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Dynamic analysis of floors subjected to rhythmic human activities based on the use of biodynamic models: numerical and experimental modelling

Análise dinâmica de pisos submetidos a atividades humanas rítmicas com base no uso de modelos biodinâmicos: modelagem numérica e experimental

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Received 08 January 2023 Revised 05 July 2023 Accepted 06 July 2023 Corrected 27 March 2024	Abstract: This work aims the assessment of the dynamic structural response of reinforced concrete floors subjected to rhythmic human activities, based on the use of biodynamic models, in order to consider the people-structure interaction effect. Initially, an experimental modal analysis on the investigated floor, with dimensions of 16m x 35m, was performed, in order to identify and assess the global dynamic behaviour of the structure. In sequence, a finite element model was developed and calibrated through experimental results. After that, based on forced vibration analyses, the floor dynamic response was determined (displacements and accelerations). It is concluded that the biodynamic systems modelling induced significant attenuations on the structural response when compared to those calculated based on the use of traditional "only force" models.
	Keywords: concrete floors, biodynamic systems, experimental dynamics, rhythmic human activities, human comfort assessment.
	Resumo: Este trabalho tem como objetivo a avaliação da resposta dinâmica de pisos de concreto armado submetidos a atividades humanas rítmicas, com base no uso de sistemas biodinâmicos, a fim de analisar o efeito de interação pessoa-estrutura. Inicialmente, realizou-se uma análise modal experimental sobre o piso investigado, com dimensões de 16m x 35m, visando identificar e avaliar o comportamento dinâmico global da estrutura. Em seguida, um modelo de elementos finitos foi desenvolvido e calibrado com base nos resultados experimentais. Posteriormente, a partir das análises de vibração forçada, a resposta dinâmica do piso foi determinada (deslocamentos e acelerações). Conclui-se que a modelagem dos sistemas biodinâmicos induziu atenuações significativas sobre a resposta estrutural quando comparadas àquelas calculadas via utilização de modelos tradicionais de força-rítmica.
	Palavras-chave: pisos de concreto, sistemas biodinâmicos, dinâmica experimental, atividades humanas rítmicas avaliação do conforto humano

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1 INTRODUCTION

Nowadays, there is an expansion of building projects with bold and modern architectures. These buildings have been built with the aim of optimizing execution time and flexibility in terms of final use. In this context, structural

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engineers, based on their experience and knowledge coupled with the use of newly development of the strength of materials, which has allowed the use of long spans, slender structures and lightweight, is advantageous in terms of aesthetic and design costs. On the other hand, Sousa [1] reports that this construction method can directly influence the floors dynamic response, due to the modal parameters modifications. A direct consequence of this design trend is a considerable increase in excessive vibration problems of floors structures.

Regarding building's floors subjected to human-induced vibration, several numerical and experimental investigations have been developed. Lee et al. [2] presented a numerical and experimental modelling of the dynamic structural behaviour of a 39-storey steel structure building. The numerical model results indicated that the floors presented the global vertical vibration mode with natural frequency equal to 2.7 Hz and damping factor of 0.3%. Thus, the upper floors were easily excited due to the resonance between the human activities and the building's structure, because on the twelfth floor there is a gym center. The AISC [3] Design Guides 11 related problems with excessive vibration due human movement caused by aerobic dancing and pop concerts in building's floors with larger span and low mass and structural damping. The study was conducted on buildings with vibration problems in Canada, USA and Switzerland.

Therefore, the evaluation of the floors dynamic structural behaviour is crucial, aiming to establish the dynamic properties (mass, stiffness and damping), in project phase, to avoid the resonance phenomenon that can occur due to proximity (or equality) between the structural system natural frequencies and the excitation frequencies. In this context, in agreement with the experimental results of Varela and Battista [4] the continuous slabs present vibration problems due to the fact that the dynamic excitations produced by human activities are more acute and frequent, and these structural systems present a multimodal behaviour.

In the investigation developed by Gaspar [5], experimental tests were performed on rigid and flexible floors, and the structural system flexibility influence during the practice of rhythmic activities was evaluated. The results showed that the floor flexibility has a significant influence on the human comfort during rhythmic activities. Furthermore, the author noted that the accelerations tended to decrease as the frequencies of activity and their respective harmonics moved away from the floor fundamental frequency.

Another relevant aspect that has been investigated is regarding the human action modelling used to evaluate the dynamic behaviour of floors. In the past, most studies on human-induced vibration have used a deterministic approach to describe human load. However, this approach is not accurate because human loads are not really point loads or line loads. In reality, human loads are random in nature and their intensity and direction vary with time. In this way, the response of a structure to human-induced vibration is also random in nature.

The present research considers the dynamic loads representing human rhythmic activities, based on the use of traditional "only force" models and also the mathematical formulation associated to the biodynamic systems. The traditional models can be found in the design guides AISC [3], SCI [6], and the work developed by Faisca [7]. On the other hand, having in mind the most realistic representation of the human rhythmic activities, the biodynamic systems modelling is based on the dynamic properties of each individual (Sousa [1]).

