



ORIGINAL ARTICLE

Study on the compatibilization of hierarchical models for tunnel design

Estudo para a compatibilização de modelos hierárquicos no projeto de túneis

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Abstract: This work presents a study on the compatibilization of the lining load results between continuous ground mass and bedded beam models for tunnel design, through the calibration of the loads imposed to the bedded beam models. A review on compatibilization premises and the computation of “ideal” compatibilization loads, which yield identical results between model hierarchies is presented. A case study was developed, illustrating that even with significant simplification of the calibrated loads, if they bear magnitude and distribution that resembles those of the “ideal” compatibilization load, reasonable compatibility, potentially better than that of usual generic imposed loads, may be obtained. Motivated by this observation, a parametric study on the magnitude and distribution of the “ideal” compatibilization load was performed, yielding conclusions that foresee the estimation of simplified compatibilization loads directly from the physical problem definition.

Keywords: tunnels, hierarchical modeling, numerical modeling.

Resumo: Esse trabalho apresenta um estudo acerca da compatibilização dos esforços solicitantes no revestimento entre modelos de maciço contínuo e de anel sobre apoios para o projeto de túneis, a partir da calibração de carregamentos impostos aos modelos de anel sobre apoios. Uma revisão das premissas de compatibilização e do cálculo de carregamentos de compatibilização “ideal”, que levam a resultados idênticos entre hierarquias de modelos é apresentada. Um estudo de caso foi desenvolvido, ilustrando que mesmo com significativa simplificação dos carregamentos calibrados, se eles possuírem magnitude e distribuição que remetam às do carregamento de compatibilização “ideal”, compatibilidade razoável, e potencialmente melhor que aquela obtida com carregamentos impostos usais genéricos, pode ser obtida. Motivada por tal observação, uma análise paramétrica da magnitude e distribuição do carregamento de compatibilização “ideal” foi efetuada, resultando em conclusões que antevêm a estimativa de carregamentos de compatibilização simplificados diretamente da definição do problema físico.

Palavras-chave: túneis, modelagem hierárquica, modelagem numérica.

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1. INTRODUCTION

Duddeck [1] formalized a conceptual sequence of the basic routine structural engineering activities involved in the analysis and design process of a structure. Most of said activities necessarily imply a simplification and idealization of reality, with the definition of mathematical models to assess relevant physical phenomena within the design process. Two main model type groups are highlighted, Research Models and Technical Models. In summary, Research Models aim at representing as many physical phenomena with as much accuracy as possible. Technical Models strive to cover the essential physical phenomena with adequate accuracy to allow for design with adequate safety and performance

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with the lowest possible cost and effort. However, the formulation of adequate Technical Models requires the use of sound Research Models.

These reflections closely dialogue with Bucalem and Bathe's [2] definition of hierarchical modelling. A same set of physical phenomena may be represented by a series of hierarchical models, where increasing hierarchy leads to increasing amount and accuracy of physical phenomena representation, but also to greater modelling effort.

Due to the multitude, complexity and interdisciplinarity of physical phenomena involved in tunnel analysis, hierarchical modelling and various model hierarchies and methods are relevant within the design practice. The International Tunneling Association [3] lists 4 groups of models/methods for the structural design of tunnels, namely: continuum models; bedded-beam models; empirical approaches; and the observational method.

The continuum models may allow for the potential representation of complex physical phenomena, but also lead to greater effort / complexity in input data estimation / postulation and output data interpretation. Thus, it is common and may be advantageous in the design practice to apply the lower hierarchy bedded beam models together with the higher hierarchy continuum models, either for quick estimates, model validation, and, in some cases, direct application after calibration [4].

This paper follows up on Działoszynski and Stucchi [5] with an investigation of the potential result compatibilization between continuum models – henceforth referred to as hierarchy H1 models – and bedded-beam models – henceforth referred to as hierarchy H2 models. Specifically, the calibration of simplified hierarchy H2 model imposed loads based on the “ideal” compatibilization load (as defined in Działoszynski and Stucchi [5]) are analyzed, with a parametric study of the “ideal” compatibilization load in relation to the variation of reference hierarchy H1 model's parameters and hypotheses.

2. COMPATIBILIZATION OF RESULTS BETWEEN MODEL HIERARCHIES

Illustrating the common parallel application of model hierarchies in the tunnel design practice routine, many authors, like Duddeck [1], Duddeck and Erdmann [6], Van der Poel et al. [7], Prado and Waimberg [4], Vu et al. [8] and Działoszynski and Stucchi [5] have compared the application and results of different model hierarchies when representing the same physical problems.

Działoszynski and Stucchi [5] compared model hierarchies H1 and H2 from a conceptual standpoint, establishing basic premises for the potential result compatibilization of model hierarchies. In summary:

- The main difference between model hierarchies is their respective representation of the ground mass and the associated consequences to the manifestation soil structure-interaction.
- It is clear that the intended result compatibilization must refer to results that are common to both model hierarchies. Thus, model hierarchy result compatibilization shall refer to computed beam loads and displacements.
- Compatibilization refers herein to the calibration of input parameters of model hierarchies which lead to similar results. Due to the nature of the respective model inputs, compatibilization shall be achieved through the calibration and estimation of model hierarchy H2 input parameters, rather than model hierarchy H1 input parameters.
- From a conceptual standpoint, perfect and generalized compatibilization would imply that all the complex behaviors of the deformable solid ground mass of hierarchy H1 were fully translated at the soil-structure interface through the links / “springs” and loads imposed to model hierarchy H2. Thus, in practice, compatibilization should only be viable in an approximate, case-by-case basis (that is, for each type of problem, compatibilization rules may vary).

Under these premises, model inputs may be calibrated to achieve result compatibilization between models.

2.1. “Ideal” compatibilization load

One of the hierarchy-H2 models main advantages is its simplicity, and it is usually desirable that its imposed loads and links / “springs” have a rather simplified definition. Nonetheless, for the investigation on the postulation of simplified compatibilization loads, it is interesting to know the exact value of “ideal” compatibilization loads that yield identical results for analogous models in both hierarchies, despite of its potential complexity.

Działoszynski and Stucchi [5] formalized and validated a practical procedure for computing “ideal” compatibilization loads within the Finite Elements Method (FEM). The procedure may be easily implemented in usual available FEM applications.

In summary, the application of the procedure presumes:

- A given physical problem is defined and solved through a hierarchy H1 model, for which the results are considered to be known beforehand in all nodes and calculation stages.
- The definition of the hierarchy H1 model implies the definition of a beam / plate type element finite elements mesh for the lining, as well as a nodal force vector due to self-weight applied to the lining, f_n^w .
- Links / “springs” must be postulated to the hierarchy H2 model. The definition is arbitrary, and the procedure will yield “ideal” compatibilization loads regardless of the quality of the hypotheses. However, the compatibilization loads shall be more intuitive / representative for better link / “spring” hypotheses.

Under these premises, the procedure yields a nodal force vector due to the ground mass¹, f_n^s , which leads to identical displacement and beam load results between model hierarchies if imposed to a hierarchy H2 model with: analogous finite mesh to that of the reference H1 model; the same implied nodal forces due to self-weight, f_n^w ; and the arbitrarily defined links / “springs”. A nodal force vector due to links / “springs”, f_n^l , will also indirectly be calculated.

Of course, the procedure implies that the results for the hierarchy H1 model are already known, so the “ideal” compatibilization loads thusly computed have little direct practical application. However, the “ideal” compatibilization loads can be used to investigate patterns and trends for the calibration of simplified compatibilization loads, as presented in the sections hereafter.

2.2. Simplified load calibration through the “ideal” compatibilization load

The “ideal” compatibilization load may be used to calibrate simplified imposed loads for hierarchy H2 models. Hierarchy H2 model imposed loads may be represented with vertical and horizontal components. As illustrated by Blom [9], they are usually defined as distributed loads applied to the tunnel area projected to a plane perpendicular to the respective load component direction. Loads shall be defined in such manner in this paper, and the calibration of loads from the “ideal” compatibilization nodal forces imply their distribution in the abovementioned projected areas.

