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Application of general method for design of concrete columns and encased composite steel and concrete columns

Aplicação do método geral para dimensionamento de pilares de concreto e pilares mistos de aço e concreto totalmente revestidos

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Abstract

The purpose of this work is the development and implementation of a numerical procedure for the design of reinforced concrete columns (RC) and composite columns with fully concrete encased steel I-section (SC). For this purpose, a computer program was developed to determine the load capacity of these columns, applying the General Method adopted by ABNT NBR 6118:2014. The program is based on an iterative process from integration of the curvatures along the column, obtained by determining the moment-curvature relationship of the cross section by the Newton-Raphson method. Several experimental and numerical results are compared to the program's to demonstrate the accuracy of the procedure.

Keywords: reinforced concrete columns, composite steel and concrete columns, general method.

Resumo

O propósito deste trabalho é o desenvolvimento e implementação de um procedimento numérico voltado para o dimensionamento de pilares de concreto armado e pilares mistos de aço e concreto totalmente revestidos. Assim, foi desenvolvido um procedimento computacional para a determinação da capacidade resistente desses pilares, a partir do Método Geral adotado pela ABNT NBR 6118:2014. O programa emprega um processo iterativo a partir da integração das curvaturas ao longo do pilar, obtidas através da determinação da relação momento-curvatura da seção transversal pelo método de Newton-Raphson, com base em um modelo discreto de fibras. Diversos resultados numéricos e experimentais são utilizados em comparações para comprovação da eficácia do procedimento implementado.

Palavras-chave: pilares de concreto armado, pilares mistos de aço e concreto, método geral.

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1. Introduction

The purpose of this work is the development and implementation of a general procedure for the design of reinforced concrete columns (RC) and composite columns with fully concrete encased steel I-section (SC).

A calculation model in accordance with the ABNT NBR 6118:2014 [1] General Method was implemented in order to evaluate the bearing capacity of reinforced concrete columns and composite columns according to the same parameters comparisons.

The program is based on an iterative process from integration of the curvatures along the column, obtained by determining the moment-curvature relationship of the cross section by the Newton-Raphson method, based on a discrete fiber model. Several experimental and numerical results are compared to the program's to verify the efficiency of the procedure.

2. Computational implementation

The program CSTMI, *Calculation of Composite Cross Sections*, was developed by Caldas and Sousa Jr. [2] with the purpose of analyzing generic composite cross sections at ambient (Caldas [3]) and elevated temperature (Caldas [4]).

The main algorithms implemented in CSTMI are: algorithms for obtaining moment-curvature relationship; algorithms for obtaining the deformed cross section according to the applied axial force and moments; algorithms for obtaining axial force-moment interaction surface and interaction surface of moments for a given axial force; and algorithms for obtaining 3D surface interaction to axial force and moments.

A fiber model is applied in the program for the integration of stresses and determination of resistances. In general, this method consists of discretization of the cross section in small elements (called fibers), and each of them can assume a constitutive model.

The stress-strain relationships for concrete and steel are considered according to ABNT NBR 6118:2014 [1]. The reinforcing steel properties are considered separately and are not included in the fiber mesh. Simply put, the concrete areas occupied by reinforcements are not deducted from the discretized cross section. All meshes are generated with the assistance of GMSH program [5], which uses a text output from CSTMI as geometry data entry and discretizes the cross section to elements of three and four sides. The program interface for columns calculation developed for this study and the discretized cross section are shown in Figure 1.

The solution by the General Method consists of finding the equilibrium of internal forces to external forces applied in all cross sections along the column, considering the effects of material and geometric nonlinearity.

Only columns subjected to axial load and moment about one axis were considered in this study. The methodology to calculate columns is based on the numerical integration of the curvatures obtained from moment-curvature relationship. Thus, the program obtains an initial displacement of the column, leading to its deformed shape. The process is then repeated, considering the second order effects, until the convergence of displacements, subsequently getting a final equilibrium setting.