Thus, considering the increasing number of reported excessive vibration problems in building's floors, this study aims to study a real structural system (fitness centre) with dimensions of 16 m x 35 m, and total area of 560 m², subjected to rhythmic human actions. The floor modal properties are determined by experimental and numerical analysis, with subsequent comparison between them. Considering the floor dynamic response evaluation, in time and frequency domain, and having in mind qualitative and quantitative comparisons, the peak acceleration, and Root Mean Square (RMS) acceleration, and Vibration Dose Values (VDV) were calculated based on the use of different modelling strategies to simulate the rhythmic human dynamic loads.

Finally, the human comfort assessment indicated relevant peak and RMS accelerations and VDV values, when the "only force" models were used in the dynamic analysis. However, based on the investigated floor dynamic response, it was verified that the use of biodynamic systems to represent the human dynamic characteristics have produced lower structural responses, when compared to the dynamic effects related to the "only force" models.

2 INVESTIGATED CONCRETE FLOOR AND FINITE ELEMENT MODELLING

The investigated reinforced concrete floor represents a real structural system and corresponds to a fitness centre located on the eighth story of the State University of Rio de Janeiro (UERJ), Rio de Janeiro/RJ, Brazil, see Figure 1. It is important to emphasize that Figure 2 presents a top view of the Department of Physical Education at UERJ, consisting of 24 (twenty-four) concrete slabs. However, this investigation focused on examining the dynamic structural behaviour of a gym that presents dimensions equal to 16m x 35m, and total area of 560 m², divided in 12 panels of concrete slabs with thickness equal to 12 cm. The concrete material properties were obtained based on the building's original structural project, provided by UERJ. The concrete presents compressive strength of 13.7 MPa and modulus of elasticity equal to

17.6 GPa. It should be noted that this floor was constructed at the end of the 70's and these material properties are in fact real and were widely used in the design practice at Rio de Janeiro/RJ, Brazil, at that time. Furthermore, it was adopted a Poisson's ratio equal to 0.2, and specific weight of 25 kN/m³, according to the criteria established by ABNT NBR 6120 standard [8].

The computational model developed for the floor dynamic analysis adopted the usual mesh refinement techniques present in Finite Element Method (FEM) simulations using the ANSYS [9] program release 12.1. In this numerical model, the concrete slab was simulated based on finite shell elements SHELL63 (ANSYS [9]). The beams and columns were represented by three-dimensional beam finite elements BEAM44 (ANSYS [9]), which considers flexural and torsion effects. It should be emphasized that the investigated reinforced concrete floor was considered working in elastic-linear regime. The floor structural cross sections remain plane after deformation (Bernoulli's hypothesis). It must be emphasized that the developed finite element model of the investigated concrete floor was calibrated based on the use of experimental results. Figure 3 illustrates the developed floor finite element model.



(a) Inside view. (b) Figure 1. Investigated reinforced concrete floor.



Figure 2. Investigated structural model: top view [dimensions in cm].



Figure 3. Finite element model of the investigated reinforced concrete floor.

3 NUMERICAL MODAL ANALYSIS

The natural frequencies (eigenvalues) and vibration modes (eigenvectors) of the reinforced concrete floor were determined based on a free vibration analysis (modal analysis) through the use of the ANSYS [9] software. The investigated first six floor vibration modes presented predominance of flexural behaviour, as illustrated in Figures 4 and 5.

The numerical analysis carried out by Sousa [1] has shown that the concrete floor fundamental frequency f_{01} is equal to 7.89 Hz. It is important to emphasize that according to the Brazilian concrete code NBR 6118 [10] this frequency value should be higher than the critical frequency value equal to 9.60 Hz (=1.2 ×8.0 Hz) [(f_{01} = 7.89 Hz < f_C = 9.60 Hz): rhythmic human activities]. Thus, based on the Brazilian concrete code NBR 6118 [10] recommendations (see section 23.3 and Table 23.1 presented in reference [10]), excessive vibrations could be perceived by users. It is also noteworthy that the floor fundamental frequency (f_{01} = 7.89Hz) and the other five natural frequencies are in the dynamic excitation frequency range (human rhythmic activities), of the second and the third harmonics, according to the ranges defined by Faisca [7] (5.66 Hz to 8.57 Hz), and Littler and Ellis [11] (4.50 Hz to 8.40 Hz), respectively. Therefore, initially, it can be concluded that the investigated floor can be susceptible to excessive vibration and human discomfort.



Figure 4. Investigated concrete floor vibration modes: 1st and 2nd vibration modes.



Figure 5. Investigated reinforced concrete floor vibration modes: 3rd to 6th vibration modes.

4 EXPERIMENTAL MODAL ANALYSIS

The reinforced concrete floor experimental modal analysis was conducted through dynamic monitoring. Initially, the behaviour of the main structure vibration modes was investigated through the numerical modal analysis (see Section 3). This way, it is important to point out that the floor experimental dynamic monitoring was concentrated mainly at the central section of slab concrete slab L3 (slab L3: see Figures 2 and 5), associated to the maximum modal amplitude, also responsible for the maximum energy transfer of the system dynamic response, and related to the third floor vibration mode ($f_{03} = 8.00$ Hz: see Figure 5).