Działoszynski and Stucchi [5] presented a case study evaluating the beam load result compatibility between analogous hierarchy H1 and H2 two-dimensional models. In the case study, the imposition of a load calibrated from the “ideal” compatibilization load yielded beam load results with a significantly better compatibility than the imposition of usual loads from the bibliography / design practice.

Of course, various simplified compatibilization loads may be calibrated from the same “ideal” compatibilization nodal forces, depending on their degree of complexity. It is expected that increasing complexity leads to more compatible results, whereas decreasing complexity leads to simpler load definition.

Under these premises, consider the same hypotheses as those of the abovementioned case study (see Działoszynski and Stucchi [5] for details) with a single stage excavation calculation sequence (excavation and support application are modelled as simultaneous, in a simplified manner) and both normal and tangential hierarchy H2 links / “springs”.

Two imposed loads were calibrated by “fitting” from the “ideal” compatibilization load:

- Imposed load C-A was simplified form the “ideal” compatibilization load, but no restriction for its degree of complexity was imposed.
- Imposed load C-B was also simplified form the “ideal” compatibilization load, but its distribution was imposed as bilinear, that is, substantially simplified.

Figure 1 illustrates the proposed loads. Only the modelled symmetrical part is represented.

¹ Corresponding to the “ideal” compatibilization load.

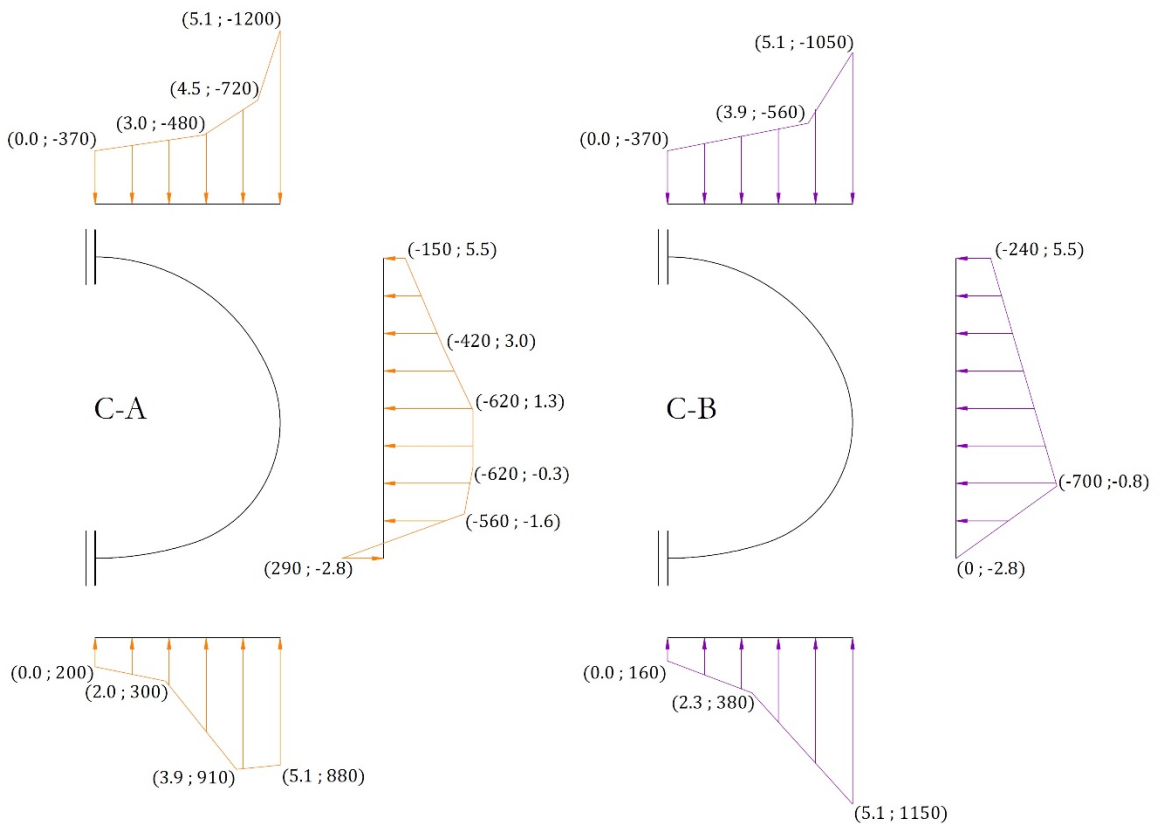


Figure 1. Loads calibrated from “ideal” compatibilization load proposed for comparison of compatibility with varying loading complexity.

Figures 2 and 3 show the axial force and bending moment beam load results for the hierarchy H2 model with the proposed loads, as well as for the reference hierarchy H1 model, for the sake of comparison. Results were plotted along the developed tunnel length of the modelled symmetrical part, s , starting from the base of the invert (see figures detail).

As illustrated by the figures, both proposed loads yielded results that are considerably compatible with those of the reference hierarchy H1 model, especially for the axial force, with practically identical maximum values, even for the greatly simplified load C-B. It should be highlighted that bending moments for the investigated problem were relatively low.

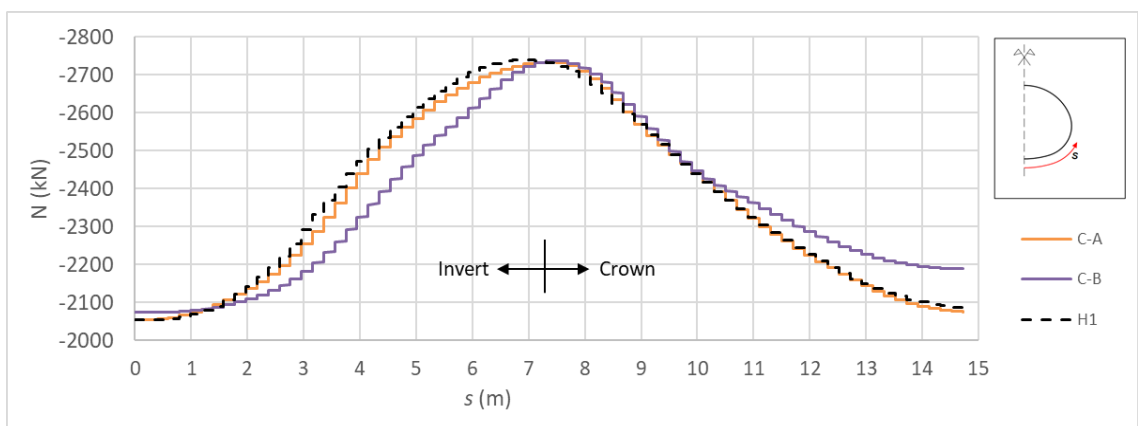


Figure 2. Comparison of axial force beam load results for varying complexity of calibrated load.

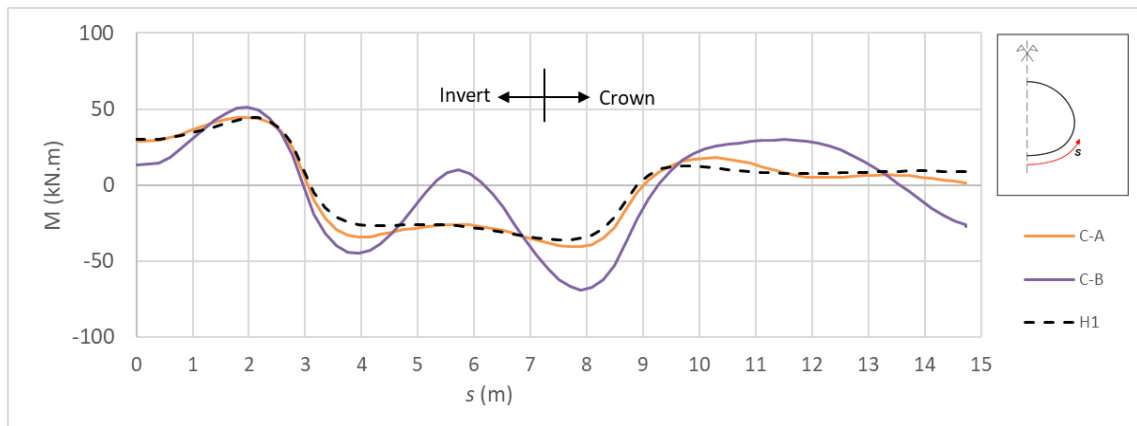


Figure 3. Comparison of bending moment beam load results for varying complexity of calibrated load.