**Figure 1 – Columns calculation interface of CSTMI**
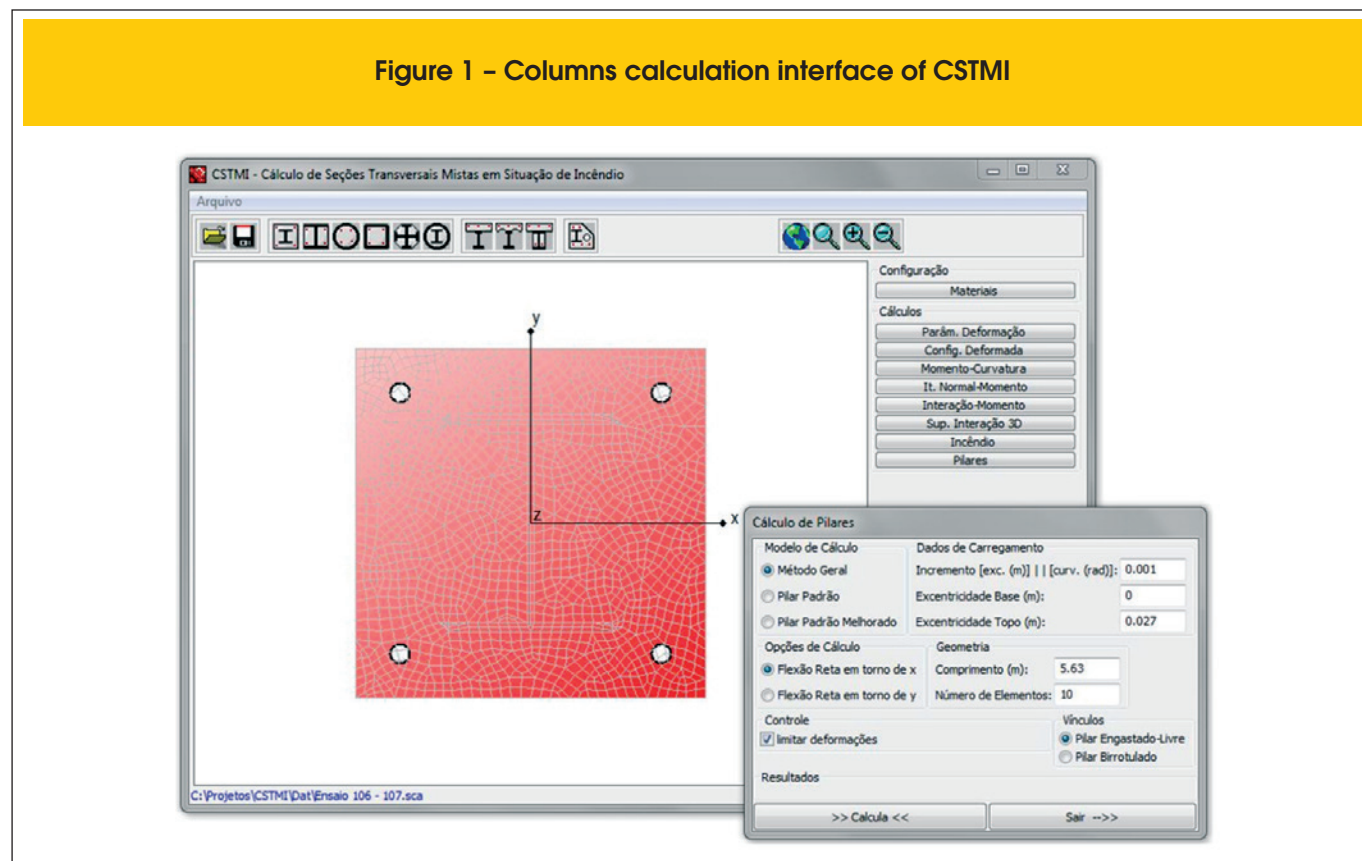
Figure 2 – Bar discretization and applied moments
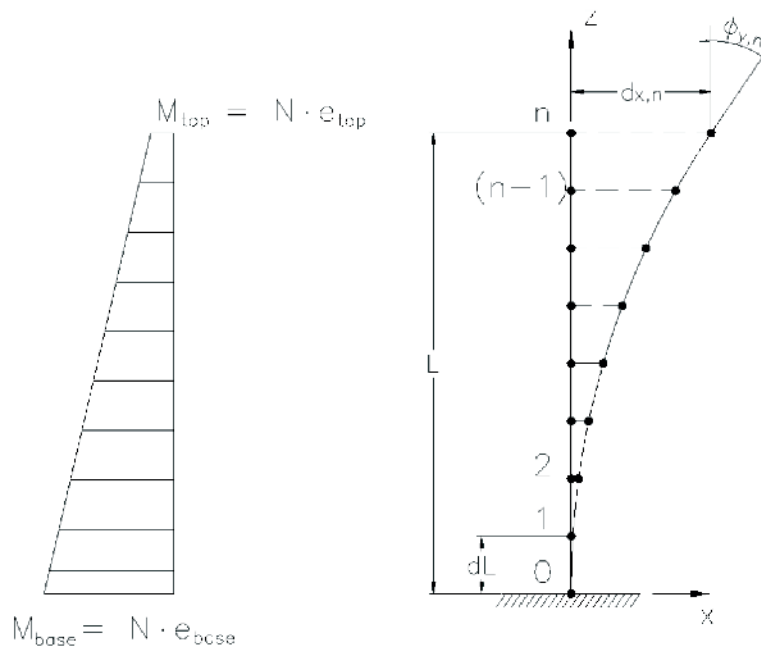
Using the Newton-Raphson method, Caldas [3] has implemented an algorithm for determining the moment-curvature relationship by the control of applied moments (load control), given by increments of applied moment in the section.

In the implementation of this work the geometry of the column is simplified as a bar formed by a finite number of elements informed by the program user, as shown in Figure 2. The calculations of the equilibrium section are performed for each node of the bar.

Figure 3 – Moment-curvature relationship for a load N_{sz}

Consider a restrained-free column with predefined cross-section and reinforcement subjected to an axial compressive load N_{sz}. By the load control algorithm, the program obtains a moment-curvature relationship as shown in Figure 3.

It is considered that the moments are applied only at the ends of the bar, by top and base eccentricity as shown in Figure 2. The axial force and moments (only about one axis) are applied to the geometrical center of the cross section and balanced on each node of the bar, so the accuracy of the model increases as more elements are used.

The applied moment of the bar at undeformed condition, on each node i, can be written as:

$$M_i = M_{base} + \left(M_{top} - M_{base}\right)\frac{z_i}{L} \tag{1}$$

where M_{base} and M_{top} are, respectively, the moments applied at the base and top of the column; z_i is the element position; and L is the length of the bar.

The program builds a position vector with the coordinates of the nodes and a vector with the applied moments corresponding to them. By introducing each point of the vector of applied moments at the moment-curvature relationship and interpolating linearly, the program obtains a vector with the corresponding curvatures for each node of the bar.

The rotation $\phi_y(z)$ of the cross section at any point can be obtained by integrating the curvatures k(z) along the length of the bar:

$$\phi_y\left(z\right) = \int_0^L k_y\left(z\right)dz + C_1 \tag{2}$$

Applying the boundary condition $\phi_y(0) = 0$ in equation (2) leads to $C_1 = 0$. Numerically, the integration of curvature is obtained by the sum of the areas under the graph formed by the points of the curvatures vector.

From the integration of the curvatures the program builds a vector with the rotations ϕ_i of the nodes obtained with equation (3):

$$\phi_i = \phi_{i-1} + \frac{k_i + k_{i-1}}{2}(z_i - z_{i-1}) \tag{3}$$

where k_i and k_{i-1} are, respectively, the curvatures at the availed and at the previous node; z_i and z_{i-1} are respectively the positions of the availed and of the previous node; and ϕ_{i-1} is the rotation of the previous node. The equation (3) is valid for $i \geq 1$. The node 0 has null rotation, $\phi_y(0)$, which is determined by the boundary conditions. The transversal displacement $d_x(z)$ of the column's axis is obtained by double integration of the curvatures, ie., by simple integral of the rotations of the cross section, according to equation (4):

$$d_x(z) = \int_0^L \phi_y(z)dz + C_2 \tag{4}$$

Applying the boundary condition $d_x(0) = 0$ in equation (4) leads to $C_2 = 0$. The reasoning for obtaining numerical integration of the rotations is identical to that used for the integration of curvatures.