In modal analysis identification techniques there are methodologies that use experimental data just from one response location and others that simultaneously use data from several response locations. In each of those situations there may be one force location or various force locations. Therefore, two different techniques currently used in structures dynamic experimental monitoring are related to the single-input multiple-output (SIMO) and single-input single-output (SISO) (Cunha et al. [12], Debona [13], Brandt [14], Gülbahçe and Çelik [15], Cao et al. [16], Chen et al. [17] and Brownjohn et al. [18]). In SIMO methodology, the frequency responses between various structure sections and the excitation point are measured. The system identification is made based on the Fast Fourier Transforms (FFTs) between each section and the excitation point. On the other hand, in the SISO test the frequency response functions (FRFs) between the excitation force and the only monitored structure section are determined.

In order to obtain the floor experimental modal parameters like natural frequencies, vibration modes and experimental modal damping, the following methodologies were utilised: Test I (experimental structural damping), Test II (impact hammer on the floor) and Test III (modal shaker). In sequence, the Fast Fourier Transforms (FFTs), the Frequency Response Functions (FRFs) and the structural damping of the investigated concrete floor were obtained based on experimental modal analysis.

4.1 Test I: experimental structural damping

The first experimental modal analysis test (Test I: experimental structural damping) consists of evaluating the floor dynamic response resulting from the accelerometer placed at the central section of slab L3, see Figure 6. The results were measured based on the use of a piezoelectric uniaxial accelerometer (PCB Piezotronics Model 393B04), with sensitivity of 998 mV/g. A data acquisition system ADS-2500 manufactured by LYNX Electronic Technology was

used in this investigation. This system is based on signal conditioners that turn the sign of the variation in electrical engineering value (specific deformation, acceleration and force), controlled by a computer.

It should be noted that 5 (five) human jumps were performed in order to obtain the results, where the individual jumped on the floor, repeating the same procedure, as presented in Figure 6. After that, the damping ratio was obtained experimentally in Test I, based on the use of the logarithmic decrement method filtering the respective floor vibration modes. The damping ratio ξ for a lightly damped system can be determined by Equation 1.

$$\xi = \ln \frac{a_i}{a_{i+i}} \frac{1}{2\pi j} \tag{1}$$

Where ξ = structural damping coefficient; terms " a_{i+j} " and " a_{i+j} " mean the ith and (i + j)th measured peak accelerations, respectively. Based on the experimental results, the structural damping mean value is equal to 2.77% (ξ = 2.77%) (see Table 1). It must be emphasized that Cao et al. [16] determined an experimental modal damping ratio equal to 2.3% to a reinforced concrete floor. On the other hand, the AISC Design Guide [3] recommends structural damping coefficient values between 2% and 5% to concrete floors. Several research works frequently suggested damping ratio of 5% for reinforced concrete structures. (Chen et al. [17], Brownjohn et al. [18], Chopra [19], Mario and Leigh [20], Agarwal and Shrikhande [21], and Prislan and Svensek [22]). Thus, the floor damping ratio determined in this research work is relatively low for reinforced concrete structures, indicating that the investigated structural system may not be able to efficiently absorb and dissipate energy associated to the dynamic excitation.



(a) Jump from the platform.
 (b) Floor acceleration in time domain (m/s²).
 Figure 6. Experimental Test I: structural damping assessment.

Table 1.	Experimental	structural	damping	coefficient	values (see Fig	ure 6).
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Excitation	1° jump	2° jump	3° jump	4° jump	5° jump	Mean value
Structural damping	2.75%	2.82%	2.80%	2.77%	2.73%	2.77%

4.2 Test II: impact hammer on the floor

The second test (Test II: impact hammer on the floor) is based on a single-input single-output technique (SISO), combining the Polytec vibrometer PDV-100 and the Dytran impact hammer [22]. This way, the experiment consists of evaluating the floor dynamic structural response (velocity), based on the results determined via laser signal applied on the investigated concrete slab (slab L3 central section: see Figures 2 and 5). Furthermore, it should be noted that the floor was excited by the force application provided by the Dytran impact hammer located near the concrete slab L3 central section, as illustrated in Figure 7.



Figure 7. Experimental Test II: impact hammer on the floor concrete slab.

F. A. Sousa, G. L. Debona, and J. G. S. SilvaColarRunningAuthors

The basic functioning of the Laser Doppler Vibrometry (LDV) analysis methodology is related to a laser beam focused on the tested structure, so that the relative movement between the laser and the structure causes the presence of the Doppler effect, i.e., the relative change in wavelength and frequency of a wave when the observer and the source are moving [4]. The Frequency Response Function (FRF) was determined based on impacts on the concrete floor, associated to the output responses determined in the structure experimental modal analysis (see Figure 8). The FRF function show that the floor experimental frequency ($f_{2exp} = 7.80$ Hz) is close to the third natural frequency calculated in the numerical modal analysis ($f_{03num} = 8.00$ Hz), see Figure 8. The other energy transfer peak related to 6.50 Hz ($f_{1exp} = 6.50$ Hz) is associated to the impact hammer excitation frequency, see Figure 8.



Figure 8. Experimental Test II (impact hammer on the floor concrete slab): FRF assessment.