Of course such comparison is not totally balanced, as the loads C-A and C-B are formulated specifically for the physical problem / tunnel being analyzed, and from the reference hierarchy H1 model results, which must be known beforehand. Usual imposed loads from bibliography and design practice are generically formulated, without any previous knowledge or necessary relation to the reference hierarchy H1 model results.

Naturally, the abovementioned observations refer to a single case study. Still, they may be seen at least as illustrative of the notion that even after great simplification, loads calibrated from the “ideal” compatibilization load can yield reasonable - and potentially better than usual generic imposed loads - result compatibility with the reference hierarchy H1 model.

That is, as long as the hierarchy H2 imposed load bears magnitude and distribution that adheres in a general manner to that of the “ideal” compatibilization load, the aforementioned reasonable result compatibility may be achieved between models.

However, there is an essential limitation to the practical application of “ideal” loads, as its computation requires solving beforehand the reference hierarchy H1 model, after which the practical significance of solving the analogous hierarchy H2 model is mostly lost. Such notion motivated the parametric study described hereafter, envisioning the estimation of compatibilization loads without the need of knowing beforehand the solution of the reference hierarchy H1 model.

It should be highlighted that the abovementioned discussion refers to result compatibility between models, and not necessarily between model and actual physical behavior. Even the hierarchy H1 models bear idealizations and simplifications that may lead to less accurate results. The “ideal” compatibilization loads essentially “inherit” those potential limitations. Of course, different hierarchy H1 models may be formulated to better represent physical behavior, with recalculation of the “ideal” compatibilization load.

3. “IDEAL” COMPATIBILIZATION LOAD PARAMETRIC STUDY HYPOTHESES

The conclusions from item 2.2 motivated a parametric study on the “ideal” compatibilization load response to the variation of hypotheses of the reference hierarchy H1 model, which reflect a variation on the definition of the physical problem. The present analysis focuses more directly on New Austrian Tunnelling Method (NATM) primary lining, but many conclusions apply broadly to tunnels in general.

The aim of the parametric study was to identify general patterns and trends in the magnitude and distribution of the “ideal” compatibilization load according to the hierarchy H1 model / physical problem definition. As concluded in item 2.2, as long as a hierarchy H2 model imposed load bears magnitude and distribution that generally resemble that of the “ideal” compatibilization load, reasonable - and potentially better than usual generic imposed loads - result compatibility may be obtained, even if it is greatly simplified.

The present study investigates the abovementioned patterns through simple models, foreseeing the potential postulation of compatibilization loads directly from the physical problem definition, without the need to know beforehand the results from the hierarchy H1 model, as the “ideal” compatibilization load computation would require.

3.1. Basic premises and disclaimers

As explained in item 2.2, the present study aims at investigating compatibility between models, and not between model and actual physical behavior measured from instrumentation.

Due to the intrinsic idealizations, even within the hierarchy H1 models, sometimes less realistic results were obtained (for example, exaggerated invert heave when linear-elastic behavior is considered). Nonetheless, the hypotheses in the study are similar to usual design practice considerations, and thus, those less realistic results are part of the routine engineering design challenges.

Analogously, due to its diversity, the combinations of hypotheses from the parametric analysis include some simulations for which structural analysis indicated inadequate safety or performance of the tunnel. In actual design, the dimensioning would be duly adjusted. Still, those simulation were considered conceptually adequate, enriching the spectrum of evaluated hypotheses.

In summary, for each combination of model H1 hypotheses, henceforth referred to as scenarios, the following activities were performed:

- Solve the hierarchy H1 model defined with the postulated combination of the hypotheses.
- Compute the hierarchy H2 model's nodal components of the "ideal" compatibilization load due to the soil mass f_n^s through the procedure outlined in item 2.1. The computation is performed for a postulated link / "spring" hypothesis applied for all scenarios.
- Distribute and normalize the nodal forces f_n^s into the normalized "ideal" compatibilization load q_n^s .

It must be acknowledged that the activity sequence presumes the postulation of the hierarchy H2 links / 'springs' through bibliography, with compatibilization achieved through the imposed load. Thus, the present study implies protagonism of the imposed load and its calibration for compatibility, despite acknowledged limitations of the link / "spring" hypotheses of the bibliography.

3.2. Modeling hypotheses

Both model hierarchies were defined and solved through the FEM applying the software Midas GTS NX. Models were two-dimensional under plane strain hypothesis and considered only the righthand side symmetrical part of the physical problem.

For both hierarchies, the lining was modelled with 2-nodded Bernoulli-Euler beam elements. For the tunnel lining, constitutive behavior was assumed as homogeneous, isotropic and linear elastic, i.e., plasticity, and its associated parameters – e.g. friction angle and cohesion – were considered only for ground materials. It is acknowledged that some scenarios yielded results that would lead to the yielding of the lining, but to maintain homogeneity and the intended degree of complexity of the parametric analysis, elastic behavior was considered even in these cases.

For hierarchy H1, ground was modelled with 3-noded triangular plane strain elements, with various material hypotheses, depending on the scenario. No interface elements were considered.

For hierarchy H2, links / "springs" were applied discretely in the nodes along the full tunnel perimeter. Links / "springs" in the normal direction were considered with linear-elastic behavior in compression and null reaction to tension. The compressive stiffness was considered for each material as recommended by Martinek and Winter [10] for tunnels with cover equal or higher to 3 times its diameter. It must be highlighted that for the present study the scenarios with cover of 1 equivalent diameter do not adhere to such hypothesis, but the link / "stiffness" was maintained nonetheless, to allow for homogeneity along the analyses. Tangential links / "springs" were considered as linear elastic with stiffness estimated according to Plizzari and Tiberti [11].

3.3. Basic structure of the scenarios

The varying scenario hypotheses combinations were applied over a common basic model structure, as illustrated in Figure 4. The figure illustrates some parameters that are varied for the parametric analysis and some parameters that are kept constant throughout.

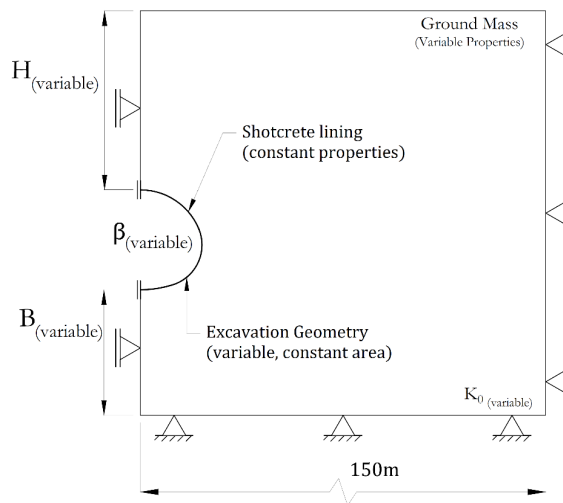


Figure 4. Basic hierarchy H1 model structure for the parametric analysis.

The basic structure reflects simple models, aiming at keeping the parametric analysis at a relatively low complexity level to allow better evaluation of fundamental physical behaviors.