Figure 4 – Symmetry condition for simple supported columns
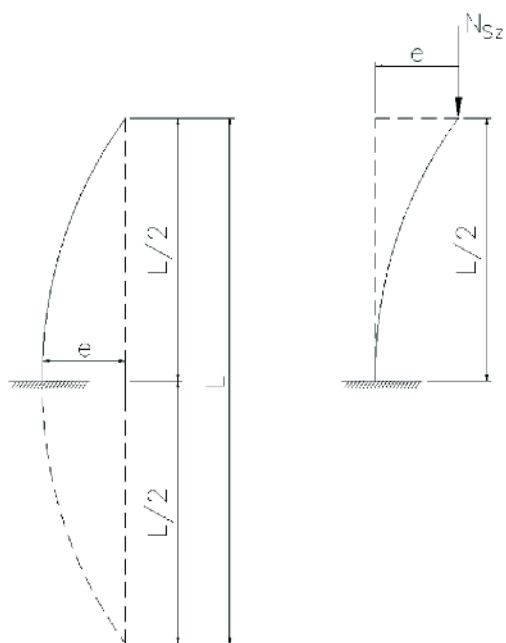
The program builds a vector with the displacements of the nodes, given by:

$$d_i = d_{i-1} + \frac{\phi_i + \phi_{i-1}}{2}(z_i - z_{i-1}) \tag{5}$$

where ϕ_i and ϕ_{i-1} are, respectively, the rotations at the availed and at the previous node; z_i and z_{i-1} are, respectively, the positions of the availed and of the previous node; and d_{i-1} is the displacement of the previous node.

Then, it is obtained by this process the deformed position of the cross sections at the nodes from the undeformed position. A second iteration is executed from the deformed position of the column, now considering the additional effects due to the column's displacement (second order effects). The applied moment in the deformed condition at node i is given by:

$$M_i = M_{base} + \left(M_{top} - M_{base}\right)\frac{z_i}{L} + N_{Sz}\left(d_n - d_i\right) \tag{6}$$

where d_i is the displacement obtained at the previous iteration for the node under consideration; and d_n the displacement obtained at the previous iteration to the top of the column.

The increase of the applied moments in the second iteration results in a new deformed configuration of each cross section of the bar, which, in turn generates a new displaced position of the nodes and increases their moments. This process is repeated until the occurrence of one of the following situations:

a) convergence of the procedure for a given tolerance;
b) instability of the bar by excessive displacement;
c) failure of the material due to the occurrence of stresses exceeding the standards limits.

It was observed that the occurrence of items b and c is perceived by the fact that the applied moment considered is above the moment-curvature relationship, not being possible in this case to get a curvature corresponding to the moment.

The same formulation of restrained-free columns is adopted for the calculation of simple supported columns, but it uses half of the bar length value in the calculations (Figure 4), since the buckling coefficient K of those columns is equal to 1.0, compared to the value of K = 2.0 in restrained-free columns. It is emphasized that this symmetry condition must also be assumed to the applied diagram of bending moments.

Based on this iterative process, it is possible to identify if the column resists the axial force and moments applied.

To obtain the column resistance, it was developed a second algorithm based on the plastic resistance of cross section and repeating the foregoing process. First, the plastic resistance of the cross section in axial compression, N_{REF}, is obtained by:

$$N_{REF} = A_a f_{ya} + A_c f_{ck} + A_s f_{ys} \tag{7}$$

Table 1 – Comparison of resistance for reinforced concrete columns by tests and CSTMI