4.3 Test III: modal shaker

After that, the third experimental test (Test III: modal shaker) consists of modelling the floor dynamic behaviour through the monitoring of the concrete slab acceleration (slab L3 central section), while the floor is excited by the shaker model TV 51140-M (rated force of 400 N), located near slab L3 centre, see Figure 9. The acceleration in the time domain was measured using the accelerometer (PCB Piezotronics) connected to the data acquisition system (ADS 2500). The NCH Tone Generator app controlled by a computer was used to generate square wave signals connected to the vibration exciter (shaker). The harmonic dynamic excitations were applied on the concrete slab considering a frequency range between 1 Hz and 10 Hz, having in mind the floor natural frequencies (see Figures 4 and 5). The reinforced concrete floor experimental acceleration in time and frequency domain is illustrated in Figure 10.



Figure 9. Experimental Test III: modal shaker on the floor concrete slab.





Based on the experimental results, it was observed that the main energy transfer peak associated to 8.05 Hz ($f_{2exp} = 8.05$ Hz) corresponds to the investigated floor third natural frequency ($f_{03num} = 8.00$ Hz), and the other energy transfer peak related to 6.64 Hz ($f_{1exp} = 6.64$ Hz) is associated to the impact shaker excitation frequency applied on the concrete floor, as illustrated in Figure 10.

4.4 Experimental modal analysis: results comparison

It is well known that the numerical modelling of the floor modal parameters (mass and stiffness) can be modified by non-structural elements, human occupation or even boundary conditions. This way, as mentioned before, the finite element model of the studied concrete floor was calibrated based on the use of experimental results. To do this, it was considered on the numerical model, an addition of mass around 5% of the structure total mass, associated to the slab's coating.

Therefore, the investigated reinforced concrete floor natural frequencies values calculated based on the experimental dynamic monitoring and finite element modelling were compared aiming to calibrate the results. Having in mind that the main objective was the identification of the concrete floor maximum modal amplitude associated to the third vibration mode ($f_{03num} = 8.00$ Hz: see Figure 5), it was concluded that the numerical and experimental responses are in good agreement, due to the fact that the experimental and numerical floor natural frequencies approached very well, with differences between 2.56% (Test II) and 0.62% (Test III), respectively, as presented in Table 2.

Tests		Natural Frequency (Hz)	
Tests	Experimental Tests	Finite Element Model	Differences (%)
II	$f_{exp} = 7.80 \text{ Hz}$	$f_{03} = 8.00 \text{ Hz}$	2.56
III	$f_{exp} = 8.05 \text{ Hz}$	$f_{03} = 8.00 \text{ Hz}$	0.62

Table 2. Experimental modal analysis versus and numerical results.

5 MODELLING OF THE BIODYNAMIC SYSTEMS

It is noteworthy the several scientific works (Campista [23], Gaspar et al. [24] Matsumoto and Griffin [25], Shahabpoor et al. [26] and Toso et al. [27]) indicated that the most realistic representation of human rhythmic activities is related to the biodynamic systems defined based on each individuals dynamic properties. Sousa [1] formulated a biodynamic model considering a single degree of freedom (SDOF), consisting of a mass-spring-damper system. This model's choice is aligned with the strategy proposed by Shahabpoor et al. [26].

Thus, the biodynamic systems parameters were calculated based on the dynamic equilibrium equation solution and the classical optimization problem [1], [23], see Equations 2 to 6. The optimization process objective function is the function on the decision variables to be minimized (F_{obj}); see Equation 3. The individual experimental and optimized forces were mathematically correlated; see Equation 4, and the experimental and the optimized forces are calculated in Equations 5 and 6.

$F_i(t) = k_i x_i(t) + c_i v_i(t) + m_i a_i(t)$	(2)
$F_{obj} = 1 - \left(corr_1^2\right)$	(3)
$coor_1 = corr(F_d, F_{d1})$	(4)
$F_{d1} = F_{exp} - m a celfpa1$	(5)
$F_d = x(1)$ velfpa1 + $x(2)$ delsfpa1	(6)

γ

Concerning the parameters presented in Equations 2 to 6; $F_{i(l)}$: force produced by the individual *i* (N); m_i (kg); c_i (Ns/m); k_i (N/m): mass, damping and stiffness of the individual *i*, respectively; $a_i(t)$ (m/s²); $v_i(t)$ (m/s); $x_i(t)$ (m): acceleration; velocity and displacement of the individual *i*, respectively; F_{obj} : function object; *corr_1*: correlation between forces (F_{d} , $x F_{d1}$); F_{d1} (N): experimental force of the individual *i*, excluding the parcel referring to the acceleration multiplied by mass; F_{exp} (N): experimental force of the individual *i*; *m* (kg): mass of each person; *acelfpa1*(m/s²); *velfpa1*(m/s); *deslfpa1*(m): experimental acceleration, experimental velocity and experimental displacement of the individual *i*, respectively; F_d (N): optimized force of the individual *i*; x(1) (Ns/m): optimized damping of the individual *i*; x(2) (N/m): optimized stiffness of the individual *i*.

It is important to point out that, the stiffness (k_i) and damping (ci) were calculated based on the information determined from the experimental tests by knowing characteristics like force, acceleration, velocity and displacement from each tested individual through the optimization process, genetic algorithm (GA). The method has been widely used in order to determine the characteristics of individuals, as observed in the following research works: Abbas et al. [28], Yu et al. [29], Marzbanrad and Afkar [30] and Campista [23].