A rectangular finite element mesh was considered, with horizontal ground surface and homogeneous stratigraphy. Mesh refinement was imposed such that the elements close to the tunnel lining had edges about 0,2m long, and the largest elements, close to the model boundary had edges about 1,0m long. This setup yielded result fields with acceptably smooth aspect and lower sensitivity to further refinements.

The external rightmost boundary of the finite elements mesh was considered at a distance where disturbances due to the modelled physical problem are negligible. The sole load applied to the model was self-weight, and the water table was considered to be below the lower boundary of the model.

The hierarchy H1 models were formulated as a common sequence of elastoplastic equilibrium calculations. Stage sequence was considered as:

- (E1) Initialization of the in-situ stress field prior to excavation through the K_0 procedure.
- (E2) Excavation of the full geometric section of the tunnel, with partial excavation stress relief on a fraction of $\omega = 1 - \beta, (0 \leq \beta \leq 1)$, with the excavation still unlined.
- (E3) Excavation stress relief of the remaining fraction of β with application of the full tunnel lining.

Figure 5 schematically illustrates the calculation sequence.

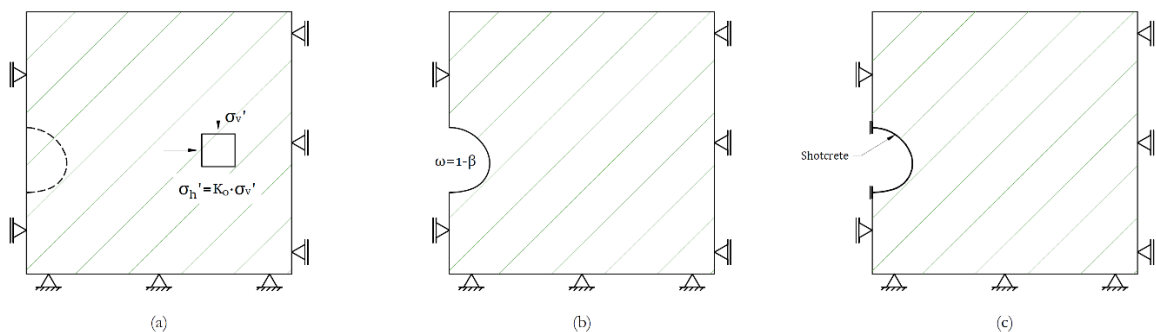


Figure 5. Calculation stages of hierarchy H1 model. (a) Stage (E1); (b) Stage (E2); (c) Stage (E3).

It must be highlighted that the proposed calculation sequence represents a significant simplification of the NATM excavation sequence and its tridimensional aspects. Nonetheless, the tridimensional effects of the lag between excavation and lining installation were considered in a simplified manner analogously to the β -method (Schikora and Fink², apud Plaxis [12]), allowing for the consideration and investigation of the basic associated physical behaviors.

Tunnel support was considered as a uniform 0.25m thick shotcrete layer applied along the full tunnel perimeter. As previously described, linear-elastic behavior was considered for the lining, with unit weight of 25kN/m³, Young's modulus of 25GPa and Poisson's ratio of 0.25.

4. "IDEAL" COMPATIBILIZATION LOAD PARAMETRIC STUDY SCENARIOS

The hypotheses combined to form the evaluated scenarios are described in this section.

4.1. Tunnel geometry

Three tunnel geometries reflecting different types of tunnels were evaluated, as illustrated in Figure 6, from left to right, geometries G1, G2 and G3. All geometries bear the same area / equivalent diameter, seeking to normalize the excavated mass and, to a certain degree, the net stress relief.

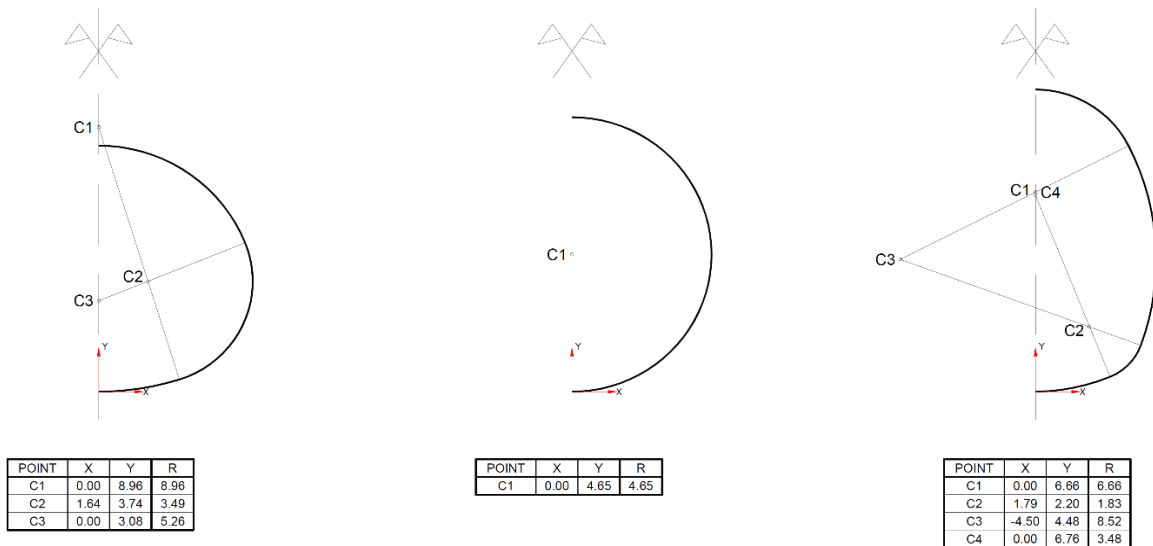


Figure 6. Tunnel geometries considered in the parametric analysis.

4.2. Tunnel cover

Three cover dimensions were evaluated (classification according to Chapman et al. [13]), namely: low cover, measuring 1 equivalent diameter; medium cover, measuring 3 equivalent diameters, and high cover, measuring 8 equivalent diameters.

4.3. Distance between invert bottom and model bottom

Three dimensions of the vertical distance between the bottom of the invert and the bottom of the model were considered, measuring 1, 2 and 5 equivalent diameters.

² K. Schikora and T. Fink, "Berechnungsmethoden moderner, bergmännischer Bauweisen beim UBahn-Bau," *Bauingenieur*, vol. 57, pp. 193-198, 1982.

4.4. In situ stress state prior to excavation

Under the basic structure of the scenarios, the in-situ stress state is fully defined through tunnel cover and the coefficient of horizontal stress K_0 , which was considered in the values of 0.5, 0.8 and 1.5. For specific cases, the value of 0.3 was also considered.

4.5. Unlined stress relief fraction

The unlined stress relief fraction ω may be defined with relation to the lined stress relief fraction, β . The value of β was evaluated as 0.25, 0.5 and 1.

4.6. Soil mass materials

For a given constitutive model, material behavior is given through a series of independent parameters. However, to maintain the degree of complexity and scenario count intended for the present study, parametric analysis was performed varying full sets of material parameters, rather than varying material parameters individually. Parameter sets were stipulated to be coherent to usual geomaterials – according to common practice and technical publications, such as FHWA [14] - and with mutual compatibility (e.g. materials with higher Young’s Modulus are more resistant, etc.).

A total of 5 possible geomaterials mA, mB, mC, mD1 and mD2 were considered. In each scenario a single isotropic material is applied homogeneously to the full ground mass. Self-weight was considered constant throughout the materials so that the initial in situ stress state was independent of the material.

Materials mA and mC were modelled as linear elastic perfectly plastic with a Mohr-Coulomb yielding criterion. Material mB was considered as linear-elastic, seeking to represent a material with the same constitutive laws as mA and mC but with strength high enough to prevent any yielding. Table 1 illustrates the parameters for materials mA, mB and mC.