Authors	Column (exp1; exp2)		Base (cm)	Height (cm)	Number of bars	Unitary As (cm²)	Total As (cm²)	Cover (cm)	L (cm)	fc (MPa)	fy (MPa)	e1 (cm)	Fu,exp 1 (kN)	Fu,exp 2 (kN)	Fu,CSTMI (kN)	Fu,CSTMI/Fu,exp1	Fu,CSTM/Fu,exp2	Fu,CSTMI/Fu,exp,mean
Goyal e Jackson (7)	A1	A2	7,62	7,62	4	0,355	1,42	1,27	182	19,9	352	3,81	33,1	33,4	33,1	1,00	0,99	1,00
Goyal e Jackson (7)	C1	C2	7,62	7,62	4	0,355	1,42	1,27	182	23,3	352	2,54	44,5	46,8	48,2	1,08	1,03	1,06
Goyal e Jackson (7)	E1	E2	7,62	7,62	4	0,355	1,42	1,27	182	21,9	352	1,27	66,7	65,4	67,3	1,01	1,03	1,02
Goyal e Jackson (7)	G1	G2	7,62	7,62	4	0,355	1,42	1,27	182	22,2	352	1,91	55,4	53,0	55,5	1,00	1,05	1,02
Goyal e Jackson (7)	I1	I2	7,62	7,62	4	0,25	1,00	1,27	182	22,7	310	1,27	60,0	57,4	58,6	0,98	1,02	1,00
Goyal e Jackson (7)	K1	K2	7,62	7,62	4	0,25	1,00	1,27	182	22,8	310	1,91	46,6	45,6	45,7	0,98	1,00	0,99
Goyal e Jackson (7)	M1	M2	7,62	7,62	4	0,25	1,00	1,27	182	22,9	310	2,54	37,1	37,0	34,4	0,93	0,93	0,93
Goyal e Jackson (7)	O1	O2	7,62	7,62	4	0,25	1,00	1,27	122	23,6	310	1,27	82,3	92,4	84,0	1,02	0,91	0,96
Goyal e Jackson (7)	P1	P2	7,62	7,62	4	0,25	1,00	1,27	122	23,6	310	1,91	64,5	72,7	65,5	1,02	0,90	0,96
Goyal e Jackson (7)	Q1	Q2	7,62	7,62	4	0,25	1,00	1,27	122	19,9	310	2,54	51,4	48,9	45,4	0,88	0,93	0,91
Goyal e Jackson (7)	R1	R2	7,62	7,62	4	0,25	1,00	1,27	274	21,4	310	1,27	33,5	31,1	31,1	0,93	1,00	0,96
Goyal e Jackson (7)	S1	S2	7,62	7,62	4	0,25	1,00	1,27	274	20,9	310	1,91	23,0	24,3	24,4	1,06	1,00	1,03
Goyal e Jackson (7)	T1	T2	7,62	7,62	4	0,25	1,00	1,27	274	20,7	310	2,54	19,4	20,6	21,2	1,09	1,03	1,06
Kim e Yang (8)	L2-1	L2-2	8,00	8,00	4	0,3175	1,27	1,50	144	25,5	387	2,40	63,7	65,7	65,8	1,03	1,00	1,02
Kim e Yang (8)	L2-3	L2-4	8,00	8,00	4	0,3175	1,27	1,50	240	25,5	387	2,40	38,2	35,0	36,1	0,95	1,03	0,99
Kim e Yang (8)	L4-1	L4-2	8,00	8,00	8	0,3175	2,54	1,50	24	25,5	387	2,40	109,5	109,3	102	0,93	0,93	0,93
Kim e Yang (8)	L4-3	L4-4	8,00	8,00	8	0,3175	2,54	1,50	240	25,5	387	2,40	49,0	47,0	47,1	0,96	1,00	0,98
Kim e Yang (8)	M2-1	M2-2	8,00	8,00	4	0,3175	1,27	1,50	24	63,5	387	2,40	179,0	182,8	168,6	0,94	0,92	0,93
Kim e Yang (8)	M2-3	M2-4	8,00	8,00	4	0,3175	1,27	1,50	144	63,5	387	2,40	102,8	113,5	100,2	0,97	0,88	0,93
Kim e Yang (8)	M2-5	M2-6	8,00	8,00	4	0,3175	1,27	1,50	240	63,5	387	2,40	45,2	47,6	54,7	1,21	1,15	1,18
Kim e Yang (8)	M4-1	M4-2	8,00	8,00	8	0,3175	2,54	1,50	24	63,5	387	2,40	207,7	204,6	186,7	0,90	0,91	0,91
Kim e Yang (8)	M4-3	M4-4	8,00	8,00	8	0,3175	2,54	1,50	240	63,5	387	2,40	59,6	60,5	70,7	1,19	1,17	1,18
															Global Mean			1,00
															Standard Deviation			0,07

where A_a, A_c and A_s are, respectively, the areas of steel I-section, concrete and reinforcements; f_{ya}, and f_{ys} are, respectively, the yield stresses of steel I-section and reinforcements; and f_{ck} is the compressive characteristic strength of concrete.

To determine the moment-curvature relationship the program starts considering an applied axial force N_{Sz} equal to 10% of the plastic resistance N_{REF}. If the process results in convergence, the program increases the applied force in more 10% of N_{REF}, and so on until convergence is not reached. From this point, cuts of 1% of N_{REF} are made until there is convergence of the model again, yielding then, with 1% precision of N_{REF}, the column axial force resistance.

3. Validation of results

3.1 Reinforced concrete columns

To validate the procedure for calculation of reinforced concrete columns implemented via CSTMI, the authors have modeled columns subjected to combined compression and uniaxial bending studied by Araújo [6], who compared theoretical results obtained from a computer program implemented by him to results of several experiments by other researchers.

It is emphasized that, as a comparison with tests, all partial factors were considered unit. He also specified as unitary the value of α, reduction factor on the characteristic strength of the concrete due to the long-term effects (Rüsch effect). The tested columns were subjected to constant bending moments. All these considerations have also been adopted in the numerical model.

Table 1 shows the test results compared to the results obtained by CSTMI.

In both tests performed by Goyal and Jackson [7], and in those performed by Kim and Yang [8], two columns with the same geometric characteristics and material (for example, the first line of the Table, columns A1 and A2) were evaluated for each series. Thus, there are two experimental results, $F_{u,exp1}$ and $F_{u,exp2}$, for each column series.

Samples of columns of the same series can be considered dependent on each other, thus should not be evaluated separately in order to not violate the principle of statistical independence. In this case, it may be used only the value of $F_{u\,exp1}$, or only $F_{u,\,exp2}$, or the average of the two values. It was evaluated the data of the ratios $R = F_{u,\,CSTMI}/F_{u,\,EXP1}$ and $R = F_{u,\,CSTMI}/F_{u,\,EXP2}$ for the couple of tests with the same characteristics, and it can be concluded that it is possible to take the average between the tests since they do not differ (p-value = 0.357, from the paired t-test at 5% significance).

The histogram of Figure 5 shows the frequency distribution for the R mean values obtained for each tested pair of columns (for example, A1 and A2). It was observed that the distribution is normal (p-value > 0.15, using the Kolmogorov-Smirnov test).

The average value of the ratios $R = F_{u,\,CSTMI}/F_{u,\,EXP}$ was 1.0, with a standard deviation of 0.07. In order to verify that the theoretical value calculated by CSTMI is equivalent to the test values, it was performed the t-test to compare if the average is equal to 1.0. The p-value found was 0.887, indicating that the average value does not differ from 1.0 so it can be stated that the column's resistance determined by CSTMI is equivalent to the test values.