In addition, there was a correlation between the experimental and optimized forces based on Person's correlation coefficient (PCC), see Equation 7. This coefficient represents a linear correlation between two obtained datasets to evaluate the effectiveness of the performed optimization process.

$$= \frac{\sum(x_i - \bar{x}).(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2).(\sum(y_i - \bar{y})^2)}}$$

$$(7)$$

In Equation 7, *r*: Pearson's correlation coefficient; x_i : optimized force (N); y_i : experimental force (N); \bar{x} : average values of x_i (N); \bar{y} : average values of y_i (N). The representation of the biodynamic system (SDOF) is illustrated in Figure 11.



Figure 11. Modelling of the biodynamic systems (SDOF).

In this work, the experimental tests were developed at the Laboratory of the Graduate Program in Civil Engineering of the Faculty of Engineering of UERJ (PGECIV/FEN/UERJ) [1], [23]. The biodynamic systems dynamic properties were determined based on several experimental tests executed by one hundred and fifty people, considering the individuals jumping on a developed dynamic loading platform. The equipment selected for the experimental tests were a computer, ADS-2002 data acquisition system, an accelerometer, three load cells, a metronome and an MDF platform. Figure 12 illustrates the utilised equipment and the general procedures to the experimental tests development.

This way, considering the experimental response obtained based on the use of the data acquisition system, the dynamic response of each individual (acceleration and dynamic force) was calculated through the use of accelerometers placed on the individual's body and load cells located under the load platform. After that, based on the accelerations collected experimentally, the velocities and displacements were calculated integrating the signal using the MATLAB program. Aiming to eliminate unwanted noise "high pass" and "low pass" filters were used for filtering (clearing) the signals related to dynamic forces, accelerations, velocities, and displacements [1 Hz < fs < 8 Hz; fs represents the signal frequency].

It must be emphasized that the experimental tests were developed aiming to provoke resonance on the floor. In this context, the dynamic excitation frequency (rhythmic human loading: f = 2.2 Hz) investigated in the Campista's research work [23] was chosen to mobilize the third harmonic of the floor dynamic response ($f = 4 \times 2.2 = 8.8 \text{ Hz}$), susceptible to excessive vibrations due to the equality (or even proximity) between this excitation frequency and the investigated structure natural frequencies. Therefore, this excitation frequency of 2.2 Hz [132 bpm (beats per minute)] was used and controlled based on the use of a metronome.



Figure 12. Description of the experimental tests: equipment and general procedures.

The biodynamic systems variables were determined considering the solution of the dynamic equilibrium equation and the classical optimization problem, obtained through the use of genetic algorithms (GA) (Rao [31]). Due to space limitations, Table 3 presents only ten results determined based on the experimental tests and the optimization process. It is noteworthy that in all experimental results the Pearson's correlation coefficient between experimental force and the optimized force have presented a very good relationship.

To calibrate and validate the experimental results, the load platform numerical model was developed based on the use of ANSYS program [9], considering the exact same condition of the experimental tests [23]. The individual was represented by a biodynamic system and modelled based on the use of a mass-spring-damper system, with one degree of freedom (SDOF). Figure 13 presents the comparison between the platform experimental dynamic responses with the same monitored section of the developed platform finite element model.

Person	Mass (kg)	Damping (Ns/m)	Stiffness (N/m)	Pearson's Correlation ($F_{d1} \ge F_d$)
1	92.70	820.94	42812.81	0.98
2	81.95	657.02	37521.26	0.97
3	67.80	547.48	34311.23	0.99
4	56.40	589.41	39755.25	0.95
5	57.60	1019.22	42314.31	0.95
6	81.00	1806.75	29018.39	0.97
7	69.70	645.85	38085.19	0.98
8	88.55	636.79	40037.26	0.99
9	64.80	768.36	37653.04	0.97
10	68.65	678.62	43094.19	0.98

Table 3. People dynamic characteristics: experimental tests.



Figure 13. Numerical versus experimental: acceleration in time and frequency domain.

6 MATHEMATICAL MODELLING OF THE RHYTHMIC HUMAN ACTIVITIES

The human activities impact assessment on building floors is proving to be a difficult subject. The traditional representation of these dynamic loads is based on mathematical models usually known as "only force" models, where the dynamic force component is applied directly on the floor over time in resonance with one of the structure natural frequencies. Thus, the dynamic loading model proposed by Bachmann *et al.* [32], cited in AISC [3] considers the harmonics associated with the excitation frequency due to human dynamic actions, considering a dynamic coefficient for each harmonic, see Equation 8.

$$\mathbf{F}(\mathbf{t}) = \mathbf{Q} + \sum_{i=1}^{N} \alpha_i \mathbf{Q} \sin(2\pi \mathbf{i} \mathbf{f}_{\mathbf{p}} \mathbf{t} - \boldsymbol{\phi}_n) \tag{8}$$

Where, F(t): dynamic excitation in (N); Q: person's weight in (N); f_p : step frequency in (Hz); t: time in (s); i: harmonic number; α_i : dynamic coefficient; ϕ_n : phase difference; N: number of considered harmonics.