Materials mA, mB and mC were formulated to investigate different materials under the same constitutive model representation. It must be acknowledged that the considered constitutive laws are a great simplification of the actual geomaterial physical behavior, but as the applied hypotheses are relatively common in design practice, they are considered pertinent to the present study.

To evaluate said limitations, some scenarios applied materials mD1 and mD2, which considered the modified Cam Clay constitutive model [15]. Despite some limitations regarding quantitative representativity, this constitutive model reproduces various additional geomaterial behaviors. Materials mD1 and mD2 were considered with the same parameters except for the hardening parameter hypothesis as summarized in Table 2. For usual soil mechanics equivalent parameters, mD1 and mD2 would have C_c of 0.1, C_r of 0.01 and a friction angle of 30°.

Table 1. Parameters for materials mA, mB and mC.

Material	Unit weight	Young’s modulus	Poisson’s ratio	Cohesion	Friction angle	Dilatancy angle
	γ kN/m ³	E MPa	ν	c' kPa	ϕ' °	ψ °
mA	20	100	0.3	75	22	0
mB	20	1000	0.3	-	-	-
mC	20	20	0.3	0	30	0

Table 2. Parameters for materials mD1 and mD2.

Material	Unit weight	Poisson’s ratio	Initial void ratio	Stiffness parameter 1	Stiffness parameter 2	Critical state line slope	Over Cons. Ratio
	γ kN/m ³	ν	e_0	λ	κ	M	OCR
mD1	20	0.3	0.5	0.04342	0.00434	1.2	1.1
mD2	20	0.3	0.5	0.04342	0.00434	1.2	6

4.7. Scenario summary

A total of 93 scenarios were analyzed. Scenarios were named according to their hypothesis combination as [Tunnel geometry]-[Cover]-[Distance from base of invert to base of model]-[K_0 value]-[β value]-[Ground mass material]. This scenario labeling system is applied to refer to scenarios henceforth in this paper.

Scenario analysis and global result inspection was performed individually, rather than with an automated routine. To make such approach feasible, the scenarios do not contemplate all possible combinations between the postulated hypotheses, and only the 93 combinations were selected and analyzed.

5. “IDEAL” COMPATIBILIZATION LOAD PARAMETRIC STUDY RESULTS

The 93 scenarios were divided into subgroups to evaluate the effects of the variation of each parameter regarding the “ideal” compatibilization loads, which are the focus of the study. Other outputs were also evaluated to interpret the physical meaning of the patterns observed for the “ideal” compatibilization loads.

To allow interpretation with the usual design practice form of hierarchy H2 models’ imposed load [9], the “ideal” compatibilization loads were decomposed in horizontal and vertical components and distributed along the projected lining area perpendicular to the direction of the respective component direction.

Additionally, to allow better comparison, loads were normalized by the pre-excavation in situ stress in the respective direction. Regarding this hypothesis, it is crucial to highlight that the normalization factor varies for each position – due to varying cover – and each direction – due to the K_0 coefficient.

Translating the abovementioned hypotheses mathematically, the horizontal component of the normalized “ideal” compatibilization load in a given node / position i , denoted by $[q_n^s]_{x,i}$ may be calculated by (Equation 1):

$$[q_n^s]_{x,i} = \frac{[f_n^s]_{x,i}}{\frac{|y_i - y_{i-l}|}{2} + \frac{|y_{i+l} - y_i|}{2}} * \frac{l}{(y_{terr} - y_i) * \gamma * K_0} \tag{1}$$

Where $i+l$ and $i-l$ denote the nodes adjacent to node i , $[f_n^s]_{x,i}$ is the horizontal component of the “ideal” compatibilization nodal force in node / position i ; y_j is the vertical coordinate of an arbitrary node j ; γ is the unit weight of the homogeneous hierarchy H1 ground mass; and y_{terr} is the vertical coordinate of horizontal ground surface.

Analogously, the vertical component of the normalized “ideal” compatibilization load in a given node / position i , denoted by $[q_n^s]_{y,i}$ may be calculated by (Equation 2):

$$[q_n^s]_{y,i} = \frac{[f_n^s]_{y,i}}{\frac{|x_i - x_{i-l}|}{2} + \frac{|x_{i+l} - x_i|}{2}} * \frac{l}{(y_{terr} - y_i) * \gamma} \tag{2}$$

Where x_j is the horizontal coordinate of an arbitrary node j .

Near the edges of the respective projected area “spans”, the projected area magnitude is close to zero. Thus, even low nodal forces may yield high distributed loads, tending to highlight small disturbances from the numerical approximation. These potential high values are not considered representative or relevant for the actual load applied to the lining, because the area where these distributed loads are applied is practically null (i.e. these high values near the edges are usually not observed for actual the nodal forces). These values shall be nonetheless presented, for the sake of transparency, but result plot scale shall prioritize the representative / relevant results.

5.1. Variation of tunnel cover

The effects of tunnel cover variation on the normalized “ideal” compatibilization load were assessed based on the comparison of 27 scenarios. The normalized “ideal” compatibilization loads for 18 of those scenarios are presented in Figure 7 for illustration.

In summary it could be observed that:

- On the one hand, an “arching” effect is observed due to normalized values practically always lower than 1. On the other hand, cover variation usually did not lead to great / generalized variation in magnitude or distribution of the normalized “ideal” compatibilization loads. This behavior could be attributed to limitations of the constitutive model applied, and similar observations are also made in other publications, such as Hejazi et al. [16] or Chakeri and Unver [17].
- Usually slightly higher values were observed for normalized vertical loads applied to the crown and slightly lower in the invert for the higher cover scenarios.
- Plastification tends to cause local disturbances in the normalized load distribution. When the plastification pattern significantly shifts with cover, a trend of increase of normalized magnitude and homogenization of distribution is observed as elastoplasticity increases.
- Ascending / “floating” movements of the lining seem to be related to significant variations in normalized magnitude and distribution of the “ideal” compatibilization load, mainly for vertical loads applied to the crown. This occurs for low cover scenarios, especially for the vertically elongated geometry G3 and lower relative groundmass stiffness, as illustrated in Figure 8. These movements lead to compressive reaction in the normal hierarchy H2 links / “springs” in the crown, with otherwise null values for tension.

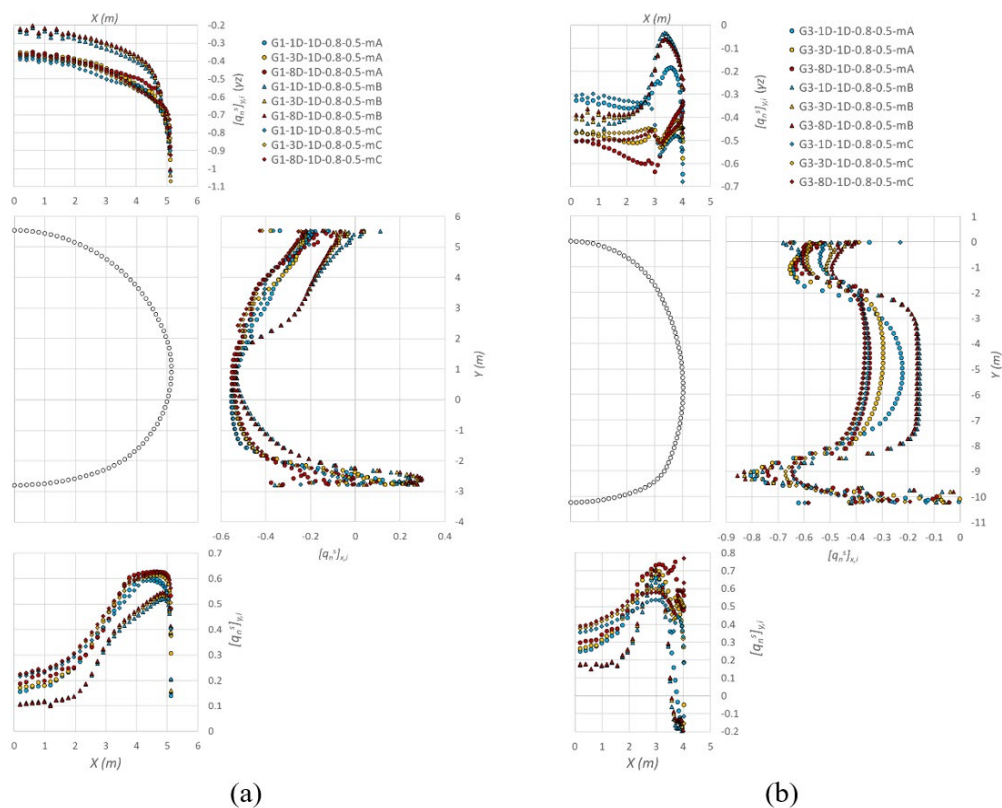


Figure 7. Normalized “ideal” compatibilization load for scenarios: (a) varying cover and materials in geometry G1; (b) varying cover and materials in geometry G3.

