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ORIGINAL ARTICLE

Distribution of load effects and reliability of reinforced concrete frames: intact and with columns removed

Distribuição de esforços e confiabilidade de pórticos de concreto armado: íntegro e com remoção de colunas

Pedro Henrique Preto Facholli[a]
André Teófilo Beck[a]

[a]Universidade de São Paulo – USP, Escola de Engenharia de São Carlos, São Carlos, SP, Brasil

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Abstract: The design of reinforced concrete (RC) frames is made on a member-by-member basis. Similarly, in the literature, the reliability of RC beams and columns is often studied in isolation from the rest of the structure. Yet, in the construction of regular frames, symmetry and regularity are often exploited, resulting in the same design for each element type. This is despite of different load effects on different parts of the structure, which leads to significant variations in the failure probability of the elements. Hence, in this paper, we investigate the reliability of members and the distribution of load effects in regular RC frame buildings, considering intact and column loss cases, where symmetry is lost. Results show that the ratios of normal-to-bending loads change significantly along building height, and this has a significant impact on reliability of individual columns.

Keywords: structural reliability, regular frame structures, reinforced concrete structures, discretionary column removal.

Resumo: O projeto de estruturas aporticadas de concreto armado é feito elemento a elemento. Da mesma forma, a confiabilidade de vigas e pilares de concreto armado é frequentemente estudada, na literatura, considerando elementos isolados do restante da estrutura. Entretanto, na construção de pórticos regulares, é usual explorar a simetria e regularidade, resultando no mesmo dimensionamento para cada tipo de elemento. Porém, com esforços solicitantes diferentes atuando em diferentes partes da estrutura, variações significativas podem ocorrer na probabilidade de falha dos membros. Neste trabalho, é investigada a confiabilidade dos elementos e a distribuição de esforços em estruturas aporticadas de concreto armado, considerando casos intactos e com remoção de colunas, nos quais a simetria é perdida. Os resultados mostram que as razões normal – momento fletor variam significativamente ao longo da altura do edifício, exercendo um impacto considerável na confiabilidade dos pilares de cada lance.

Palavras-chave: confiabilidade estrutural, estruturas aporticadas regulares, estruturas de concreto armado, remoção discricionária de coluna.

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1 INTRODUCTION

In the design of building structures, one needs to consider the uncertainties affecting the strength of structural materials, the expected loads on the structure, and the accuracy of engineering calculation models. The design of building structures is made using design codes, which employ partial safety factors to overcome the uncertainties and produce safe and reliable structures. More recently, structural reliability theory has allowed a more comprehensive understanding of the impacts of uncertainties in structural design.

Corresponding author: André Teófilo Beck. E-mail: atbeck@sc.usp.br
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Loading in structures include those that the structure will certainly be exposed to, although of unknown intensity, but also exceptional loads due to accidental or malevolent actions, to which the structure is exposed to with smaller probability. One exceptional loading condition is loss of a load-bearing element due to accidental or malevolent action. Accidental actions include traffic accidents (impact of moving truck on a wall or column), accidental explosions (like gas, for instance), major fire, or gross human error. Malevolent actions include bomb detonations, as in terrorist acts. Some recent occurrences of whole frame collapses, following initial damage of limited proportions, have raised the necessity for robust structural design. A structure is said to be robust if it can withstand local damage, which does not propagate in a manner which is disproportional to the extent of the initiating damage event.

Brazilian design codes do not have specific requirements with respect to structural robustness, other than the requirement for appropriate binding of slabs, and for continuity of lower beam reinforcements [1]; see a review of Brazilian normative in [2]. Yet, progressive collapse events have been observed in Brazil, for example: rupture of balconies of 15 floors of Edifício Don Gerônimo, Maringá PR (2008); full collapse of Edifício Real Class, 34 floors, Belém do Pará AM (2011); full collapse of the Liberdade building in Rio de Janeiro RJ (2012); partial collapse of Edifício Senador, 13 floors, São Bernardo do Campo SP (2012); the almost-complete collapse of Poty Shopping Center, Teresina PI (2013), during construction; and more recently, the full collapse of Andrea building in Fortaleza CE (2019) due to inadequate maintenance of columns.

Under multiple hazards, the probability of structural collapse is given by [3]–[5]:

$$p_c = P\left[C\right] = \sum_{H}\sum_{LD} P\left[C|LD,H\right]P\left[LD|H\right]P\left[H\right] \tag{1}$$

where C is for collapse, LD is local damage, and H is a hazard, like fire, traffic accident, explosion, etc. In Equation 1, $P\left[H\right]$ is the probability of occurrence of hazard H, $P\left[LD|H\right]$ is the conditional probability of local damage, given occurrence of a hazard, and $P\left[C|LD,H\right]$ is the conditional probability of collapse, given hazard and local damage. The sum over H and LD considers all relevant hazards and local damages a structure may be exposed to. Local damage may be in terms of localized strength reduction, due to a fire or vehicle impact; or the loss of load-bearing structural elements, such as walls or columns.

Following Equation 1, it is possible to reduce the probability of collapse in three ways: a) by limiting the threat probability (installing screening barriers, limiting speed of vehicles, prohibiting inflammable and explosive substances); b) by limiting the probability of local damage, given hazard (safety barriers for impact, local strengthening measures); and c) by reducing or arresting damage propagation (by proper design including alternate load paths). This paper addresses damage propagation by looking into the conditional failure probability of neighboring beams and columns, considering discretionary column loss events.

Many works in the published literature have addressed the resisting mechanisms, and formulated methodologies for modelling and verification of structures exposed to progressive collapse due to local damage: Starossek [6] studied and classified the damage propagation in buildings, Izzuddin et al. [7] proposed a simplified dynamic assessment for progressive collapse, Khandelwal and El-Tawil [8] investigated the robustness of building systems by the *pushdown* analysis, Masoero et al. [9] presented an analytical model for PC of 2D frames, Oliveira et al. [10] used a high-fidelity FE model to investigate the safety of structures after the loss of a column.

International design codes present both direct and indirect measures for design against progressive collapse. Indirect measures address the problem qualitatively, by prescribing minimal levels of ductility, strength, and continuity [11]. Direct design measures against progressive collapse are presented in North-American guidelines such as [12] and [13], which include local strengthening and design for alternate load paths. In the so-called *Alternate Load Path Method* (APM), it is warranted that the structure has the required strength to re-distribute loads initially sustained by the damaged column, bridging over it, for a period sufficient for repair action to be taken. This "bridging" strength refers to the term $P\left[C|LD,H\right]$ in Equation 1. Following [3]–[5], the conditional collapse probability can be accepted to be between about 0.01 and 0.1 since, even if local damage is certain given hazard ($P\left[LD|H\right]=1$), hazard probabilities are usually small (between 10^{-6} and 10^{-5} for typical hazards like explosions and fire). References [14]–[16] address the hazard probabilities for which the APM design becomes cost-effective, from a risk-management perspective.

Based on APM requirements, many studies have addressed the behavior of beams and slabs, responsible for the reserve strength, and considered as the last line of defense against progressive collapse [17]. However, in column loss situations, the overload on adjacent columns is also significant. As the main elements in bearing the vertical loads, special attention also needs to be given to columns in PC situations.

Interestingly, most studies on the reliability of RC beams and columns address individual elements which are "detached" from the main structure. This may be a consequence of the usual design procedures, where elements are designed on an element-by-element basis. Focusing on results published in Brazil, studies on the reliability of "detached" beam elements include [18]–[23] and "detached" columns include [24]–[29]. Studies addressing "detached" beams and/or columns, based on Brazil and elsewhere, include [30]–[32]. Yet, to facilitate construction of regular RC buildings, it is usual to consider the same beams and slabs throughout the building, and to produce similar columns (same cross section dimensions) over the height of the building, perhaps differentiating interior and facade columns. As load effects and normal-to-bending action ratios are not the same throughout the building, design for beam and column regularity produces variations in the reliability of elements located in different parts of the building. These variations have not been thoroughly studied in the literature. Importantly, building regularity is lost in case of a column failure.

Within this framework, this paper presents a study on the spatial distribution of load effects in regular RC frames, and the impact on the reliability of beams and columns located in different parts of the structure. We address the cases of intact frames, and the situation of discretionary column removal. The study includes frames of four and eight stories, with removal of internal and external (facade) columns. Under column removal, we consider usual frames (not strengthened), and strengthened following ASCE 7-16 [33] recommendations for abnormal loading condition. Our study is limited to gravitational loads and to linear structural behavior. Albeit simple, the linear model captures the distribution of load effects in the structure, and the interaction between normal and bending load effects in columns. Non-linear material modelling will be considered in future research. Horizontal loading and out-of-plumbness will also be addressed in the future.

2 RELIABILITY ANALYSIS BY FORM

Probabilistic analysis of structures is a field of research that has been extensively developed since the second half of the 20th century, when works of Freudenthal [34], Cornell [35] and Hasofer and Lind [36] revolutionized the assessment of structural safety and laid the foundations for structural reliability.