Figure 5 – Histogram of R ratios
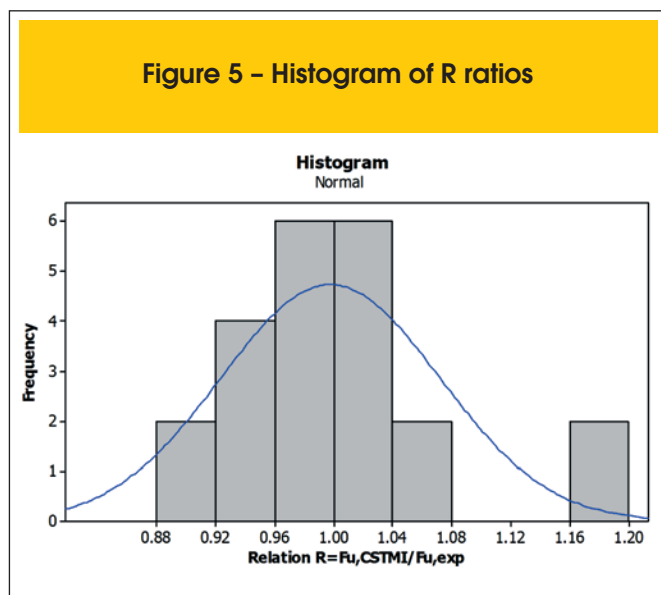
Figure 6 – Box-plot of R ratios for each method
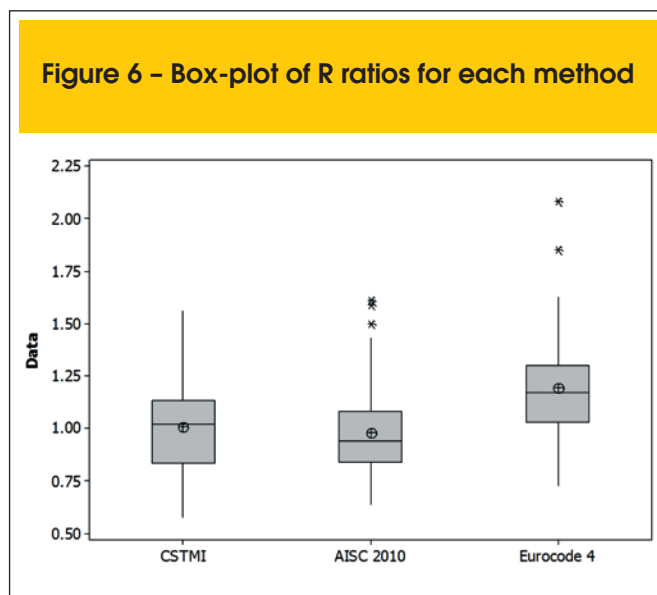
3.2 Fully encased composite columns

Composite columns subjected to combined compression and uni-axial bending studied by Kim [9] were modeled to validate the procedure of calculation of fully concrete encased steel I-section (SC) implemented at CSTMI program. The author raised an extensive database of experimental results of several researchers [10 to 16] with samples of fully encased composite columns, concrete-filled circular, and rectangular hollow sections.

The results of fully encased composite columns were divided into two categories: *columns* with 119 tests of bars subjected to axial compression; and *beams-columns*, with 136 tests of bars subjected to combined compression and uniaxial bending.

Since it is required an initial eccentricity to start the iterative process for the calculation of the column resistance, the category *columns* samples were disregarded. It was also removed the tests of *beams-columns,* beams with low density concrete, without reinforcement bars, and specimen with the occurrence of biaxial bending, remaining a sample of 87 experiments.

The tests were performed in simply supported columns with axial compression and lateral forces producing a constant bending moment along the bar. As the tests of reinforced concrete columns, it was also considered for the concrete of the composite columns the factor α = 1.0. All columns were divided into 10 longitudinal ele-

ments and the cross section was divided into elements of 3 and 4 sides of approximately 10 mm.

Kim has compared the experimental data with the theoretical calculation from the procedures of ANSI/AISC 360-05 [17] (identical to ANSI/AISC 360-10 [18]) and EN 1994-1-1:2004 [19]. Table 3 lists the tests evaluated, with the characteristics of the columns, the applied eccentricities and the experimental resistance. It is also presented the R = $N_{R,EXP}/N_{R,TEO}$ ratios for each of the standards and with the use of CSTMI program.

It is observed that the R ratios for all of the evaluated methods presented great variability (for CSTMI the lowest value was R = 0.58 and the highest, R = 1.56). This shows the difficulty to calibrate a numerical method to results so different. With the elimination of outliers in the same test group (tests 106 to 109 of Han, 1992), the sample was reduced from 87 to 83 columns.

Table 2 lists the mean and standard deviation for the R ratio for the methods evaluated by Kim and the CSTMI. Figure 6 presents a box-plot illustrating this information.

For the CSTMI, it was obtained an average value of the ratio R equals to 1.01 with a standard deviation of 0.20. Figure 7 shows a dispersion of the R values for all tests calculated by CSTMI. The 95% confidence interval was set between 0.60 and 1.41, ie., 95% of the R values are in this range.

To verify the equivalence of theoretical to the test values, it was performed a z test to compare if the mean is equal to 1.0. The

Table 2 – Mean values of the R ratio for comparisons between the composite columns tests, implemented, and standards procedures

Methods	N	Mean	Standard-deviation	Minimum	1st Quartile	Median	3rd Quartile	Maximum
CSTMI	83	1.01	0.20	0.58	0.83	1.02	1.13	1.56
AISC 2010	83	0.98	0.22	0.64	0.84	0.94	1.08	1.61
EN 1994-1-1:2004	83	1.19	0.22	0.73	1.03	1.17	1.30	2.08