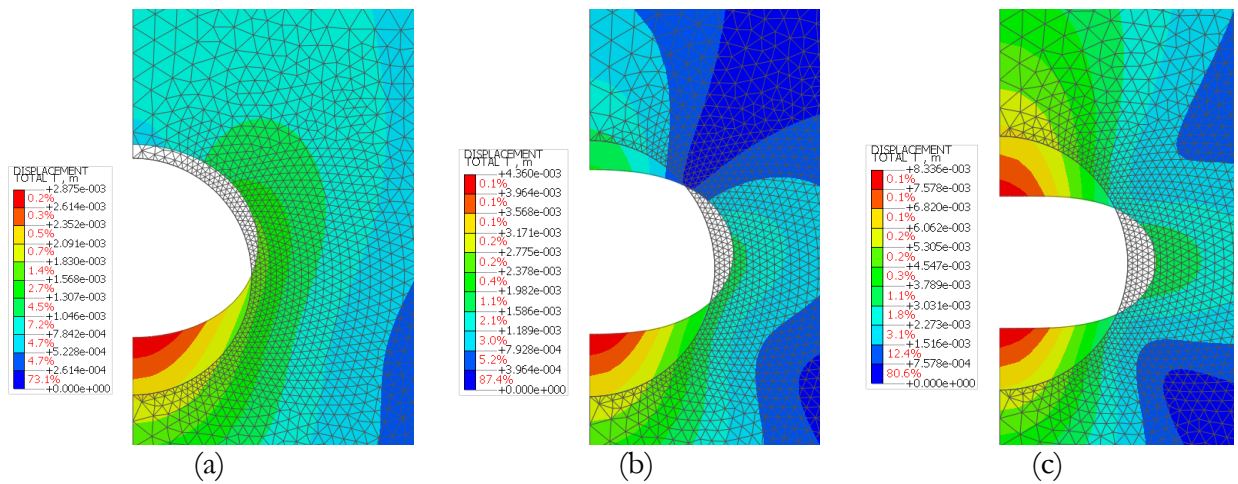


Figure 8. Variation of displacements in calculation stage (E3) (lined displacements) for scenarios varying cover in material with lower stiffness. (a) low cover; (b) medium cover; (c) high cover.

Regarding these “ascending” movements, it is considered that they may be exaggerated due to limitations of the applied constitutive models, and thus, the associated variation of the “ideal” compatibilization load may also not be representative of actual physical behavior.

5.2. Variation of soil mass materials

The effects of soil mass material variation on the normalized “ideal” compatibilization load were also assessed based on the comparison of 27 scenarios. The normalized “ideal” compatibilization loads for 18 of those scenarios are presented in Figure 7 for illustration.

It could be observed that:

- The evaluation was performed based on a limited amount of simulations, and with a rather simple constitutive model. Thus, conclusion must be considered in the light of this limitations. The observation regarding the applied constitutive model shall be explored in item 5.6.
- Relatively trivial observations were verified: higher ground stiffness usually leads to lower normalized loads and lower ground strength leads to more plastification, which cause higher normalized magnitude and more homogeneous distribution.
- The effects of the “ascending” movements of the lining crown described in item 5.1 were once again observed.

5.3. Variation of tunnel geometry and in situ stress state prior to excavation

As described hereafter, it was noticed that the way in which the shape of the tunnel influences the general distribution of normalized “ideal” compatibilization loads bears a relationship with the proportion between horizontal and vertical in situ stresses prior to excavation, quantified by K_0 . The parameters were thus analyzed together in this section, based on 51 scenarios.

Figure 7a shows various results for scenarios with geometry G1. It may be compared to Figure 7b which shows results for analogous scenarios with geometry G3, respectively, to illustrate the effects of the variation of tunnel geometry. Figure 9a presents results for a collection of scenarios that illustrate the effects of the variation of K_0 .

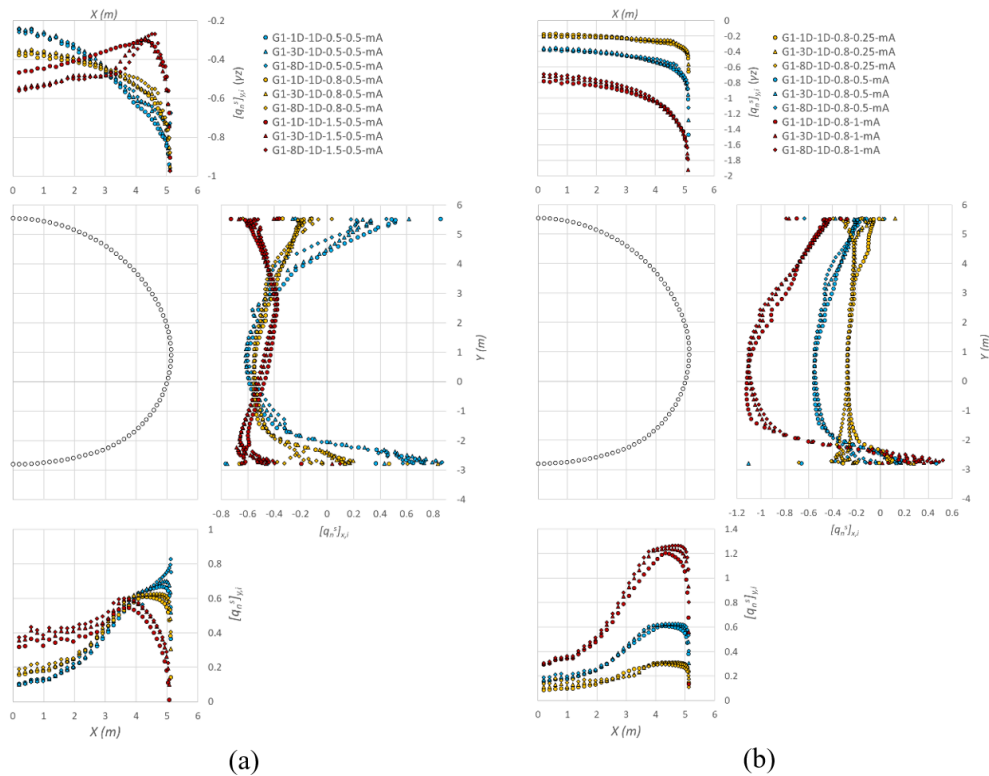


Figure 9. Normalized “ideal” compatibilization load for scenarios varying: (a) cover and K_0 in geometry G1; (b) cover and β for geometry G1.

Regarding the normalized vertical loads applied to the crown:

- For the horizontally elongated geometry³ normalized loads tend to concentrate in the edge of the “span”⁴, with reduction in the direction towards the symmetry axis. Contrarily, for the vertically elongated geometry⁵ normalized loads tend to concentrate “midspan”⁶.
- Analogous behavior was observed for the variation of K_0 , where higher values led to the tendency of normalized loads being concentrated “midspan” and lower values led to the tendency of normalized loads concentrating in the edge of the “span”.