The reliability of a structure refers to a degree of belief that it meets its technical design requirements during a specified lifetime and respecting operational conditions [37]. The technical design requirements can be defined by limit state equations, which represent a boundary between desirable (safety or service) and undesirable state (failure) of the structure. These limit states are given for each failure mode of each element of the structural system:

$$g(\mathbf{X}) = g(X_1, X_2, \dots, X_n) = 0 \tag{2}$$

where $\mathbf{X}$ is a random variable vector. Negative values of the limit state equation represent failure, whereas positive values represent survival. The boundary between failure (Ω_f) and survival (Ω_s) domains is given by $g(\mathbf{x}) = 0$, such that:

$$\Omega_f = \{\mathbf{x} \mid g(\mathbf{x}) \le 0\} \tag{3}$$

$$\Omega_s = \{x \mid g(x) > 0\} \tag{4}$$

Thus, the failure probability, which is the probability of limit state violation, can be evaluated by a multi-dimensional integration of the joint probability density function $f_{\mathbf{X}}(\mathbf{x})$ over the failure domain:

$$p_f = P[X \in \Omega_f] = P[g(X) \le 0] = \int_{\Omega_f} f_x(x)\,dx \tag{5}$$

The solution of Equation 5 can be performed by Monte Carlo simulation or transformation methods. In this paper, the First Order Reliability Method (FORM) is used, by way of the UQLab software [38] in a MATLAB environment.

The FORM method consists in mapping the joint probability density function from the original design space $\mathbb{X}$ to the standard Gaussian space $\mathbb{Y}$ through the Principle of Normal Tail Approximation [39] and the Nataf transformation [40]. In this space, the limit state equation is approximated by a tangent hyper-plane at the design point. This point represents the most probable point in the failure domain and is obtained by the constrained optimization problem:

$$\begin{aligned}&\text{find}: \mathbf{y}^* \\ &\text{which minimizes}: d = \|\mathbf{y}\| = \sqrt{\mathbf{y}^{\mathbf{t}}\mathbf{y}} \\ &\text{subject to}: g(\mathbf{y}) = 0\end{aligned} \tag{6}$$

where $\mathbf{y}$ is the random variable vector in the standard Gaussian space, $\mathbf{y}^*$ is the design point and d is the distance of a point to the origin. The solution to this problem yields the Hasofer and Lind's [36] reliability index, β, commonly used as a safety metric in structural codes.

The transformation from the original space to the standard space is done as the optimization algorithm moves towards the design point. In this paper, the improved Hasofer-Lind-Rackwitz-Fiessler algorithm [41], [42] is used to ensure unconditional convergence. At the design point, the limit state equation is approximated by a tangent hyper-plane. Using the property of radial symmetry of the normal standard distribution, a first order approximation of the failure probability is given by:

$$p_f \cong \Phi(-\beta) \tag{7}$$

where β is the reliability index and Φ(.) is the standard gaussian cumulative probability distribution function.

The contribution of each random variable to the calculated failure probabilities can be found by:

$$\boldsymbol{\alpha}\left(\mathbf{y}^*\right) = \frac{\nabla g\left(\mathbf{y}^*\right)}{\|\nabla g\left(\mathbf{y}^*\right)\|} \tag{8}$$

where $\nabla g = \left\{\frac{\partial g}{\partial y_i}\right\}$ is the gradient vector containing the partial derivatives of the limit state equation with respect to each random variable. The component α_i^2 provides a linear approximation of the sensibility of the p_f to random variable X_i, which is composed by a combination of its distribution, mean value and standard deviation [37]. Through the sensibility, it is possible to identify more or less significant variables in a structural reliability problem in order to inform relevant aspects that deserve more attention of the designer, mainly in the statistical treatment of the variables, which can reduce the uncertainties and increase the accuracy of the analysis.

2.1 Limit state equation for beams

The rectangular beams considered herein are subjected to pure bending caused by gravity loads. The resistance to formation of a plastic hinge is given by the classical equations of equilibrium and recommendations of the ABNT NBR 6118 [1]:

$$M_R = A_s\, f_y\, (d - x) + 0.408\, x^2\, f_c\, b_w \tag{9}$$

$$x = 1.25\left(\frac{A_s f_y}{0.85\, f_c\, b_w}\right) \tag{10}$$

where M_R is the bending moment resistance of the cross section, A_s is the steel reinforcement area, f_y is the yield strength of steel, d and b_w are the effective height and width of the cross section, f_c is the compressive strength of concrete and x is the neutral axis position, given by Equation 10. Equation 9 is valid for concrete with less than 50 MPa of characteristic compressive strength and simple reinforced beams.

Thus, the limit state equation for beams (g_B) is given by:

$$g_B(\mathbf{X}) = E_B\, M_R\left(f_c, f_y, d, b_w\right) - M_s\left(d, b_w, D, L\right) \tag{11}$$

where E_B is the model error variable for beam bending, M_s is the maximum bending moment on the beam, given by the FE model, and the random variable vector is $\mathbf{X} = \left\{E_B, f_c, f_y, d, b_w, D, L\right\}$, where D and L are the dead and live loads, respectively.

2.2 Limit state equation for columns

The rectangular columns were subjected to normal loads combined with bending. Unlike beams, column resistance cannot be assessed directly as it represents a nonlinear system resulting from the imposition of the equilibrium equations between the internal load effects and strengths [43]. These relations are considered by shifting the neutral axis position in the cross section, allowing the construction of an interaction curve that separates the safe and failure regions, as depicted in Figure 1, evaluated following prescriptions of [1].

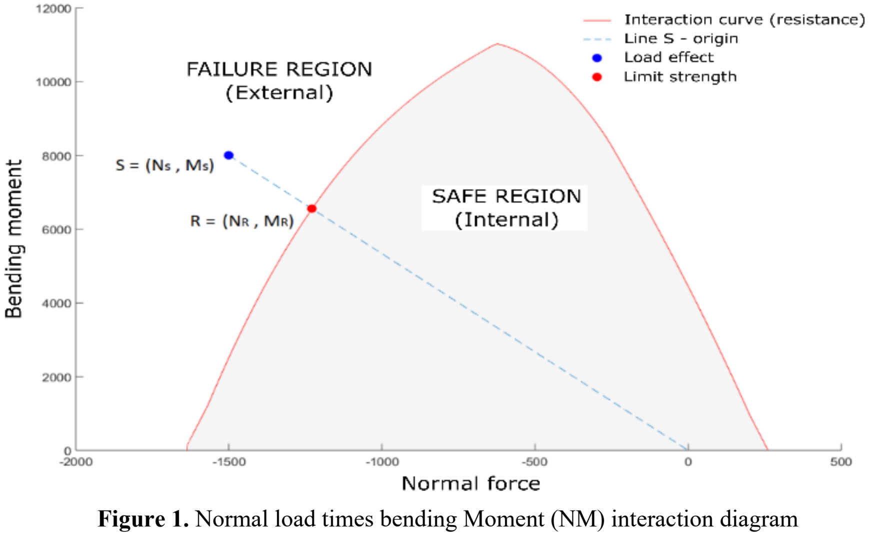

Figure 1. Normal load times bending Moment (NM) interaction diagram for strength of RC columns.

Formulation of the limit state for RC columns follows [30], and is based on the shortest load path criterion. Let *S* be the load effect, given as a point of normal force and bending moment (N_S , M_S) in the diagram (Figure 1). The line joining this point to the origin is the "load path". This line defines the limit strength point *R*, given as (N_R , M_R), and which can be obtained by a curve intersection algorithm. The limit state function for columns (g_C) is established by comparing the distance between points *R* and *S* to the origin:

$$g_C(\mathbf{X}) = E_C\sqrt{(N_R)^2 + (M_R)^2} - \sqrt{(N_S)^2 + (M_S)^2} \tag{12}$$

where E_C is the model error variable for columns. For a point inside the safe region, the distance from *S* to the origin is smaller than the distance from *R* to the origin, leading to a positive value for the limit state function. In Equation 12, the strength is a function of resistance variables ($N_R(f_c, f_y, d, ...)$, $M_R(f_c, f_y, d, ...)$), and the load effects are functions of the loads: $N_S(D, L, d, b_w)$, $M_S(D, L, d, b_w)$.

The formulation presented herein assumes that load effects N_S , M_S vary in proportion to the loads (D, L) , which is valid in case of dependent gravity loads. When lateral (wind or earthquake) loads are considered, load effects may change independently, and a more complex "out-crossing rate" formulation is required [44].

2.3 Collapse probabilities

In this paper, we conduct reliability analyses of intact frames and frames with discretionary column removals, representing situations in which local damage has occurred. The failure probabilities given through the limit state equations for beams and columns (Equations 11 and 12) for the damaged structures represent the conditional collapse probability $P[C \mid LD, H]$, following Equation 1. The $P[C]$ term needs to be evaluated considering a risk analysis to determine $P[H]$ and $P[LD \mid H]$ terms. For the intact structures, in which local damage has not occurred, the failure probabilities represent the term $P[C]$ related to the usual loading condition.

3. FINITE ELEMENT MODELLING FOR LOAD EFFECTS

The load effects in the studied frames are assessed through a linear static analysis. A mechanical model based on the finite element method (FE) is used, with elastic material behavior and 2D frame elements. A linear static analysis is sufficient for an approximate study of the load distribution in regular frames, and for its redistribution across the floors and spans. A study of the ultimate collapse loads requires mechanical models which incorporate geometric and material nonlinearities behavior and are out of the scope of this paper.

Each frame element in the model has two nodes with three degrees of freedom in each node: one rotation and two displacements, as depicted in Figure 2.