The mathematical model proposed by SCI [6] was developed based on experimental tests considering groups of individuals performing rhythmic activities on the test structure. It is noteworthy that the parameters used in this model are related to the number of participants performing human rhythmic activities on the floor, as shown in Equation 9.

$$F(t) = G\{1 + \sum_{n=1}^{\infty} r_{n,v} \sin(2n\pi f_{p}t + \phi_{n})\}$$
(9)

In Which, F(t): dynamic excitation in (N); G: person's weight in (N); $r_{n,v}$: Fourier coefficient induced by v people; n: number of terms of the Fourier series; v: number of people; ϕ_n : phase difference; f_n : step frequency in (Hz); t: time in (s).

The third loading model was developed by Faisca [7] and formulated based on experimental tests considering the Hanning function. Equation 10 represents the parameters considered in the modelling, such as the influence of the human activity impact on the structure.

$$F(t) = CD\left\{K_P P\left[0.5 - 0.5\cos\left(\frac{2\pi t}{T_c}\right)\right]\right\} \to t \leq T_c \text{ or } F(t) = 0 \to T_c \leq t \leq T$$

$$\tag{10}$$

Regarding Equation 10, F(t): dynamic excitation in (N); P: person's weight in (N); K_p : impact coefficient; t: time in (s); CD: lag coefficient; T: activity period in (s); T_c : activity contact period in (s).

7 FORCED VIBRATION ANALYSES

In this research work, the floor dynamic structural response was calculated based on the use of 18 (eighteen) people practising human rhythmic activities arranged on different areas of the concrete slabs. The dynamic loading functions associated to the "only force" models (Faisca [7], AISC [3] and SCI [6]), and also based on the use of biodynamic systems [1], see Equations 2 to 10, will be considered as dynamic excitations on the concrete floor, aiming to represent the rhythmic human actions.

Aiming to assess the people-structure interaction effect, it must be emphasized that in the forced vibration analyses, only the rhythmic human dynamic loads were considered, acting on the calibrated floor structure (calibrated numerical model: see section 4.4), together with the floor self-weight (permanent loads), without the addition of any variable loads.

The dynamic response $[a_p: peak$ accelerations; $a_{w,rms}$: RMS accelerations; VDV: vibration dose values] calculated on the floor structural sections (SS: sections A to F) was calculated considering the "Loading Model I" (LM-I), "Loading Model II" (LM-II) and "Loading Model III" (LM-III). It is also noteworthy that the investigated excitation frequency (f = 2.20 Hz) can induce the resonance phenomena, see Figures 14 to 16 and Tables 4 to 6.

Initially, it becomes evident that the structural system critical sections were determined on the concrete slab floor areas close to the dynamic loads application (see Tables 4 to 6). Nevertheless, the modal response directly affects the investigated reinforced concrete floor dynamic structural response (see Tables 4 to 6).



Figure 14. Loading model I applied on the reinforced concrete floor [dimensions in cm].



Figure 15. Loading model II applied on the reinforced concrete floor [dimensions in cm].



Figure 16. Loading model III applied on the reinforced concrete floor [dimensions in cm].

\mathbf{S}	Bi	Biodynamic [1] Faisca [7] AISC [3]						dynamic [1] Faisca [7] AISC [3]				
S	ap	aw,rms	VDV	ap	aw,rms	VDV	ap	aw,rms	VDV	ap	aw,rms	VDV
А	0.102	0.031	0.072	0.122	0.083	0.163	0.665	0.176	0.476	0.424	0.168	0.399
В	0.103	0.033	0.076	0.122	0.083	0.163	0.487	0.168	0.425	0.293	0.166	0.378
С	0.024	0.009	0.021	0.017	0.012	0.027	0.302	0.200	0.400	0.153	0.040	0.133
D	0.008	0.003	0.008	0.008	0.002	0.007	0.130	0.088	0.174	0.082	0.013	0.051
Е	0.010	0.002	0.005	0.004	0.001	0.002	0.070	0.047	0.094	0.034	0.005	0.021
F	0.004	0.001	0.003	0.002	0.000	0.001	0.035	0.024	0.048	0.017	0.003	0.011

Table 4. Floor dynamic response (LM-I: 18 people): ap (m/s²); aw,rms (m/s²); VDV (m/s^{1.75}).

Table 5. Floor dynamic response (LM-II: 18 people): a_p (m/s²); a_{w,rms} (m/s²); VDV (m/s^{1.75}).

S	Bio	odynamic	[1]		Faisca [7]			AISC [3]			SCI [6]	
S	ap	aw,rms	VDV	ap	aw,rms	VDV	ap	aw,rms	VDV	ap	aw,rms	VDV
А	0.010	0.004	0.011	0.015	0.002	0.008	0.180	0.134	0.266	0.107	0.018	0.069
В	0.024	0.009	0.021	0.039	0.012	0.027	0.341	0.239	0.477	0.214	0.040	0.129
С	0.110	0.035	0.080	0.131	0.086	0.168	0.865	0.467	0.978	0.371	0.170	0.389
D	0.083	0.029	0.068	0.123	0.081	0.158	0.821	0.436	0.916	0.349	0.160	0.366
Е	0.023	0.008	0.018	0.037	0.011	0.025	0.340	0.239	0.477	0.221	0.038	0.130
F	0.008	0.003	0.009	0.015	0.002	0.008	0.180	0.134	0.266	0.107	0.018	0.069

Table 6. Floor dynamic response (LM-III: 18 people): ap (m/s²); aw,rms (m/s²); VDV (m/s^{1.75}).