Regarding the normalized horizontal loads applied to the side of the tunnel:

- For the horizontally elongated geometry normalized loads tend to concentrate “midspan”⁷, with reduction in the direction towards the edges. Contrarily, for the vertically elongated geometry normalized loads tend to concentrate near the edges of the “span”⁸.
- Analogous behavior was observed for the variation of K_0 , where higher values led to the tendency of normalized loads being concentrated in the edge of the “span” and lower values led to the tendency of normalized loads concentrating “midspan”.

Regarding the normalized vertical loads applied to the invert:

³ That is, the horizontal axis is longer than the vertical.

⁴ That is, the outer edge of the area projection where the load is applied.

⁵ That is, the horizontal axis is shorter than the vertical.

⁶ That is, the symmetry axis of the area projection where the load is applied.

⁷ That is, close to the middle of the height of the area projection where the load is applied.

⁸ That is, the top and bottom of the area projection where the load is applied.

- In practically all scenarios lower normalized loads are observed “midspan”⁹ and increase as the edge of the “span”¹⁰ approaches.
- Still, trends like those observed for the vertical loads applied to the crown were observed. Higher K_θ led to higher values “midspan” and vice versa.

It must also be highlighted that variations of the arc radius along tunnel perimeter led to variation of the normalized load magnitude and distribution, especially when the transition is abrupt.

The analogy between behavior due to geometry elongation and K_θ is backed up by the physical interpretation that rotating by 90° the section of an oval tunnel that is very far from the model’s boundaries would yield the same result as rotating the pre-excitation *in situ* stress field (lining self-weight neglected, ground self-weight proportionally adjusted). For the modelled scenarios, this correspondence is not exact, but the fundamental physical behavior still applies.

Under these premises, and despite specific differences regarding magnitude and trend intensity, as well as limited exception cases, two general distribution patterns were identified for the normalized “ideal” compatibilization loads:

- General distribution pattern I: vertical normalized loads applied to the crown concentrate in the edge of the “span”; horizontal normalized loads applied to the side of the tunnel concentrate “midspan”.
- General distribution pattern II: vertical normalized loads applied to the crown concentrate “midspan”; horizontal normalized loads applied to the side of the tunnel concentrate in the edges of the “span”.

For both patterns the normalized vertical loads applied to the invert concentrate in the edge of the “span”, with varying intensity.

It was observed that the definition of the normalized load distribution pattern bears a relationship with the geometry – which may be quantified in a simplified manner by the section’s aspect ratio¹¹ - and the initial *in situ* stress state - quantified by K_θ – as illustrated in Figure 10. General pattern I is observed for lower aspect ratios and lower K_θ , and General pattern II is observed for higher aspect ratios and higher K_θ .

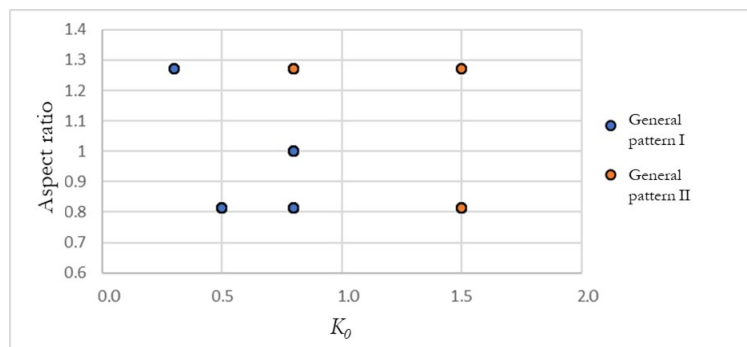


Figure 10. Normalized load distribution pattern depending on section’s aspect ratio and K_θ value.

The models also showed a relation between the distribution pattern and the variation of displacements during calculation stage (E3), as illustrated in Figure 11 with some typical results. It may be observed that General distribution pattern I is related to variation of displacements that lead to horizontal “flattening”¹² of the section. General distribution

⁹ That is, the symmetry axis of the area projection where the load is applied.

¹⁰ That is, the outer edge of the area projection where the load is applied.

¹¹ Largest vertical dimension divided by largest horizontal dimension.

¹² That is, the horizontal axis length increases and the vertical axis length decreases, or the horizontal axis length increases more than the vertical axis length, or the horizontal axis length decreases less than the vertical axis length.

pattern II is related to variation of displacements that lead to vertical “flattening”¹³ of the section. These trends are coherent with those related to tunnel section aspect ratio and K_0 values, which may be associated as relative stiffness and loading in relation to the aforementioned displacement patterns.

In some scenarios the displacements are less characteristic of the postulated definitions. Consistently, the general distribution patterns of the normalized loads are also somewhat less characteristic.

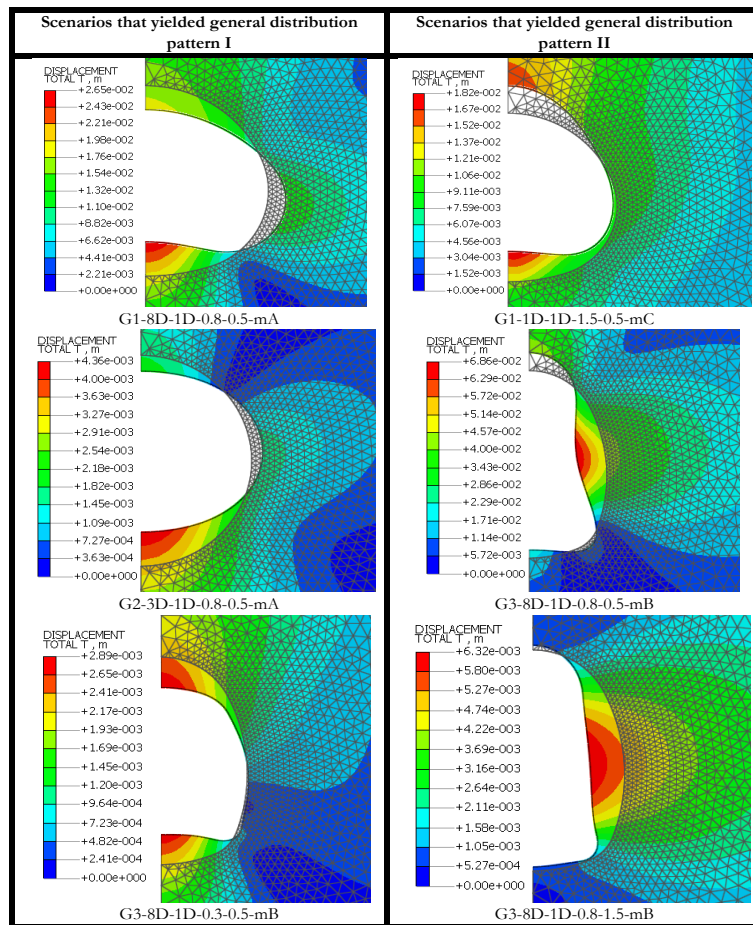


Figure 11. Variation of displacements in calculation stage (E3) in relation to normalized load general distribution patterns.

5.4. Variation of unlined stress relief fraction

The effects of the β value on the normalized “ideal” compatibilization load were assessed based on the comparison of 27 scenarios. The normalized “ideal” compatibilization loads for 9 of those scenarios are presented in Figure 9b for illustration.

In summary, it was observed that:

- The variation of β leads to a variation, from a general standpoint, of the same proportion of the normalized magnitude of the “ideal” compatibilization loads. The distribution pattern of the normalized loads does not vary greatly in most cases.

¹³ That is, the vertical axis length increases and the horizontal axis length decreases, or the vertical axis length increases more than the horizontal axis length, or the vertical axis length decreases less than the horizontal axis length.

- Such trend, however, is not exact. Deviations from this behavior are as great as the variation of the plastification pattern within the ground mass as β varies. Major variations on the plastification pattern in the ground mass led to shifts in the normalized load distribution for specific cases.
- Nonetheless, despite the effects of plasticity, a general trend of proportionality between normalized magnitudes and β could be observed in most cases.