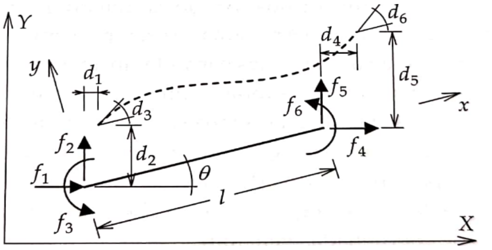

Figure 2. Degrees of freedom of the frame element [45].

The nodal displacements vector $\mathbf{u}$ is obtained by a system containing the global stiffness matrix $\mathbf{K}$ and the nodal forces vector $\mathbf{f}$:

$$\mathbf{K} \cdot \mathbf{u} = \mathbf{f} \tag{13}$$

With the displacements, the load effects can be found through equilibrium equations. The model used herein was validated by comparing the results with academic software Ftool [46]. In the FE model, one element is used for each column in each floor and one element is used for each spam of the beams.

3.1 Limitations of linear analysis

The mechanical model used herein is limited to elastic behavior of the materials and does not consider geometric nonlinearity and dynamic effects. Therefore, as a phenomenon that causes large deformations in the structure, the collapse mechanism is considered in a simplified and conservative approach.

Figure 3 depicts the nonlinear behavior of a reinforced concrete beam which suffered a midspan column removal. The first part of the load-displacement curve is the linear elastic stage or approximated as such; this is followed by the arch effect, the snap-through and the catenary (or membrane) action. Catenary action only occurs if columns at both ends of the beam element provide enough horizontal restraint. The end of the curve, point D, represents the ultimate load capacity of the beam, which can be quantified by nonlinear procedures. The bending moment resistance in the linear stage, used herein as the limit state (Equations 9–11), corresponds to point A, where the limits of material and geometrical linearities are valid. Therefore, a great reserve of strength is still available in the elements, which is not captured in the simplified analyses carried out in this paper. Hence, the reliability index values computed in this paper are minimum values, corresponding to the strength at point A in Figure 3.

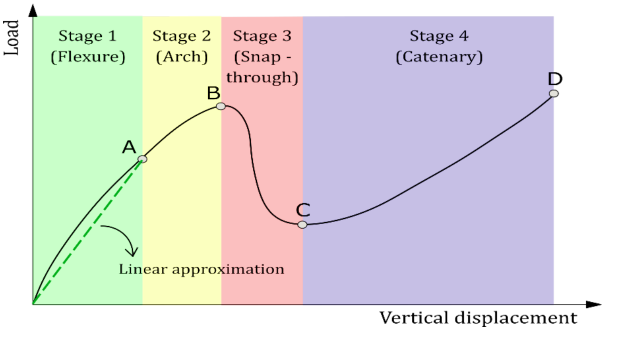

Figure 3. Nonlinear behavior in the collapse of a reinforced concrete beam.

4. STUDIED FRAMES, DESIGN AND LOAD CASES

The analyses presented in this paper are based on the buildings illustrated in Figure 4: a 4-story and an 8-story reinforced concrete frames with four bays, spans of 5.0 meters and story height of 3.0 meters. Five different situations were analyzed in each frame, one intact, two with removal of an external column and two with removal of an internal column, as shown in Figure 5. For the frames with column removal, we consider a normal design (no strengthening for column loss) and a strengthened APM design, i.e., a frame reinforced to bridge over a failed column, as detailed in the next section.

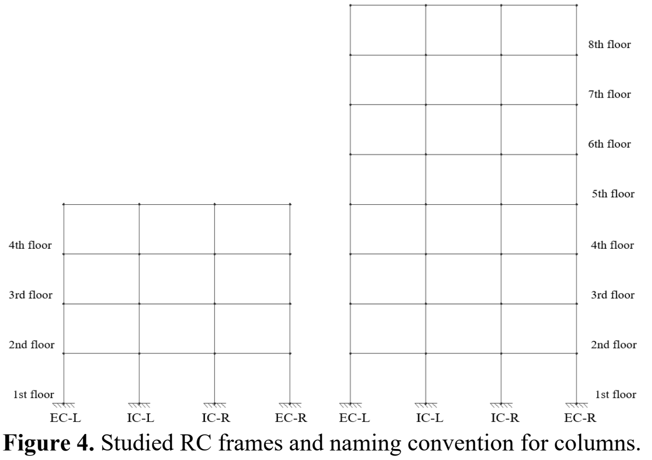

Figure 4. Studied RC frames and naming convention for columns.

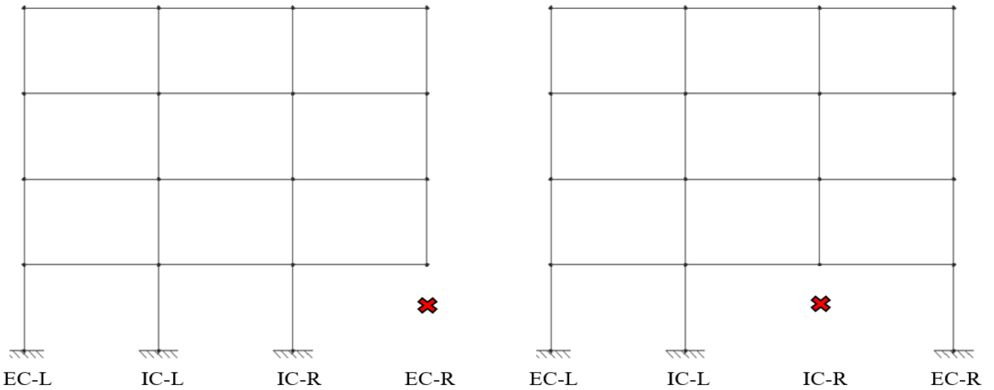

Figure 5. Illustration of the considered column loss conditions: external (left) and internal (right).

4.1 APM design

The Alternate Path Method is specified in documents UFC 4-023-03 [13] and GSA [12]. In this direct design approach against progressive collapse, the structure is designed such as to be able to bridge over a removed load-carrying element. This is achieved by ensuring that alternate load paths are available in the structure in cases where local damage occurs, confining disproportionate collapse.

The method is applied by notionally removing key load-bearing structural elements, followed by a structural analysis. One at a time, internal and external columns are removed, as well as columns at critical locations, as determined by engineering judgment. The elements which do not resist the load redistribution need to be strengthened. The analysis can be linear static, nonlinear static or nonlinear dynamic. The use of linear static procedures is limited to regular structures that are 10-stories or less.

In light of Equation 1, the APM design is done by considering $P\left[LD|H\right]=1$, in which the local damage corresponds to a column loss. Recall that the failure probabilities obtained by a reliability analysis in a damaged structure (column loss) refers to the term $P[C\,|\,LD,H]$. To find the term $P\left[C\right]$, one needs to use Equation 1, as commented in Section 2.3.

4.2 Load combinations and design

The loads applied in the structures are quantified based on an influence area with a total depth of 8.0 meters, 4.0 meters in each side of the frame. All nominal values of the loads are based on ABNT NBR 6120:2019 – Design loads for structures [47]. The live loads are established considering a residential building, with the rooms classified as "Pantry and laundry area", with 2.0 kN/m^2. For the dead loads, only the self-weight of the structural elements is considered, with slabs of 0.12 meters height.

The design code ABNT NBR 8681 [48] specifies an exceptional loading combination arising from the occurrence of actions that can cause catastrophic effects. These actions are transitory and extremely brief in duration. Following ABNT NBR 8681, the required design strength R_D is given by:

$$R_D \geq 1.2\,D_n + 1.0\,L_n \tag{14}$$

where D_n is the nominal dead load and L_n is the nominal live load.

Although the definition of exceptional loads in the Brazilian standard [48] may be in convergence with events that can cause severe local damage, it does not specifically recommend any loading condition for design or strengthening of structures that already suffered damage, which is the case of the Alternate Path Method. In the APM design, the residual capacity of the structure must be assessed, which is conditional to the occurrence of local damage (column loss). Hence, to maintain coherence between our analysis for the cases of intact frame and column loss, the design factors for loads are taken from ASCE 7 [33], for usual loading (intact frame) and abnormal loading (column loss – residual capacity).

For demonstration purposes, in Section 5.2 we show how use of load combinations given in ABNT NBR 8681 (Equation 14) results in overdesigned elements when an external column is removed. Although Equation 14 may not be fitted for the progressive collapse assessment through the Alternate Path Method, it may be well suited for the Enhanced Local Resistance method, where collapse is prevented by reinforcing specific elements to resist initial abnormal loads.

As a consequence of using ASCE 7 load combinations, the reliability index results obtained herein for the intact structure are not the same as those obtained using design factors of codes [1] or [48]. Yet, the analysis is justified as we are more interested in the variation of reliability indexes for different points of the structure. Moreover, the recent study on reliability-based calibration of Brazilian design codes [49] has suggested that the partial load factor for dead load should be reduced (to $\gamma_D = 1.25$) and for live load should be increased (to $\gamma_L = 1.70$). These values are somewhat closer to the values given by ASCE 7, as reported below.

Following ASCE 7, under normal loading condition, the required design strength R_D is given by:

$$R_D \geq 1.2 D_n + 1.6 L_n \tag{15}$$

The design strength R_D is evaluated following ABNT NBR 6118:2014, with $\gamma_c = 1.4$ and $\gamma_S = 1.15$.