🔊 Biodynamic [1] Fa		Faisca [7]	aisca [7] AISC [3]			SCI [6]						
\mathbf{v}	ap	aw,rms	VDV	ap	aw,rms	VDV	ap	aw,rms	VDV	ap	aw,rms	VDV
А	0.003	0.001	0.002	0.002	0.000	0.001	0.035	0.024	0.048	0.017	0.003	0.011
В	0.004	0.001	0.003	0.004	0.001	0.002	0.071	0.047	0.093	0.033	0.005	0.021
С	0.009	0.003	0.007	0.009	0.002	0.007	0.138	0.094	0.186	0.088	0.014	0.055
D	0.027	0.008	0.018	0.016	0.011	0.025	0.280	0.187	0.373	0.145	0.037	0.124
Е	0.099	0.030	0.070	0.119	0.081	0.159	0.472	0.163	0.414	0.284	0.161	0.368
F	0.109	0.034	0.079	0.122	0.083	0.163	0.665	0.176	0.476	0.424	0.168	0.399

In sequence of the investigation, Table 7 presents the recommended human comfort criteria limits when the human rhythmic activities are considered, based on the criteria defined by: Setareh [33], Littler and Ellis [11], AISC [3] and SCI [6], and Table 8 presents the studied floor dynamic response maximum values.

aw,rms (m/s ²) [6]	VDV (m/s ^{1.75}) [33]	VDV (m/s ^{1.75}) [11]	Person's reaction
< 0.35	< 0.50	< 0.66	Reasonable: passive people
0.35 - 1,27	0.50 - 3.50	0.66 - 2.38	Disturbing
1.27 - 2.47	3.50 - 6.90	2.38 - 4.64	Unacceptable
> 2.47	> 6.90	> 4.64	Probably causing panic

Table 7. Human comfort criteria limits: rhythmic human activities.

Peak accelerations: $a_{lim} = 0.50 \text{ m/s}^2$ [3].

Table 8. Floor dynamic response maximum values: LM-I; LM-II; LM-III (Figures 14 to 16).

Load model	a _p (m/s ²)	a _{w,rms} (m/s ²)	VDV (m/s ^{1.75})	SS
LM I	0.665	0.176	0.476	Section A
LM II	0.865	0.467	0.978	Section C
LM III	0.665	0.176	0.476	Section F

Based on the concrete floor dynamic structural response, calculated using the LM-I; LM-II and LM-III (Figures 14 to 16), it can be concluded that the "only force" models [3], [6], [7] have produced higher dynamic responses when compared to the dynamic effects related to the utilisation of the biodynamic systems [1], as shown in Tables 4 to 6. The peak accelerations calculated based on the use of the AISC loading model [3] presented the floor dynamic response maximum values, surpassing the human comfort limit ($a_{lim} = 0.50 \text{ m/s}^2$ [3]): [LM I: $a_p = 0.665 \text{ m/s}^2$ (SS-A); LM II: $a_p = 0.665 \text{ m/s}^2$ (SS-F)], as presented in Tables 4 to 6. On the other hand, it must be emphasized that all investigated structural sections far from the dynamic excitation present lower peak accelerations values below de recommended human comfort limit (see Tables 4 to 6).

Furthermore, considering the RMS accelerations and VDV values, the obtained results indicated that the human comfort limits were surpassed only when LM-II (see Figure 15) was investigated, and considering the floor structural sections SS-C and SS-D, close to the dynamic loads (see Tables 4 to 6): $(a_{w,rms} = 0.35 \text{ m/s}^2 \text{ [6]})$ [LM II: $a_{w,rms} = 0.467 \text{ m/s}^2$ (SS-C); $a_{w,rms} = 0.436 \text{ m/s}^2$ (SS-D)]. (VDV < 0.50 m/s^{1.75} [33]) [LM II: VDV = 0.978 m/s^{1.75} (SS-C); VDV = 0.916 m/s^{1.75} (SS-D)]. However, it must be emphasized again that all investigated floor sections far from the dynamic loads present lower RMS accelerations and VDV values below de recommended human comfort limits (see Tables 4 to 6).

It can be concluded that the concrete floor dynamic response critical section clearly is associated to the maximum modal amplitude [SS-C: see Figure 5 ($f_{03} = 8.0 \text{ Hz}$)], see Tables 4 to 6. Following the analysis, the floor dynamic structural response is presented, based on the accelerations values in time and frequency domain, respectively, see Figures 17 to 20. It must pointed out that Figures 17 to 20 show typical examples, based on the investigated floor dynamic response, considering the structural section C (LM-II: SS-C; see Figure 15).

The acceleration response spectrum in the frequency domain clearly shows the energy transfer peaks associated to the rhythmic human dynamic excitation harmonics (f = 2.20 Hz; f = 4.40 Hz; f = 6.60 Hz; f = 8.80 Hz), and also another energy transfer peak corresponding to the floor third natural frequency [$f_{03} = 8.10 \text{ Hz}$: maximum modal amplitude (section C: see Figure 5 and Figure 17)].



