting these two values in Equation 8, Equation 9 is obtained, which represents the stress limit for waiver of reinforcement to be applied in projects carried out with concrete of normal density according to the Brazilian standard.

$$\tau_{\text{lim}} = 0.15 \cdot \sqrt{f_{ck} - 2.04} \tag{9}$$

According to the Canadian standard CSA A23.3-04 [5] normal density concretes have a density greater than 2150kg/m^3 and this restriction should be added since for ABNT NBR 8953 [16] normal density structural concretes have a density greater than 2000kg/m^3.

3.4 Cases with rotation restraint

A special situation in the design of indirect supports consists of cases where a torsional moment acts on the supporting beam, where a real example is shown in Figure 26. In this example, the balcony beams (VP3 and VP4) are displaced from the columns and are supported in beam VP1, which presents an increase in section in the region close to the columns and corresponds to a fixed section in which the transfer of torsional moment to the supporting beam occurs.

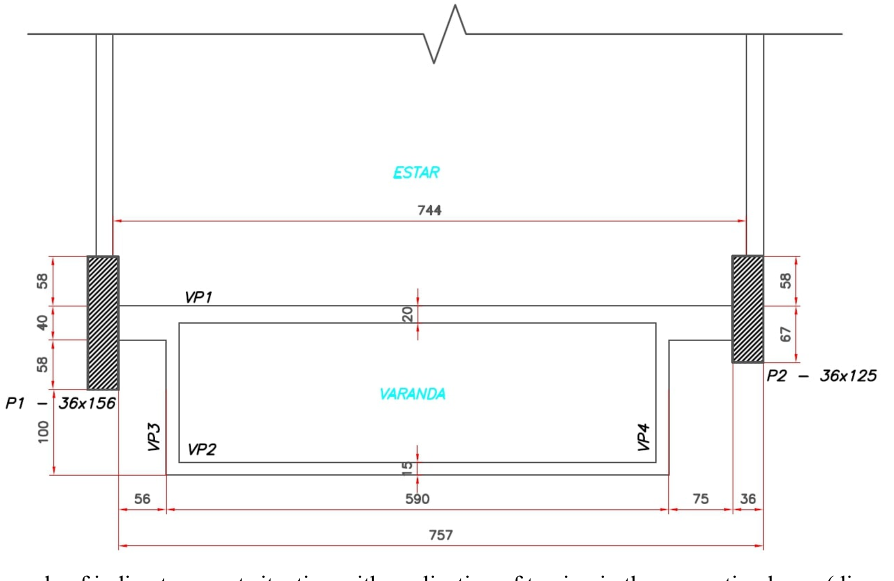


Figure 26. Example of indirect support situation with application of torsion in the supporting beam (dimensions in cm)

The study of indirect supports in situations with restraint to the rotation of the supporting beam is still very limited, and it is worth mentioning initially the test carried out by Collins and Lampert [9]. The main objective of this experimental program was to evaluate design methods for parts subjected to compatibility torsion, choosing a two-beam structure (called floor beam-spandrel beam by the author) shown in Figure 27. Although the focus of this study is the analysis of the distribution of moments in the cracking, here will be brought, mainly, the observed results referring to the behavior of the structures in the connection joints between the beams.

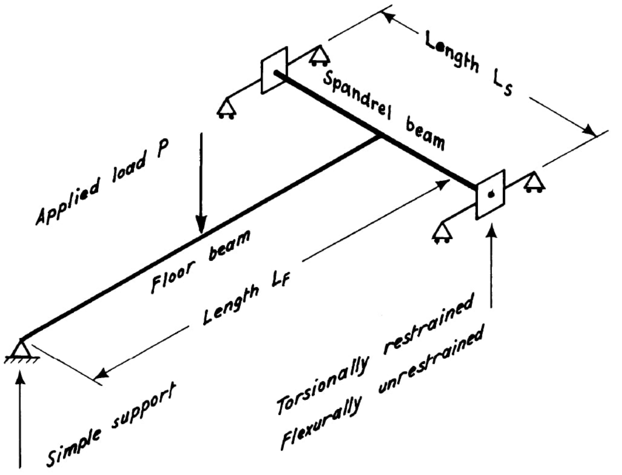


Figure 27. Structural scheme used by Collins and Lampert [9]

This structure was designed by two methods: method A (conventional design in which the supporting beam was subjected to compatibility torsion) and method B (design considering that the supporting beam does not present torsion resistance).

Thus, six specimens were tested, with specimens S1 and S3 dimensioned by method A and specimens S2, S4, S5 and S6 by method B. For the interests of this article, attention is focused on specimens S3 and S4. Specimen S3 was dimensioned by method A while specimen S4 was dimensioned by method B to establish a comparison between them.

After the tests, specimen S4 failed by crushing of the concrete compression zone. The joint in specimen S3 was the cause of the failure despite the care taken in detailing these joints. The design of this region was in accordance with the recommendations for indirect support by Leonhardt (1965) apud Collins and Lumpert [9], with the hanger reinforcement in the joint being made designed to hang 100% of the load on the supporting beam, in addition, it was arranged the longitudinal bars of the supported beam (floor beam) above the longitudinal bars of the supporting beam (spandrel beam). Despite these precautions, the failure occurred in this region, as can be seen in the case of specimen S3 shown in Figure 28.