The column loss condition corresponds to an extraordinary event. Following the Alternate Path Method (APM) and the ASCE 7 “Residual capacity” load combination, beams and columns of the regular building are strengthened such that:

$$R_D \geq 1.2 D_n + 0.5 L_n \tag{16}$$

Under column loss condition, a reduced design value is considered for the live load: the damaged frame is not expected to withstand the lifetime maximum live load, but it should support the sustained part of the live load, until repair action is taken. Accordingly, in the reliability analysis, the fifty-year extreme value of the live load is considered for the intact frame, and the arbitrary-point-in-time value of the live load is considered under column loss condition, as detailed in the sequence.

Beam elements are designed considering pure bending, and columns are designed considering normal loads plus bending. Second-order effects were not considered, to simplify the reliability analysis, and to respect the limitations of the linear finite element model. Tables 1 and 2 present the cross-section and reinforcement area for beams and columns of the 4 and 8-story buildings, respectively. In the columns, the reinforcement is placed near the top and the bottom of the cross-section. All elements are designed with concrete with 30 MPa of characteristic compressive strength (f_{ck}) and steel with 500 MPa of characteristic yield strength (f_{yk}). The structural analysis was performed with a modulus of elasticity (E) of 20 GPa. Table 3 shows the loading considered, as well as the resulting live-to-dead load ratios, which are relevant to interpret the results of reliability analysis.

Table 1. Cross-section dimensions and reinforcement for 4-story building (“CL” means column loss).

Loading case	Width (cm)	Beams		Internal columns		External columns	
		Height (cm)	Steel reinf. (cm^2)	Height (cm)	Steel reinf. (cm^2)	Height (cm)	Steel reinf. (cm^2)
Normal loading	19.0	50.0	6.80	40.0	5.00	40.0	7.50
Strengthened for external CL	19.0	70.0	16.00	50.0	8.00	50.0	13.00
Strengthened for external CL (ABNT *NBR 8681*)	19.0	80.0	18.00	50.0	10.00	50.0	17.00
Strengthened for internal CL	19.0	70.0	15.00	50.0	10.00	50.0	14.50

Table 2. Cross-section dimensions and reinforcement for 8-story building ("CL" means column loss).

Loading case	Width (cm)	Beams		Internal columns		External columns	
		Height (cm)	Steel reinf. (cm^2)	Height (cm)	Steel reinf. (cm^2)	Height (cm)	Steel reinf. (cm^2)
Normal loading	19.0	50.0	8.50	60.0	12.00	60.0	6.60
Strengthened for external CL	19.0	80.0	16.00	70.0	12.00	70.0	12.00
Strengthened for internal CL	19.0	80.0	14.00	70.0	20.00	70.0	12.00

Table 3. Nominal values for loads (in kN/m) and ratio L_n / D_n .

Frame	Case	Live load on beams (q_n)	Dead load on beams ($g_{v,n}$)	Self-weight of columns ($g_{p,n}$)	L_n / D_n on beams	L_n / D_n on columns
4-story	Intact	16.00	26.38	1.90	0.61	0.56 - 0.58
	Column loss	16.00	27.33	2.38	0.59	0.52 - 0.56
8-story	Intact	16.00	26.38	2.85	0.61	0.54 - 0.57
	Column loss	16.00	27.80	3.33	0.58	0.51 - 0.56

4.3 Random variables for reliability analysis

The reliability analysis of the frames is performed considering ten random variables, presented in Table 4. The live load random variable in column loss scenarios follows [14], taken with arbitrary-point-in-time values and Gamma distribution, as the damaged frame is not expected to withstand its lifetime maximum live load. All the beams were loaded by the same live load random variable simultaneously.

Table 4. Random variables considered in the reliability analysis and their parameters ("Variable" means "in terms of the standard deviation").

Variable (symbol)	Distribution	Mean	C.o.V.	Standard deviation	Reference
Cross-section dimensions (B, H)	Normal	d_n	Variable	$4 + 0.006 d_n$ (mm)	[50]
Self-weight of columns (G_p)	Normal	$1.06 g_{p,n}$	0.12	$0.1272 g_{p,n}$	[32]
Dead load on beams (G_v)	Normal	$1.06 g_{v,n}$	0.12	$0.1272 g_{v,n}$	[32]
Live load on beams, intact frame (Q_{50})	Gumbel	q_n	0.40	$0.40 q_n$	[32]
Live load on beams, column loss $(Q_{a.p.t.})$	Gamma	$0.25 q_n$	0.55	$0.1375 q_n$	[32]
Concrete compressive strength (f_c)	Normal	$1.22 f_{ck}$	0.15	$0.183 f_{ck}$	[32]
Steel yield strength (f_y)	Normal	$1.22 f_{yk}$	0.04	$0.0488 f_{yk}$	[32]
Model error for beam bending (E_B)	Lognormal	0.99	0.024	0.02376	[19]
Model error for columns (E_C)	Normal	1.15	0.145	0.16675	[49]

5. RESULTS FOR THE 4-STORY FRAME

5.1 Normal design, intact frame

Reliability index results for the intact 4-story frame are presented in Figure 6. Calculated values of reliability indexes are shown besides each element. The elements highlighted in yellow color are those which controlled the regular design.

Reliability indexes observed in Figure 6 are around 3.0 for beams and 4.0 for columns. Note that the element which controls design, for which design strength is smaller, is also always the element with smaller reliability index. The largest variations in reliability indexes are observed for columns of different floors. Design of internal columns is controlled by larger normal loads at the first floor. Significant reserve in safety is observed for internal columns of higher floors, when the same detailing is considered: this is expected, as normal loads are significantly smaller. Design of external columns is controlled at the highest floor, where bending moments prevail over normal loads. As observed in Figure 7, the external columns of different floors correspond to quite different points in the NM interaction diagram: the design is controlled at the fourth floor, where normal load is small, hence the load trajectory is very stepped and bending strength is limited (Figure 7 right); for the third floor (and those below), the normal load is much larger, the load trajectory is less stepped, leading to much larger bending strength. The ratio of bending moment to normal force, M/N, for external columns, is shown in Figure 8, which confirms that the load effects change significantly from the third to the fourth floor.

The results presented herein can be better interpreted by looking at the sensitivity coefficients, which show the contribution of each random variable to the calculated failure probabilities. Figure 9 shows the reliability indexes and the sensitivity coefficients for the external and internal columns of the 4-story frame. We start by noting that the model error random variable (E_C) has a greater role for all columns. For those columns which control the design, role of E_C is around 40 to 50%; for other columns, it approaches 100%. This points out to the importance of developing better probabilistic models for the model error of columns. Considering the behavior observed in Figure 7, one significant improvement would be to evaluate model error statistics for “zones”, corresponding to similar values of the ratio M/N.

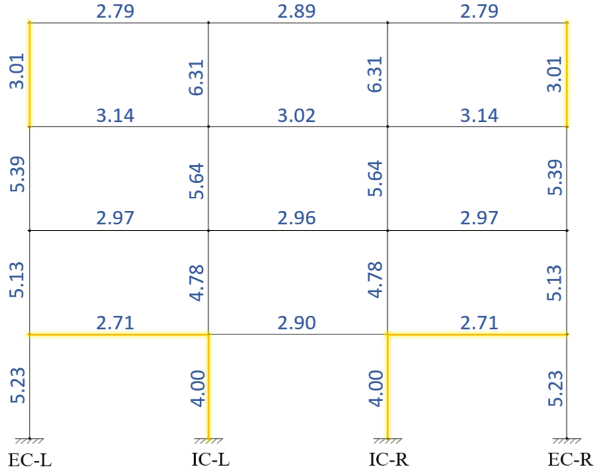

Figure 6. Reliability indexes for 4-story frame, intact condition.

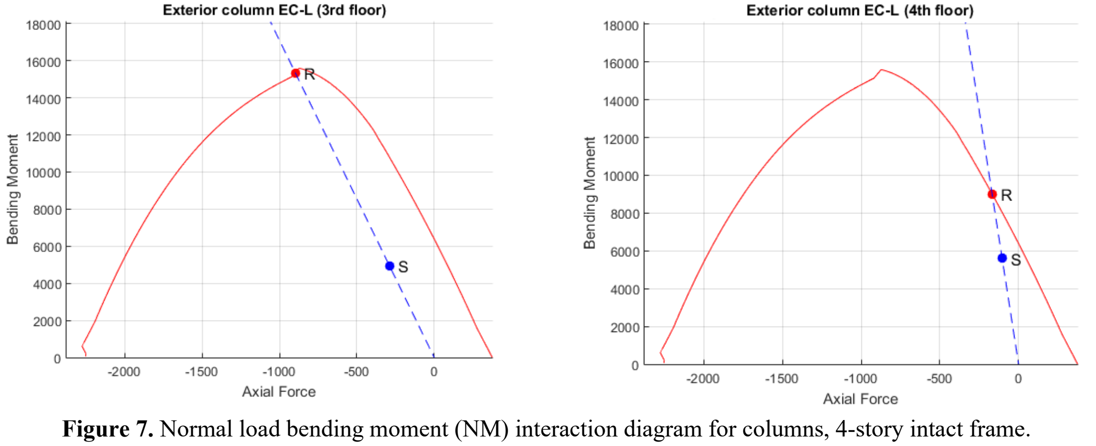

Figure 7. Normal load bending moment (NM) interaction diagram for columns, 4-story intact frame.