Figure 17. Floor dynamic structural response [Biodynamic [1]: LM-II (section C)].



Figure 18. Floor dynamic structural response [Faisca [7]: LM-II (section C)].



Figure 19. Floor dynamic structural response [SCI [6]: LM-II (section C)].



Figure 20. Floor dynamic structural response [AISC [3]: LM-II (section C)].

In addition, considering the investigated floor dynamic response, it must be emphasized as a relevant conclusion of this research work that when the people-structure dynamic interaction effect was considered in the analysis, based on the use of the biodynamic systems, the general picture is quite different, due to the fact that there are no problems related to excessive vibrations or human discomfort. It is worth to mention that the "only force" models (AISC [3], SCI [6] and Faisca [7]) are applied directly on the concrete slabs and do not include the people dynamic characteristics. On the other hand, the modelling strategy based on the use of biodynamic systems [1] incorporates the individuals dynamic characteristics (mass, stiffness and damping), modifying the floor dynamic response, see Tables 4 to 6 and Figures 17 to 20. This way, the dynamic loads generated based on the use of the biodynamic systems [1] provides a more realistic structural response and a better floor human comfort assessment. Finally, it is fair to mention that Faisca mathematical model [7] provided dynamic response values higher than those associated to the utilisation of the biodynamic models [1], but with the same order of magnitude. On the other hand, the AISC [3] and SCI [6] models provides very high dynamic structural responses, probably unrealistic, and should be used very carefully, when floors human comfort assessments are required in design practice.

8 CONCLUSIONS

In this investigation the dynamic structural behaviour of an existing reinforced concrete floor with total area of 560 m² located at the State University of Rio de Janeiro (UERJ), Rio de Janeiro/RJ, Brazil, subjected to human rhythmic activities was investigated, based on results associated to the experimental tests and finite element modelling. Therefore, the following conclusions can be drawn from the results presented in this research work:

- 1. The reinforced concrete floor modal analysis was performed numerically and experimentally. The experimental dynamic structural response was determined based on the use of accelerometers placed on the concrete slabs as well as by a vibrometer utilising a moving Laser Doppler Vibrometry (LDV) methodology, and also based on the use of a modal shaker. Therefore, the results were calibrated based on the developed floor finite element model using the ANSYS [9] software. The numerical and experimental results presented good agreement, showing that the concrete floor frequencies determined experimentally [Test II: $f_{exp} = 7.80$ Hz and Test III: $f_{exp} = 8.05$ Hz] approached the numerical frequency ($f_{num} = 8.00$ Hz) very well, with differences of 2.56% (Test II) and 0.62% (Test III), respectively, validating the developed finite element model, according to the data obtained in the experimental tests.
- 2. The forced vibration analyses were carried out on the concrete floor, when subjected to human rhythmic activities induced by 18 people considering three loading cases (LM-I; LM-II and LM-III). The dynamic analysis was conducted based on the use of mathematical functions representing the human rhythmic actions through the use of traditional "only force" models (AISC [3], SCI [6] and Faisca [7]) and biodynamic systems [1]. The floor dynamic structural response assessment indicated that the "only force" models have induced higher levels of displacements and accelerations than those provoked by the biodynamic systems [1]. This fact is associated to the difference between the mathematical models, especially the biodynamic systems that incorporate the people dynamic characteristics associated to the mass, damping and stiffness.
- 3. Having in mind the worst design situation associated to the LM-II (18 people) and considering the dynamic excitation provided by the "only force" models [3], [6], [7], based on the criteria proposed by AISC [3], it should be noted that the peak acceleration limit was surpassed ($a_p = 0.865 \text{ m/s}^2 > a_{lim} = 0.50 \text{ m/s}^2$) causing human discomfort. Considering the design limits associated to the RMS accelerations [6] and VDV values [33], the calculated results indicated that these limit values were exceeded as well: $a_{w,rms} = 0.467 \text{ m/s}^2 > 0.35 \text{ m/s}^2$ and VDV = 0.978 m/s^{1.75} > 0.50 m/s^{1.75}.
- 4. On the other hand, investigating the floor dynamic response in the same design situation associated to the LM-II (18 people), but considering the results taking into account the people-structure dynamic interaction effect through the use of the biodynamic systems, there are no problems related to excessive vibrations or human discomfort $a_p = 0.110 \text{ m/s}^2$ $< a_{\text{lim}} = 0.50 \text{ m/s}^2$; $a_{w,\text{rms}} = 0.035 \text{ m/s}^2 < 0.35 \text{ m/s}^2$; VDV = 0.080 m/s^{1.75} < 0.50 m/s^{1.75}. It is important to emphasize that the dynamic loads generated based on the use of the biodynamic systems [1] provides a more realistic structural response and a better floor human comfort assessment, due to the fact that this modelling strategy incorporates the individuals dynamic characteristics (mass, damping and stiffness), modifying the floor dynamic response.
- 5. This research work clearly has shown that the use of biodynamic mathematical models and the consideration of the human-structure dynamic interaction effect, especially to incorporate the people's damping effect in the analysis, are crucial to a more realistic floors dynamic response assessment, when the human comfort is investigated. The reinforced concrete floors dynamic structural response analysis requires that the people-structure dynamic interaction effect be included, aiming to determine more realistic results.

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