Thus, the value of β bears major importance in the normalized magnitude of the compatibilization loads. On the other hand, it has minor influence in the distribution of the normalized loads, except when severe variation of the plastification pattern of the ground mass is implicated.

5.5. Variation of distance between invert bottom and model bottom

The effects of the distance between the bottom of the invert and the bottom of the model on the normalized “ideal” compatibilization load were assessed based on the comparison of 27 scenarios. The normalized “ideal” compatibilization loads for 9 of those scenarios are presented in Figure 12a for illustration.

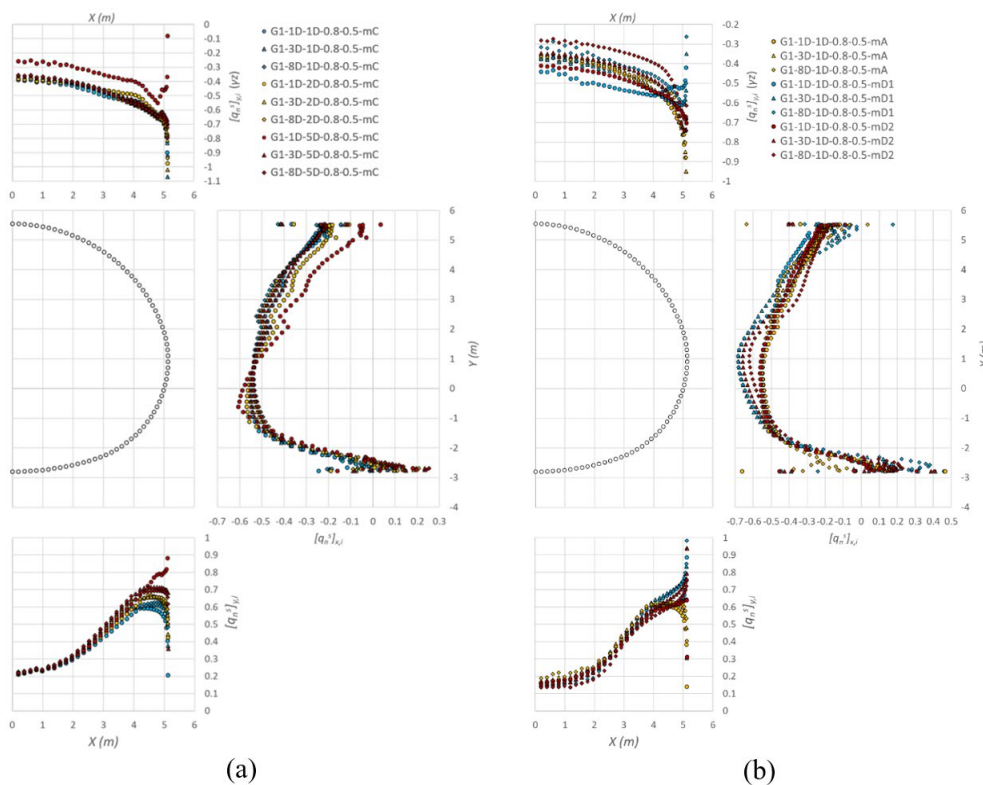


Figure 12. Normalized “ideal” compatibilization load for scenarios varying: (a) the distance between invert bottom and model bottom for geometry G1; (b) constitutive models for geometry G1.

It could be observed that:

- In comparison to the other evaluated hypotheses and parameters, the variation of the distance between the bottom of the invert and the bottom of the model showed generally a smaller importance regarding effects on magnitude and distribution of the normalized loads.
- The proportion of the effects on the normalized magnitude is low in relation to the increase in distance required to cause it. That is, higher distance variations are required to cause substantial effects in the normalized load magnitudes.

- Examples of such effects are an increase in the normalized magnitudes of the vertical loads applied close to the outer part of the invert, as well as a smaller variation of the normalized horizontal loads.
- The effects are more prominent in low cover scenarios. A significant effect was noticed for the scenarios with highest distance from invert bottom to model bottom and lowest cover, with materials mA and mC. The effect may be attributed to the previously discussed “ascending” movement of the lining.

5.6. Variation of the constitutive model selected for soil mass representation

To evaluate the influence of the constitutive model applied to represent the ground mass, scenarios with material mA – represented with a linear elastic perfectly plastic model – were compared to analogous scenarios with materials mD1 and mD2 – represented by the modified Cam Clay model.

The three materials had their parameters estimated with reference to relatively hard clays, aiming at comparing the variation of material representation, rather than the variation of the material itself.

For this analysis 9 scenarios were considered. The normalized “ideal” compatibilization loads for these scenarios are presented in Figure 12b for illustration.

The limited number of evaluated scenarios must be acknowledged, highlighting that constitutive model comparison was performed for very particular conditions. Nonetheless, for the considered scenarios, it may be observed that:

- The effects of cover variation on the normalized loads are much more pronounced for the modified Cam Clay model. This observation is coherent with the different hypotheses of the constitutive models.
- Thus, the observations from item 5.1, regarding the lower effect of cover variation on the normalized loads for scenarios applying linear elastic perfectly plastic models with Mohr Coulomb yielding criterion are not necessarily valid for more complex constitutive laws.
- The hardening parameter, defined in the evaluated scenarios by the OCR, had a significant effect on the normalized loads.
- Even with limited number of scenarios evaluated, a higher complexity on the trends of normalized “ideal” compatibilization loads in relation to hierarchy H1 input parameter variation is foreseen, with maximum normalized loads for intermediary values of input parameters.

Nonetheless, the general distribution pattern of the normalized “ideal” compatibilization loads was generally similar along the scenarios, even with the variation of the constitutive model. As previously discussed in item 5.3, the evaluated scenarios showed that this pattern bears an important relationship with tunnel geometry and *in situ* stress states.

6. CONCLUSIONS AND FUTURE WORK

Within the context of tunnel modeling and lining analysis and design, this paper presented a study on the potential compatibilization of the beam load results between continuous ground mass models and bedded beam models, through the calibration of the loads imposed to the bedded beam model. The study follows up on previous work by Działoszynski and Stucchi [5], and is based on a procedure to compute “ideal” compatibilization loads, as presented and defined in the aforementioned previous publication.

A case study presented in this paper indicated that distributed loads calibrated from the “ideal” compatibilization load yielded beam loads that had reasonable and better compatibility with the reference continuous ground mass model than those obtained when usual generic loads from the bibliography / practice are applied. This compatibilization was achieved even when a significant simplification (bi-linear distribution) was imposed to the calibrated load. Despite the fact that the observations refer to a single case study, it at least illustrates that as long as the load imposed to the bedded beam model bears general magnitude and distribution that resembles those of the “ideal” compatibilization load, the aforementioned reasonably compatible results may be obtained.

However, an essential limitation – which is not imposed to the usual generic loads from the bibliography / practice – applies to the “ideal” compatibilization load, as its computation requires knowing beforehand the results of the reference continuous ground mass model.

Motivated by such observation, and foreseeing the potential estimation of compatibilization loads directly from the physical problem definition - and without the need of knowing beforehand the results of the reference continuous ground mass model - a parametric study was performed and presented in this paper. The study was performed to investigate the effects of the variation of the reference continuous ground mass model inputs on the general magnitude

and distribution of the “ideal” compatibilization loads. The study applied a simple model structure to assess fundamental physical behaviors through the individual formulation, processing and interpretation of 93 simulation scenarios.

Qualitative / semi-quantitative trends and patterns were observed, which may serve as the basis for future work aiming at quantitative conclusions for the direct estimation of simplified compatibilization loads. Such future work would likely require a significant amount of model and result processing, probably involving routines for automated model definition and processing, as well as results processing and correlation, for example analogously to what Kung et al. [18] presented for retaining walls.

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