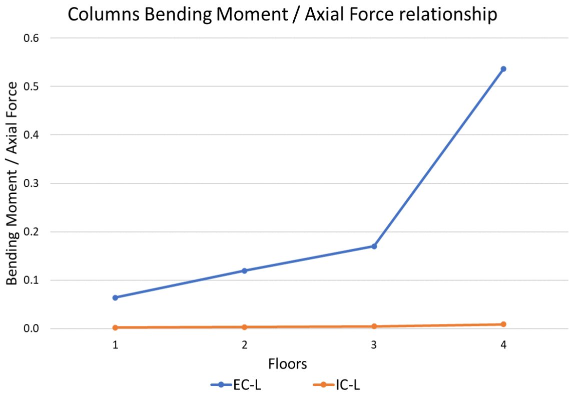

Figure 8. Ratio between bending moment and normal force (M/N) for columns at different floors, 4-story intact frame.

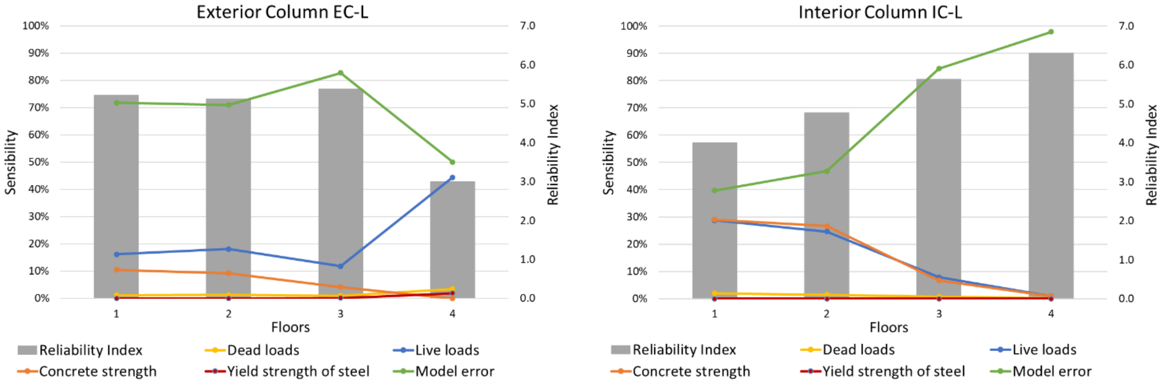

Figure 9. Reliability index and sensitivity coefficients for columns of different floors, 4-story intact frame, external columns (left) and internal columns (right).

One intuitive behavior is observed for the internal columns, with the reliability index increasing from bottom to top: for lower floor columns, with greater normal load, the contribution of concrete strength and live load rivals that of the model error; as normal loads are reduced, the importance of other variables vanishes, and E_C dominates failure probabilities, which become very small.

For external columns, the influence of live load exceeds that of concrete strength, and becomes relevant at the fourth floor, which controls the design. Recall that the normal force bending moment interaction diagram for external columns of 3rd and 4th floors is shown in Figure 7.

5.2 Discretionary removal of external column

Figure 10 illustrates the reliability indexes for the 4-story frame, when the external column is removed: left in figure, results for the original frame; right, results for the strengthened frame (APM method). All reliability indexes shown in Figure 10 are conditional on column removal (local damage). Highlighted in red are the elements for which the failure probability is larger than 0.1, which corresponds to the maximum accepted value for the conditional collapse probability (term $P[C|LD,H]$ in Equation 1). Negative reliability indexes shown in figure correspond to $P[C|LD,H] > 0.5$.

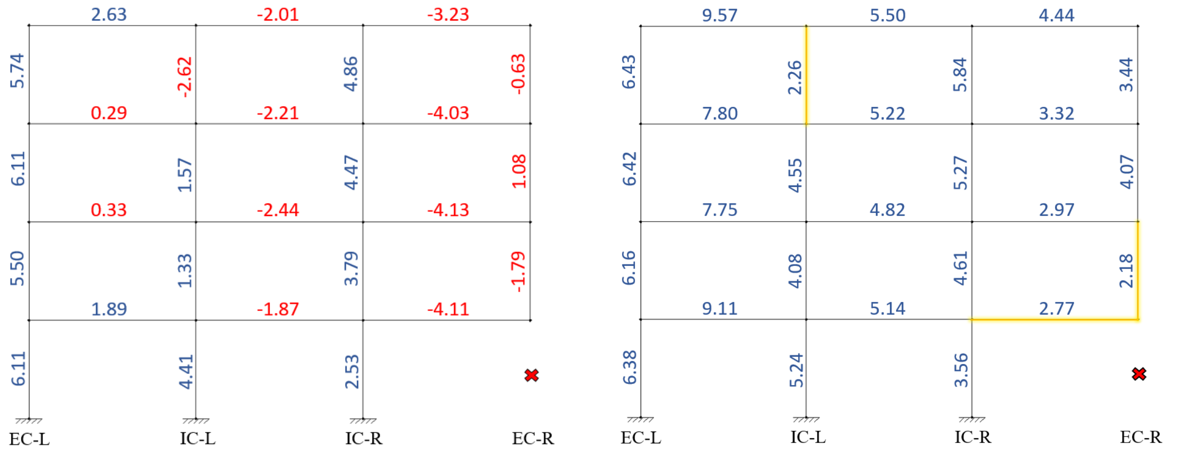

Figure 10. Reliability indexes for 4-story frame, removal of external column, original frame (left) and strengthened frame (right).

As observed in Figure 10 (left), the original (non-strengthened) frame does not withstand loss of an external column. Beams of the affected bay would most certainly form plastic hinges. External columns above the removed one (EC-R) and the internal column at the top floor (IC-L) also present unacceptably low reliability index. As noted in Figure 10 (right), the strengthened frame would likely support loss of the external column, with the weakest column presenting $\beta = 2.18$, which corresponds to a failure probability of $p_f = 0.015$.

Sensitivity indexes for the strengthened frame are shown in Figure 10. Again, the model error random variable controls failure probabilities, with around 70% dominance for those elements which control the design. For external left (EC-L) column, model error contributed alone to the failure probability: this deterred the reliability index from becoming larger than 7. Behavior of column IC-L is observed to be like the case of internal columns of the intact frame, with the reliability index decreasing at the top floor, such as the influence of the model error (Figure 11). At the top floor, an increase in the sensibility of dead and live load actions, as well as yield stress, is observed. This hints to the greater contribution of bending moments at this top column.

For column IC-R, reliability indexes become smaller for lower floors, with larger participation of concrete strength, as well dead and live loads; this occurs due to larger participation of normal loads, for this damaged structure condition. Figure 12 confirms this observation, as it shows the ratio of bending moment to normal loads, for the three remaining columns of the damaged building.

Relating results in Figure 12 to those in Figure 10, we observe that M/N ratios up to 0.4 contribute to greater strength and larger β 's, whereas values above 0.6 lead to smaller β 's (as is the case for column IC-L at the top floor).

The collapse probability given by Equation 1 for the elements with the lowest reliability indexes are shown in Table 5, for all the analyses conducted. The values were obtained considering hazard probability $P[H]$ of 10^{-5} and that local damage is certain due to hazard ($P[LD|H]=1$). As expected, the collapse probabilities are small for the strengthened structures, since the hazard probabilities are usually small. For the intact structures, the collapse probability refers to the usual loading condition, and cannot be compared directly with the cases of column loss.

At last, as commented in Section 4.2 of this paper, the reliability analysis of the building redesign with the ABNT NBR 8681 "Exceptional load combination" is shown in Figure 13. The reliability indexes of the elements which control the design are much larger than those of the frame strengthened using ASCE 7 "Residual capacity" load combination. The most critical beam's β goes from 2.77 to 4.31, whereas the internal column's goes from 2.26 to 3.94 and the external column's goes from 2.18 to 3.46. As one can observe in Table 5, the order of magnitude of the failure probability of the elements designed following ABNT NBR 8681 are around 10^{-10}, excessively lower than the 10^{-7} of the ASCE 7 load combination, which indicates that the Alternate Path method assessed with Brazilian load factors will likely overdesign the elements.