Table 3 – Database of test results of fully encased composite columns

Item	Source	Specimen	fy (MPa)	fc (MPa)	fys (MPa)	d (mm)	bf (mm)	tf (mm)	tw (mm)	bc (mm)	hc (mm)	n φ	Unitary As (cm²)	kL (m)	ey (m)	ex (m)	Nn exp (kN)	Nn CSTMI (kN)	CSTMI	AISC 2010	EN 1994-1-1:2004
6	Stevens (10)	FE3	218	17,4	270	304,8	203,2	21,22	14,15	305	406	4	1,27	4,57	0,0000	0,0254	2885	2669	1,08	1,19	1,41
7	Stevens (10)	FE4	218	16,3	270	304,8	203,2	21,22	14,15	305	406	4	1,27	4,57	0,0000	0,0508	2087	1858	1,12	1,08	1,29
8	Stevens (10)	FE5	218	27,0	270	304,8	203,2	21,22	14,15	305	406	4	1,27	4,57	0,0000	0,0508	2212	2393	0,92	0,92	1,18
9	Stevens (10)	FE6	218	18,5	270	304,8	203,2	21,22	14,15	305	406	4	1,27	4,57	0,0000	0,0762	1548	1516	1,02	0,90	1,13
10	Stevens (10)	FE7	218	18,5	270	304,8	203,2	21,22	14,15	305	406	4	1,27	4,57	0,0000	0,1016	1269	1222	1,04	0,85	1,12
11	Stevens (10)	FE8	218	19,3	270	304,8	203,2	21,22	14,15	305	406	4	1,27	4,57	0,0000	0,1270	1125	1045	1,08	0,84	1,14
12	Stevens (10)	FE9	218	18,8	270	304,8	203,2	21,22	14,15	305	406	4	1,27	4,57	0,0000	0,1524	991	886	1,12	0,84	1,17
13	Stevens (10)	FE10	218	21,2	270	304,8	203,2	21,22	14,15	305	406	4	1,27	4,57	0,0000	0,1778	856	830	1,03	0,75	1,14
14	Stevens (10)	FE11	218	20,7	270	304,8	203,2	21,22	14,15	305	406	4	1,27	4,57	0,0000	0,2032	721	719	1,00	0,70	1,11
28	Janss e Anslijn, (11)	11.1	281	34,3	270	140	140	12	7	240	240	4	1,13	3,45	0,0000	0,0399	1095	1345	0,81	0,89	1,08
29	Janss e Anslijn, (11)	11.2	281	33,1	270	140	140	12	7	240	240	4	1,13	3,45	0,0000	0,0399	1156	1286	0,90	0,96	1,16
30	Janss e Anslijn, (11)	11.3	281	28,8	270	140	140	12	7	240	240	4	1,13	3,45	0,0000	0,0399	1047	1190	0,88	0,93	1,13
31	Janss e Anslijn, (11)	12.1	372	32,2	270	140	140	12	7	240	240	4	1,13	3,45	0,0000	0,0399	1156	1290	0,90	0,90	1,08
32	Janss e Anslijn, (11)	12.2	372	31,2	270	140	140	12	7	240	240	4	1,13	3,45	0,0000	0,0399	1095	1270	0,86	0,86	1,04
33	Janss e Anslijn, (11)	12.3	372	28,8	270	140	140	12	7	240	240	4	1,13	3,45	0,0000	0,0399	973	1189	0,82	0,79	0,96
34	Janss e Anslijn, (11)	13.1	267	33,2	270	220	110	9,2	5,9	210	320	4	1,13	2,45	0,0000	0,0399	1173	1304	0,90	0,94	1,12
35	Janss e Anslijn, (11)	13.2	267	31,2	270	220	110	9,2	5,9	210	320	4	1,13	2,45	0,0000	0,0399	1021	1220	0,84	0,85	1,01
36	Janss e Anslijn, (11)	13.3	267	30,6	270	220	110	9,2	5,9	210	320	4	1,13	2,45	0,0000	0,0399	999	1205	0,83	0,84	1,00
50	Roik e Schwalbenhofer, (12)	V11	251	43,8	412	120	120	11	6,5	280	280	4	1,53	3,00	0,1600	0,0000	748	1048	0,71	0,66	1,17
51	Roik e Schwalbenhofer, (12)	V12	251	43,8	412	120	120	11	6,5	280	280	4	1,53	3,00	0,0599	0,0000	1597	2052	0,78	0,87	0,96
52	Roik e Schwalbenhofer, (12)	V13	251	46,8	412	120	120	11	6,5	280	280	4	1,53	3,00	0,1001	0,0000	1407	1469	0,96	0,92	1,26
53	Roik e Schwalbenhofer, (12)	V21	335	46,8	412	152	160	9	6	280	280	4	1,53	3,00	0,1001	0,0000	1474	1694	0,87	0,79	1,01
54	Roik e Schwalbenhofer, (12)	V22	335	37,0	412	152	160	9	6	280	280	4	1,53	3,00	0,1600	0,0000	932	1190	0,78	0,74	1,12
55	Roik e Schwalbenhofer, (12)	V23	335	37,0	412	152	160	9	6	280	280	4	1,53	3,00	0,0599	0,0000	1905	2040	0,93	0,95	1,05
56	Roik e Schwalbenhofer, (12)	V31	256	40,7	412	200	200	15,00	9,00	280	280	4	1,53	3,00	0,1001	0,0000	1674	2026	0,83	0,75	0,94
57	Roik e Schwalbenhofer, (12)	V32	256	40,7	412	200	200	15,00	9,00	280	280	4	1,53	3,00	0,0599	0,0000	2210	2633	0,84	0,81	0,91
58	Roik e Schwalbenhofer, (12)	V33	256	39,3	412	200	200	15,00	9,00	280	280	4	1,53	3,00	0,1600	0,0000	1283	1639	0,78	0,75	1,03
59	Roik e Schwalbenhofer, (12)	V41	333	39,3	412	186	200	24,00	14,50	280	280	4	1,53	3,00	0,1001	0,0000	2082	2641	0,79	0,72	0,87
60	Roik e Schwalbenhofer, (12)	V42	335	42,2	412	200	186	24,00	14,50	280	280	4	1,53	3,00	0,1600	0,0000	1503	1971	0,76	0,73	0,85
61	Roik e Schwalbenhofer, (12)	V43	335	42,2	412	200	186	24,00	14,50	280	280	4	1,53	3,00	0,0599	0,0000	2680	3467	0,77	0,73	0,85