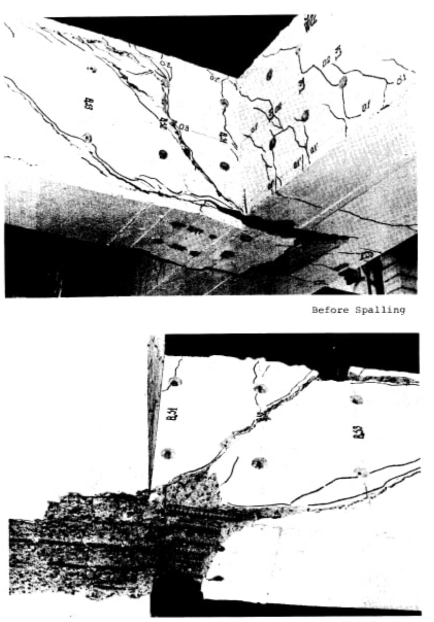


Figure 28. Joint failure in specimen S3 [9]

For the case of method A (specimen S3), the supporting beam is subjected to torsion and, as previously presented, structures in this situation present a weakening because only the inner legs are expected to act as a hanger reinforcement. While the specimen S4 does not present a torsional moment acting on the supporting beam behaved as expected. It follows that the design provisions that were made did not consider the negative effect of the presence of torsional moment in the supporting beam.

Because of this, the authors suggested that it would be practical to provide a reinforcement capable of hanging 100% of the load in the supported beam, in addition to keep the reinforcement on the supporting beam hanging also 100% of the load, with that it would be provided the double of hanger reinforcement compared to the suggestion of Leonhardt. However, it is observed that the author did not consider that the fact that there is a transmission of bending moment between the beams interferes with the behavior of the joint.

A later study carried out by Mattock and Shen [7] evaluated the behavior of indirect supports in which there is a torsional moment acting on the supporting beam. Since the experiment was previously presented from the perspective of the evaluation of the limiting stress for waiver of the hanger reinforcement, attention will now be given to specimens 2 and 5 to evaluate the interference of the torsional moment acting on the supporting beam.

For specimen 5, an unsatisfactory behavior was observed since the longitudinal reinforcement yielded with only 55% of the projected dead load and 66% of the calculated flexural strength. In this case, only 57% of the load was resisted by the hanger reinforcement. Despite the cracking scheme of the supporting beam of the specimen (Figure 29a) being very similar to that presented in specimen 2 (Figure 29b), the hanger reinforcement did not provide sufficient support from the lower part of the supporting beam at the intersection of the beams.

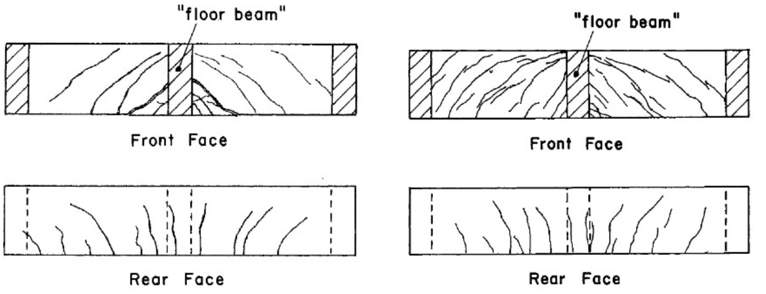


Figure 29. Cracking pattern of the supporting beam in specimens 5 and 2 [7]

As previously described, the combination of shear and torque on the supporting beam on the rear face act in the opposite direction to each other. The cracking scheme observed in Figure 29a indicates the presence of flexural cracks, so that the stirrup leg was not cut by the crack and thus did not act as a hanger reinforcement. It can be noted that the cracking scheme observed in Figure 29a is similar to the one suggested in Figure 10 for the case of the torsional moment combined with the shear.

Thus, only the inner leg is expected to act as a hanger reinforcement, which may explain the failure of this region with the detachment of the lower region of the intersection region, because the hanger reinforcement is not sufficient to hang the entire applied load. As for specimen 2 in the same study, which considered only the adjacently disposed reinforcement as hanger reinforcement, an adequate behavior was observed despite being dimensioned for a higher shear when compared to specimen 5.

Thus, in these cases, only the inner leg (adjacent to the interface between the beams) of the stirrups should be considered as hanger reinforcement, highlighting the importance of further experimental studies to better understand the behavior of the intersection zone and what is the portion of the contribution of the external legs of the stirrups in the hanging of the load.

4 CONCLUSIONS

Based on the results presented in the present work, the following conclusions can be drawn:

- The value of the load to be hang is calculated according to Equations 3 and 4;
- Hanger reinforcement must be added to the shear reinforcement;

- There is a shear stress limit transferred by the supported beam at which, for stress values lower than this limit, hanger reinforcement can be dispensed with this stress calculated by Equation 9;
- In situations where the supported beam transfers torsional moment to the supporting beam, only the stirrup legs adjacent to the interface of the connection between the beams must be considered as hanger reinforcement.

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