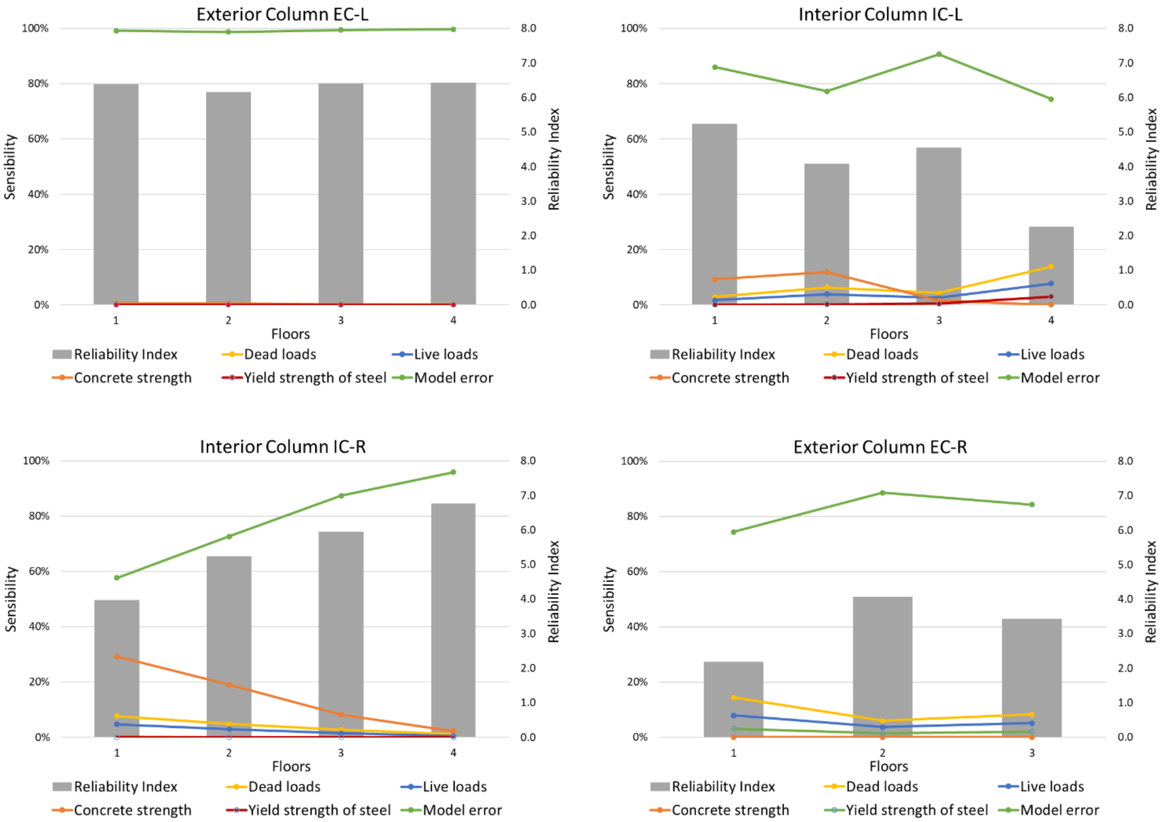

Figure 11. Reliability index and sensitivity coefficients for columns of different floors, strengthened 4-story frame with removal of external column.

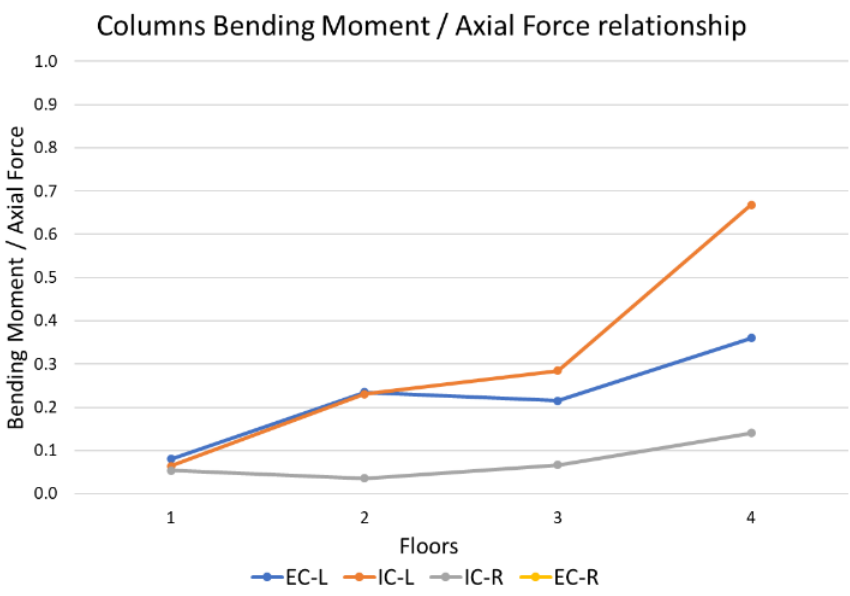

Figure 12. Ratio M/N for columns at different floors, strengthened 4-story frame with removal of external column.

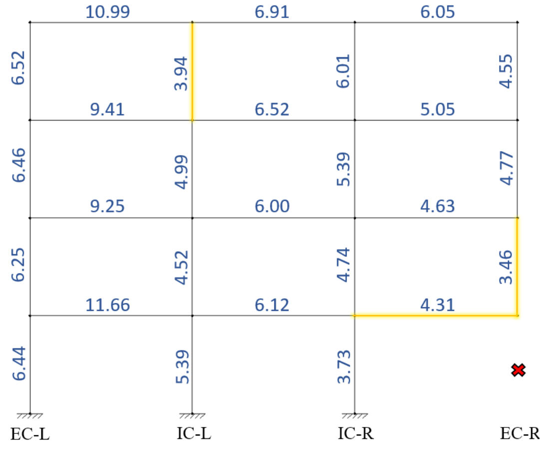

Figure 13. Reliability indexes for 4-story frame, removal of external column, designed with exceptional load combination of ABNT NBR 8681 [48].

Table 5. Collapse probability of the most critical elements, calculated through Equation 1 using $P\left[H\right]=10^{-5}$ and $P\left[LD|H\right]=1$ (in boldface, values that exceed 10^{-6} , corresponding to $P\left[C|LD,H\right]>0.1$).

Structure		Beam	External Column	Internal Column
4-story	Intact	3.36E-03	1.31E-03	3.17E-05
	ECL - O	**1.00E-05**	**9.63E-06**	**9.96E-06**
	ECL - S	2.80E-08	1.46E-07	1.19E-07
	ECR - S (ABNT NBR 8681)	8.16E-11	2.70E-09	4.07E-10
	ICL - O	**1.00E-05**	**9.98E-06**	**7.91E-06**
	ICL - S	9.35E-09	9.14E-08	4.07E-10
8-story	Intact	6.57E-03	5.39E-03	2.70E-04
	ECL - O	**1.00E-05**	**9.79E-06**	1.66E-07
	ECL - S	**1.66E-06**	**1.64E-06**	1.69E-08
	ICL - O	**1.00E-05**	**9.73E-06**	1.44E-08
	ICL - S	1.22E-08	1.99E-08	3.49E-09

ECL: external column loss; ICL: internal column loss; O: original (non-strengthened); S: strengthened

5.3 Discretionary removal of internal column

Figure 14 illustrates the reliability indexes for the 4-story frames, when the internal column is removed. Again, the original frame does not withstand column loss as beams and most of column EC-R present unacceptably low reliability indexes. However, in the first floor, the columns right next to the removed one were not significantly affected by the overload.

In the strengthened frame, all elements present acceptable reliability indexes. In this scenario, there is a reduction in the β 's of the EC-R column at the 4th floor. This again occurs due to the excessive steepness of the load path, indicating a prevalence of bending moment action. This same phenomenon is observed in the IC-L and EC-L columns, but with less intensity.

As depicted in the graphs of Figure 15, the influence of the model error is preponderant, ranging from 65 to 75% in the elements that control the design and up to 100% in the others. In the IC-L and EC-R columns, the importance of the concrete strength is significant, especially on the first floor, where the increase in compression relieves the effects of bending moment, reducing the steepness of the load path and increasing reliability. On the other hand, in the last floor, the sensitivity of the concrete drastically decreases, being surpassed by the dead and live loads. In Figure 16, we observe that M/N ratios above 1.0 lead to smaller values of reliability indexes (as is the case for column EC-R at the top floor).

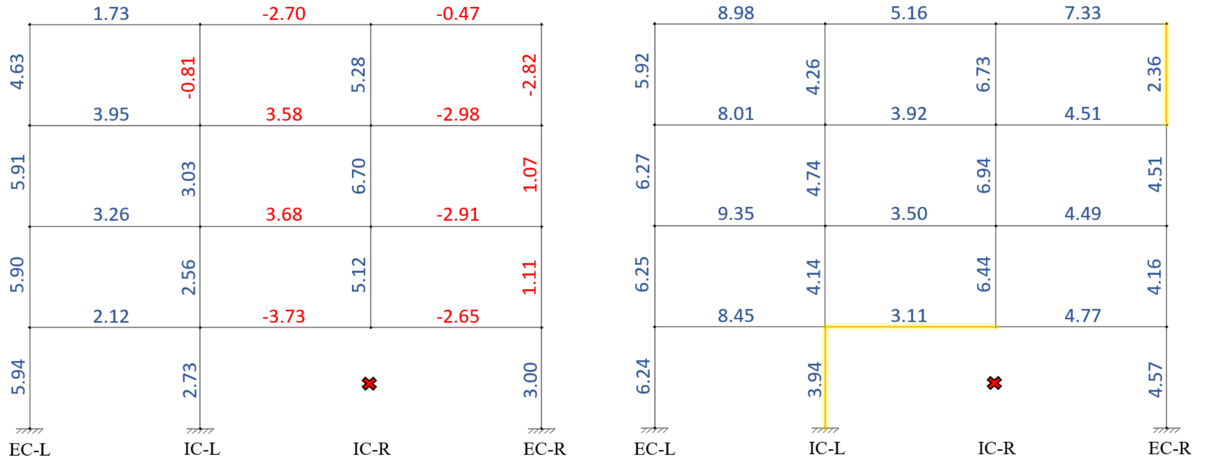

Figure 14. Reliability indexes for 4-story frame, removal of internal column, *original frame (left) and strengthened frame (right).*