65	Roik e Schwalbenhofer, (12)	V111	306	40,1	412	152	160	9,00	6,00	280	280	4	6,16	3,00	0,1001	0,0000	1721	2990	0,58	0,90	0,98
66	Roik e Schwalbenhofer, (12)	V112	306	40,1	412	152	160	9,00	6,00	280	280	4	6,16	3,00	0,0599	0,0000	2466	3773	0,65	1,03	1,01
67	Roik e Schwalbenhofer, (12)	V121	237	40,1	412	120	120	11	6,5	280	280	4	6,16	3,00	0,1600	0,0000	1116	1393	0,80	0,88	1,05
68	Roik e Schwalbenhofer, (12)	V122	237	40,1	412	120	120	11	6,5	280	280	4	6,16	3,00	0,1999	0,0000	797	1105	0,72	0,73	0,95
69	Roik e Schwalbenhofer, (12)	V123	237	40,1	412	120	120	11	6,5	280	280	4	6,16	3,00	0,1001	0,0000	1505	1922	0,78	0,91	0,97
70	Roik e Mangerig, (13)	7	265	42,0	412	200	200	15,00	9,00	300	300	4	1,13	3,00	0,0000	0,0300	4460	3726	1,20	1,43	1,50
71	Roik e Mangerig, (13)	8	265	42,0	412	200	200	15,00	9,00	300	300	4	1,13	3,00	0,0000	0,0899	2188	1976	1,11	1,07	1,34
72	Roik e Mangerig, (13)	9	265	42,0	412	200	200	15,00	9,00	300	300	4	1,13	5,00	0,0000	0,0300	3595	2992	1,20	1,50	1,55
73	Roik e Mangerig, (13)	10	265	42,0	412	200	200	15,00	9,00	300	300	4	1,13	5,00	0,0000	0,0899	1791	1524	1,17	1,05	1,34
74	Roik e Mangerig, (13)	11	265	42,0	412	200	200	15,00	9,00	300	300	4	1,13	8,00	0,0000	0,0300	1983	1807	1,10	1,61	1,39
75	Roik e Mangerig, (13)	12	265	42,0	412	200	200	15,00	9,00	300	300	4	1,13	8,00	0,0000	0,0899	976	1016	0,96	0,96	1,19
76	Roik e Mangerig, (13)	23	265	42,0	412	200	200	15,00	9,00	300	300	4	1,13	5,00	0,0899	0,0000	2294	2089	1,10	0,99	1,28
77	Roik e Mangerig, (13)	24	265	42,0	412	200	200	15,00	9,00	300	300	4	1,13	5,00	0,1501	0,0000	1605	1524	1,05	0,87	1,30
78	Roik e Mangerig, (13)	25	265	42,0	412	200	200	15,00	9,00	300	300	4	1,13	8,00	0,0899	0,0000	1647	1524	1,08	1,02	1,38
79	Roik e Mangerig, (13)	26	265	42,0	412	200	200	15,00	9,00	300	300	4	1,13	8,00	0,1501	0,0000	876	1129	0,78	0,64	1,01
80	Roik e Mangerig, (13)	27	265	42,0	412	220	206	25,00	15,00	300	300	4	1,13	5,00	0,0300	0,0000	4099	4236	0,97	1,01	1,12
81	Roik e Mangerig, (13)	28	265	42,0	412	220	206	25,00	15,00	300	300	4	1,13	8,00	0,0300	0,0000	2265	3143	0,72	0,79	0,89
82	Roik e Mangerig, (13)	29	265	42,0	412	220	206	25,00	15,00	300	300	4	1,13	5,00	0,0300	0,0000	3433	4236	0,81	0,85	0,94
83	Roik e Mangerig, (13)	30	265	42,0	412	220	206	25,00	15,00	300	300	4	1,13	8,00	0,0300	0,0000	1854	3143	0,59	0,64	0,73
84	Mirza e Hyttinen, (14)	RHB-1	287	27,2	554	96	100	8,00	5,00	240	240	4	0,79	4,00	0,0396	0,0000	933	950	0,98	1,21	1,20
85	Mirza e Hyttinen, (14)	RHB-2	287	27,2	554	96	100	8,00	5,00	240	240	4	0,79	4,00	0,0658	0,0000	540	678	0,80	0,82	0,92
86	Mirza e Hyttinen, (14)	RHB-3	287	27,7	554	96	100	8,00	5,00	240	240	4	0,79	4,00	0,1057	0,0000	560	481	1,16	1,02	1,40
87	Mirza e Hyttinen, (14)	RHB-4	305	25,9	622	96	100	8,00	5,00	240	240	4	0,79	4,00	0,2004	0,0000	302	270	1,12	0,80	1,30
88	Mirza e Hyttinen, (14)	RHB-4A	287	25,1	554	96	100	8,00	5,00	240	240	4	0,79	4,00	0,3711	0,0000	151	129	1,18	0,64	1,23
90	Mirza e Hyttinen, (14)	RNHB-1	305	27,5	622	96	100	8,00	5,00	240	240	4	0,79	4,00	0,0493	0,0000	908	841	1,08	1,18	1,21
91	Mirza e Hyttinen, (14)	RNHB-2	305	27,5	622	96	100	8,00	5,00	240	240	4	0,79	4,00	0,0577	0,0000	761	747	1,02	1,04	1,10
92	Mirza e Hyttinen, (14)	RNHB-3	287	26,8	554	96	100	8,00	5,00	240	240	4	0,79	4,00	0,1059	0,0000	530	493	1,08	0,95	1,28
93	Mirza e Hyttinen, (14)	RNHB-4	287	27,2	554	96	100	8,00	5,00	240	240	4	0,79	4,00	0,2093	0,0000	346	249	1,39	0,89	1,52
95	Mirza e Hyttinen, (14)	RHNB-1	305	27,5	622	96	100	8,00	5,00	240	240	4	0,79	4,00	0,0419	0,0000	910	934	0,97	1,13	1,12
96	Mirza e Hyttinen, (14)	RHNB-2	305	27,5	622	96	100	8,00	5,00	240	240	4	0,79	4,00	0,0594	0,0000	707	747	0,95	0,97	1,04
97	Mirza e Hyttinen, (14)	RHNB-3	305	25,9	622	96	100	8,00	5,00	240	240	4	0,79	4,00	0,0986	0,0000	530	517	1,03	0,92	1,17
98	Mirza e Hyttinen, (14)	RHNB-4	305	25,9	622	96	100	8,00	5,00	240	240	4	0,79	4,00	0,2060	0,0000	291	270	1,08	0,75	1,21
100	Roik e Diekmann, (15)	11	279	46,6	270	200	200	15,00	9,00	300	300	4	1,13	3,00	0,0500	0,0000	3902	3458	1,13	1,12	1,22