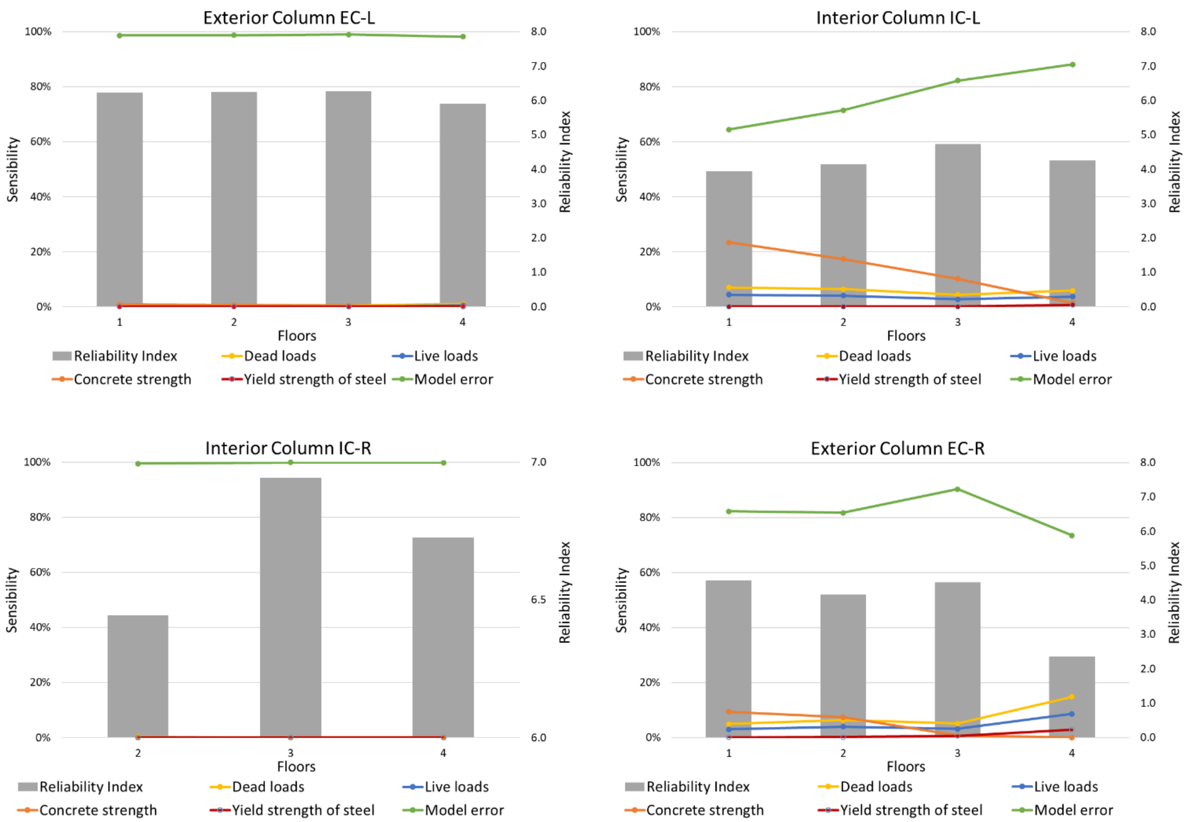

Figure 15. Reliability index and sensitivity coefficients for columns of different floors, strengthened 4-story frame with removal of internal column.

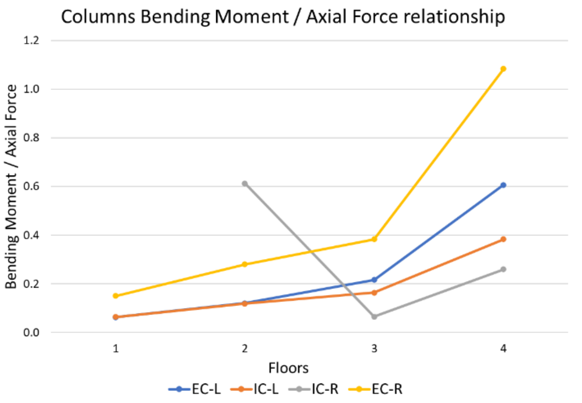

Figure 16. Ratio M/N for columns at different floors, strengthened 4-story frame with removal of external column.

6. RESULTS FOR THE EIGHT-STORY FRAME

The 8-story frame presents similar results to the 4-story frame w.r.t. the columns behavior and the inability of the non-strengthened structure to withstand column loss. Hence, due to space constraints, detailed results are not presented herein. Figure 17 depicts the reliability indexes for the intact and for the strengthened frames with column removal. The model error variable sensibility prevails in all columns. For the elements that control the design, values around 50% in the intact frame and up to 80% in the column loss frames were found.

Figure 17. Reliability indexes for the 8-story building for the intact, external column loss and internal column loss, from left to right.

7. CONCLUSIONS

In this paper, we addressed the spatial distribution of load effects and reliability index of beams and columns of RC plane frames, built considering symmetry and regularity. The analysis is limited to gravitational loads, and to linear material modelling; yet it considers usual and abnormal "column loss" loading conditions. Using load combinations recommended by ASCE and partial safety factors for concrete and steel strength by ABNT NBR 6118, we found reliability indexes around $\beta \approx 3$ for beams, and $\beta \approx 4$ for the critical columns of the intact frame.

We observed that the design of internal columns is controlled by the base (first floor) column, at which normal gravity loads are largest. The reliability index for the base column is around $\beta \approx 3$, and this value increases for upper internal columns. The design of external columns was found to be controlled by the fourth (or upper) floor, where the ratio of bending moment to normal load effect (M/N) is largest. For lower external columns, the normal load increases, the load trajectory becomes less stepped in the MN diagram, and the reliability index increases significantly.

Among all random variables considered, the model error for column strength was found to be the most significant source of uncertainty for the reliability of columns. For those columns controlling design, the contribution of model error was found to be around 40 to 70%; for other columns with greater reserve strength (due to design regularity), the contribution of model error reached 100%. This points out to the importance of developing better models for column strength, and more refined probabilistic models for the model error variable. One significant improvement would be to evaluate model error statistics for "zones", corresponding to similar values of the M/N ratio, in the MN interaction diagram.

For the discretionary column removal loading condition, it was observed that conventional design leads to unacceptable failure probabilities. The beams would most likely fail, due to exceptional bending moments generated by cantilever or double-spam effects. This conclusion does not consider the eventual compressive arch or catenary effects. For the strengthened frames, following the APM design philosophy, the conditional failure probabilities were found to be acceptable. In column loss condition, symmetry is lost, but regular design also leads to significant differences in reliability index and sensitivity coefficients of beams and columns located in different parts of the building.

Investigations are underway to include horizontal loads, non-linear material modelling, and to consider system effects like compressive arch and catenary actions, in the reliability analysis of RC frames subject to column loss loading.

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REFERENCES

[1] Associação Brasileira de Normas Técnicas, *Design of Concrete Structures – Procedure*, ABNT NBR 6118, 2014.

[2] I. V. Melo, S. H. S. Nóbrega, and P. G. B. Nóbrega, "Progressive collapse in concrete structures and the Brazilian design codes (in Portuguese)," in *An. 58º Congr. Bras. Concreto*, Belo Horizonte, MG, 2016.

[3] B. R. Ellingwood and D. O. Dusenberry, "Building design for abnormal loads and progressive collapse," *Comput. Aided Civ. Infrastruct. Eng.*, vol. 20, no. 3, pp. 194–205, 2005, http://dx.doi.org/10.1111/j.1467-8667.2005.00387.x.

[4] B. R. Ellingwood, "Mitigating risk from abnormal loads and progressive collapse," *J. Perform. Constr. Facil.*, vol. 20, no. 4, pp. 315–323, 2006, http://dx.doi.org/10.1061/(ASCE)0887-3828(2006)20:4(315).

[5] B. R. Ellingwood, "Strategies for mitigating risk to buildings from abnormal load events," *Int. J. Risk Assess. Manag.*, vol. 7, no. 6–7, pp. 828–845, 2007, http://dx.doi.org/10.1504/IJRAM.2007.014662.

[6] U. Starossek, "Typology of progressive collapse," *Eng. Struct.*, vol. 29, no. 9, pp. 2302–2307, 2007, http://dx.doi.org/10.1016/j.engstruct.2006.11.025.

[7] B. A. Izzuddin et al., "Progressive collapse of multi-storey buildings due to sudden column loss - Part I: Simplified assessment framework," *Eng. Struct.*, vol. 30, no. 5, pp. 1308–1318, 2008, http://dx.doi.org/10.1016/j.engstruct.2007.07.011.

[8] K. Khandelwal and S. El-Tawil, "Pushdown resistance as a measure of robustness in progressive collapse analysis," *Eng. Struct.*, vol. 33, no. 9, pp. 2653–2661, 2011, http://dx.doi.org/10.1016/j.engstruct.2011.05.013.

[9] E. Masoero, P. Darò, and B. M. Chiaia, "Progressive collapse of 2D framed structures: An analytical model," *Eng. Struct.*, vol. 54, pp. 94–102, 2013, http://dx.doi.org/10.1016/j.engstruct.2013.03.053.

[10] C. E. M. Oliveira, E. A. P. Batelo, P. Z. Berke, R. A. M. Silveira, and T. L. Massart, ""Computational assessment of the progressive collapse of reinforced concrete planar frames using a nonlinear multilayered beam formulation", *RIEM* -," *Rev. IBRACON Estrut. Mater.*, vol. 7, no. 5, pp. 845–855, 2014, http://dx.doi.org/10.1590/S1983-41952014000500007.

[11] B. Ellingwood and E. V. Leyendecker, "Approaches for design against progressive collapse," *J. Struct. Div.*, vol. 104, no. 3, 1978.

[12] General Services Administration, *Alternate Path Analysis & Design Guidelines for Progressive Collapse Resistance*, 2013, p. 143.

[13] Unified Facilities Criteria, Design of Buildings to Resist Progressive Collapse, UFC 4-023-03, 2009, p. 227.

[14] A. T. Beck, L. R. Ribeiro, and M. Valdebenito, "Risk-based cost-benefit analysis of frame structures considering progressive collapse under column removal scenarios," *Eng. Struct.*, vol. 225, 2020, http://dx.doi.org/10.1016/j.engstruct.2020.111295.

[15] A. T. Beck, L. R. Ribeiro, and M. Valdebenito, “Cost-benefit analysis of design for progressive collapse under accidental or malevolent extreme events,” in *Engineering for Extremes: Decision-making in an Uncertain World*, M. G. Stewart and D. V. Rosowsky, Eds., Switzerland: Springer, 2021. ISBN 978-3-030-85017-3.

[16] A. T. Beck, L. R. Ribeiro, M. Valdebenito, and H. Jensen, "Optimal design of regular frame structures subject to column loss events: a conceptual study," *J. Struct. Eng.*, 2021. https://doi.org/10.1061/(ASCE)ST.1943-541X.0003196.

[17] J. M. Adam et al., "Research and practice on progressive collapse and robustness of building structures in the 21st century," *Eng. Struct.*, vol. 173, pp. 122–149, 2018, http://dx.doi.org/10.1016/j.engstruct.2018.06.082.

[18] F. R. Stucchi and S. H. C. Santos, "Reliability based comparison between ACI 318-05 and NBR 6118," *Rev. Ibracon Estrut.*, vol. 3, no. 2, 2007.

[19] E. S. Santos, “Avaliação estatística do erro de modelos de resistência para elementos lineares de concreto armado da ABNT NBR 6118:2007,” in M.S. thesis, São Carlos Sch. Eng., Univ. São Paulo, São Carlos, 2012.

[20] D. M. Santos, F. R. Stucchi, and A. T. Beck, "Confiabilidade de vigas projetadas de acordo com as normas brasileiras," *Rev. IBRACON Estrut. Mater.*, vol. 7, pp. 723–746, 2014, http://dx.doi.org/10.1590/S1983-41952014000500002.

[21] D. A. San Martins, “Confiabilidade de vigas pré-tracionadas de concreto protendido”, M.S. thesis, Eng. Civ., Univ. Fed. Rio Grande do Sul, 2014.

[22] C. G. Nogueira and M. D. T. Pinto, "Safety variability assessment of reinforced concrete beams subjected to bending moment considering the NBR 6118:2014 safety partial factors," *Rev. IBRACON Estrut. Mater.*, vol. 9, no. 5, pp. 682–709, 2016, http://dx.doi.org/10.1590/S1983-41952016000500003.

[23] M. Scherer, I. B. Morsch, and M. V. Real, "Reliability of reinforced concrete beams designed in accordance with Brazilian code NBR-6118:2014," *Rev. IBRACON Estrut. Mater.*, vol. 12, no. 5, pp. 1086–1125, 2019, http://dx.doi.org/10.1590/s1983-41952019000500007.

[24] J. M. Araújo, "A confiabilidade no projeto de pilares de concreto armado," *Teor. Prat. na Eng. Civ.*, no. 2, pp. 1–8, 2001.

[25] H. A. T. Nogueira, “Avaliação da confiabilidade de pilares curtos em concreto armado projetados segundo a NBR 6118:2003,” in M.S. thesis, Esc. Eng., Univ. Fed. Minas Gerais, 2006.

[26] W. L. A. Oliveira, A. T. Beck, and A. L. H. C. El Debs, "Safety evaluation of circular concrete-filled steel columns de-signed according to Brazilian building code NBR 8800:2008," *IBRACON Struct. Mater. J.*, vol. 1, pp. 212–236, 2008, http://dx.doi.org/10.1590/S1983-41952008000300001.

[27] P. M. Tramontini, “Estudo de confiabilidade para seções de concreto armado submetidas a diversos tipos de esforços,” in M.S. thesis, Esc. Politéc., Univ. Fed. Rio de Janeiro, 2012.

[28] F. C. Magalhães et al., “Avaliação da confiabilidade de pilares de concreto armado com base nos resultados oriundos de distintos locais de ensaio”, in *An. 56º Cong. Bras. Concr.*, Natal, 2014.

[29] M. F. Pereira, A. T. Beck, and A. L. H. C. El Debs, "Reliability of partially encased steel-concrete composite columns under eccentric loading," *IBRACON Struct. Mater. J.*, vol. 10, pp. 298–316, 2017, http://dx.doi.org/10.1590/s1983-41952017000200003.

[30] H. P. Hong and W. Zhou, "Reliability Evaluation of RC Columns," *J. Struct. Eng.*, vol. 125, no. 7, pp. 784–790, 1989., http://dx.doi.org/10.1061/(ASCE)0733-9445(1999)125:7(784).

[31] M. M. Szerszen and A. S. Nowak, "Calibration of design codes for buildings (ACI 318): Part 2 – Reliability analysis and resistance factors," *ACI Struct. J.*, vol. 100, no. 3, pp. 383–391, 2003.

[32] W. C. Santiago, “*Reliability-based calibration of partial safety factors from main Brazilian codes*,” D.Sc. thesis, São Carlos Sch. Eng., Univ. São Paulo, São Carlos, 2019.

[33] American Society of Civil Engineers, *Minimum Design Loads and Associated Criteria for Buildings and Other Structures*, ASCE/SEI 7-16, 2016, 822 p.

[34] A. M. Freudenthal, "The safety of structures," *Trans. Am. Soc. Civ. Eng.*, vol. 112, pp. 125–180, 1947., http://dx.doi.org/10.1061/TACEAT.0006015.

[35] C. A. Cornell, "A probability-based structural code," *J. ACI*, vol. 66, no. 12, pp. 974–985, 1969.

[36] A. M. Hasofer and N. C. Lind, "An exact and invariant first-order reliability format," *J. Eng. Mech.*, vol. 100, pp. 111–121, 1974.

[37] A. T. Beck, *Confiabilidade e Segurança das Estruturas*, 1. ed. Rio de Janeiro: Elsevier, 2019.

[38] S. Marelli and B. Sudret, “UQLab: a framework for uncertainty quantification in MATLAB”, in: *The 2nd Int. Conf. Vulnerability and Risk Anal. Manage. (ICVRAM 2014)*, p. 2554–2563, 2014.

[39] O. Ditlevsen, "Principle of normal tail approximation," *J. Eng. Mech.*, vol. 107, pp. 1191–1208, 1981., http://dx.doi.org/10.1061/JMCEA3.0002775.

[40] A. Nataf, "Détermination des distributions dont les marges sont données," *C. R. Acad. Sci. Paris*, vol. 225, pp. 42–43, 1962.

[41] Y. Zhang and A. Der Kiureghian, *Finite Element Reliability Methods for Inelastic Structures – Report UCB/SEMM - 97/05*. Berkeley: Dep. Civ. Environ. Eng., Univ. California, 1997.

[42] B. Sudret and A. Der Kiureghian, *Stochastic Finite Element Methods and Reliability: A State-of-the-Art Report, Research Report No. UCB/SEMM-2000/08*. Berkeley: Dep. Civ. Environ. Eng., Univ. California, 2000.

[43] W. S. Venturini and R. O. Rodrigues, *Dimensionamento de Peças Retangulares de Concreto Armado Solicitadas à Flexão Reta*. São Carlos: São Carlos Sch. Eng., Univ. São Paulo, 1987.

[44] R. E. Melchers and A. T. Beck, *Structural Reliability Analysis and Prediction*, 3rd ed. Hoboken: Wiley, 2018.

[45] L. F. Martha, *Análise de Estruturas - Conceitos e Métodos Básicos*, 2ª. ed. Editora GEN LTC, 2017, 600 p.

[46] L. F. Martha, "Ftool: a structural analysis educational interactive tool", in *Proc. Workshop Multimed. Comput. Tech. Eng. Educ.*, 1999, pp. 51–65.

[47] Associação Brasileira de Normas Técnicas, *Design Loads for Structures*, ABNT NBR 6120, 2019, 60 p.

[48] Associação Brasileira de Normas Técnicas, *Actions and Safety of Structures – Procedure*, ABNT NBR 8681, 2003, 18 p.

[49] W. C. Santiago, H. M. Kroetz, S. H. C. Santos, F. R. Stucchi, and A. T. Beck, "Reliability-based calibration of main brazilian structural design codes," *Lat. Am. J. Solids Struct.*, vol. 17, no. 1, pp. 1–28, 2020, http://dx.doi.org/10.1590/1679-78255754.

[50] Joint Committee on Structural Safety, *Probabilistic Model Code*, 2001. [Online]. Available: https://www.jcss-lc.org/jcss-probabilistic-model-code/

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