101	Roik e Diekmann, (15)	12	279	46,6	270	200	200	15,00	9,00	300	300	4	1,13	3,00	0,0500	0,0000	3180	3458	0,92	0,92	0,98
102	Roik e Diekmann, (15)	13	279	46,6	270	200	200	15,00	9,00	300	300	4	1,13	3,00	0,0500	0,0000	3719	3458	1,08	1,07	1,16
103	Roik e Diekmann, (15)	14	242	46,6	270	152	160	9	6	300	300	4	1,13	3,00	0,0500	0,0000	2792	2770	1,01	1,12	1,25
104	Roik e Diekmann, (15)	15	392	46,6	270	200	200	15,00	9,00	300	300	4	1,13	3,00	0,0500	0,0000	4297	3804	1,13	1,09	1,19
105	Roik e Diekmann, (15)	16	279	46,6	270	200	200	15,00	9,00	300	300	4	1,13	3,00	0,1001	0,0000	2633	2698	0,98	0,98	1,26
110	Han e Kim, (16)	AH2-E2-80	310	21,4	275	100	100	8	6	160	160	4	0,71	0,70	0,0201	0,0000	1175	752	1,56	1,43	1,52
111	Han e Kim, (16)	AH2-E4-80	310	21,4	275	100	100	8	6	160	160	4	0,71	0,70	0,0399	0,0000	808	580	1,39	1,29	1,44
112	Han e Kim, (16)	AH2-E8-80	310	21,4	275	100	100	8	6	160	160	4	0,71	0,70	0,0800	0,0000	517	419	1,23	1,11	1,28
113	Han e Kim, (16)	BH2-E2-80	310	21,4	275	100	100	8	6	160	160	4	0,71	1,20	0,0201	0,0000	952	752	1,27	1,24	1,40
114	Han e Kim, (16)	BH2-E4-80	310	21,4	275	100	100	8	6	160	160	4	0,71	1,20	0,0399	0,0000	735	580	1,27	1,27	1,52
115	Han e Kim, (16)	BH2-E8-80	310	21,4	275	100	100	8	6	160	160	4	0,71	1,20	0,0800	0,0000	489	419	1,17	1,07	1,28
116	Han e Kim, (16)	CH1-E2-80	310	21,4	275	100	75	4,50	3,20	160	160	4	0,71	1,70	0,0201	0,0000	860	561	1,53	1,59	1,63
117	Han e Kim, (16)	CH1-E4-80	310	21,4	275	100	75	4,50	3,20	160	160	4	0,71	1,70	0,0399	0,0000	552	425	1,30	1,25	1,37
118	Han e Kim, (16)	CH1-E8-80	310	21,4	275	100	75	4,50	3,20	160	160	4	0,71	1,70	0,0800	0,0000	339	262	1,29	1,04	1,42
119	Han e Kim, (16)	CH2-E2-80	310	21,4	275	100	100	8	6	160	160	4	0,71	1,70	0,0201	0,0000	965	752	1,28	1,43	1,85
120	Han e Kim, (16)	CH2-E4-80	310	21,4	275	100	100	8	6	160	160	4	0,71	1,70	0,0399	0,0000	671	580	1,16	1,08	1,25
121	Han e Kim, (16)	CH2-E8-80	310	21,4	275	100	100	8	6	160	160	4	0,71	1,70	0,0800	0,0000	467	419	1,11	1,05	1,30
122	Han e Kim, (16)	DH2-E2-80	310	21,4	275	100	100	8	6	160	160	4	0,71	2,20	0,0201	0,0254	831	728	1,14	1,39	2,08
123	Han e Kim, (16)	DH2-E4-80	310	21,4	275	100	100	8	6	160	160	4	0,71	2,20	0,0399	0,0000	600	518	1,16	1,02	1,20
124	Han e Kim, (16)	DH2-E8-80	310	21,4	275	100	100	8	6	160	160	4	0,71	2,20	0,0800	0,0000	470	358	1,32	1,10	1,42
																Mean			1,01	0,98	1,19
																Standard Deviation			0,20	0,22	0,22

Figure 7 – Dispersion of the R ratio results for CSTMI calculation

CSTMI method has obtained acceptable result (p-value = 0.746 at 5% significance). It is noted that a p-value higher than 5% indicates no difference between the mean value and 1.0 so that the result obtained with CSTMI is satisfactory with a better quality than the result obtained by the ANSI/AISC 360-10 [18] (p-value equal to 0.377).

From this analysis it can be concluded that the procedure implemented to determine the ultimate axial force of fully encased composite columns is appropriate in general, with better approximations than the ANSI/AISC 360-10 [18] and EN 1994-1-1:2004 [19].

4. Conclusions

For reinforced concrete columns, CSTMI was adequate, since the resistance results proved compatible with the samples of 44 columns tested by other researchers.

For fully encased composite columns, the program CSTMI was compared with a sample of 87 tests by other researchers. The numerical results were calibrated with the experimental results for many slenderness ratios.

It is concluded that the implemented procedure provides good results for the determination of the resistance of reinforced concrete columns and fully encased composite columns. This procedure can be considered valid as a general method for calculating columns subjected to combined compression and uniaxial bending.

5. Acknowledgements

6. References

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