IBRACON
IBRACON Structures and Materials Journal
Revista IBRACON de Estruturas e Materiais
ISSN 1983-4195 ismj.org
SciELO

ORIGINAL ARTICLE

Influence of concrete strength on the distribution of bending moment in widened curved bridges

Influência da resistência do concreto na distribuição do momento fletor em pontes curvas alargadas

Yngrid Rayane Freitas Nascimento[a]
José Neres da Silva Filho[a]
Arthur da Silva Rebouças[b]
Rodrigo Barros[a]
Joel Araújo do Nascimento Neto[a]

[a]Universidade Federal do Rio Grande do Norte – UFRN, Programa de Pós-graduação em Engenharia Civil – PEC, Natal, RN, Brasil
[b]Instituto Federal de Educação, Ciência e Tecnologia do Rio Grande do Norte – IFRN, São Paulo do Potengi, RN, Brasil

Received September 13, 2021
Accepted April 22, 2022

Abstract: The study proposes to analyze the distribution of bending moment due to live load in curved bridges that have undergone a process of widening, considering the influence of concrete's strength variation. The results show that the bending moment redistribution is more significant the higher the stiffness in the widenings. In addition, the redistribution induced by the variation of stiffness depends on the live load positioning but, generally, it results in the migration of bending moments to the stiffer regions, relieving the original girders. The curvature did not significantly alter the response induced by the stiffening of the widened segments. Also, the divergences found between the MEF and the V-load Method results for models with uniform and variable stiffness were similar. Finally, the Modification Factors (MF) proved to be more sensitive to the influence of curvature than to the concrete strength in the widenings.

Keywords: bridge widening, widening stiffness, live loads, FEM, V-load method.

Resumo: Esta pesquisa propõe analisar a distribuição de momentos fletores devido à carga móvel entre longarinas de pontes curvas de concreto armado que passaram por um processo de alargamento da superestrutura. Como principais conclusões foi constatado que a redistribuição de momentos é tanto mais significativa quanto mais elevada for a rigidez dos alargamentos. Ademais também foi observado que a redistribuição induzida pela variação da rigidez está condicionada à posição do carregamento móvel, porém, em geral, resultam na migração de esforços para os trechos mais rígidos, aliviando as longarinas originais. Constatou-se ainda que a variação da curvatura pouco altera a redistribuição de esforços induzida pelo aumento da rigidez no alargamento. Ademais, as divergências entre os resultados do MEF e do Método *V-Load* para os modelos com rigidez variável não divergiram significativamente daqueles obtidos nos modelos de rigidez constante. Por fim, os Fatores de Modificação (FM) mostraram-se mais sensíveis à variação do raio de curvatura do que à resistência do concreto nos alargamentos.

Palavras-chave: pontes alargadas, resistência do concreto, cargas móveis, MEF, método V-load.

How to cite: Y. R. F. Nascimento, J. N. Silva Filho, A. S. Rebouças, R. Barros, and J. A. Nascimento Neto, "Influence of concrete strength on the distribution of bending moment in widened curved bridges," *Rev. IBRACON Estrut. Mater.*, vol. 16, no. 1, e16105, 2023, https://doi.org/10.1590/S1983-41952023000100005

Corresponding author: Yngrid Rayane Freitas Nascimento. E-mail: yngrid.rayane.eng@gmail.com
Financial support: None.
Conflict of interest: Nothing to declare.
Data Availability: The data that support the findings of this study are available from the corresponding author, Y. N., upon reasonable request.

1 INTRODUCTION

The structural analysis of horizontally curved bridges and viaducts presents a higher complexity level when compared to similar structures with straight layouts since, due to the curvature and stiffness of the deck, some geometric parameters directly affect the structural behavior. Regardless, curved bridges have become increasingly competitive, given their structural efficiency, stability, economy, and aesthetics.

One of the most prominent subjects related to the analysis of curved bridges is the live load distribution on the deck, particularly the influence of the curvature on this mechanism. Studies such as Kim et al. [1] and Zhang et al. [2] highlight the curvature as a key parameter in the distribution of bending moments. Although relevant advances have been achieved in this field, the collected research concerning load distribution in widened curved structures is still significantly limited.

In Brazil, the geometric standards for highway bridges have been evolving since the 1940s. For the first bridges, the recommended total width was 8.30 m (27.3 ft), without considering the addition of shoulders. Nowadays, the standard width for bridges of one roadway is 12.80 m (42 ft), comprehending two lanes as well as shoulders on each side [3]. In addition to changes in the standards governing the geometric characteristics of highway bridges, the constant increase in traffic volume and, therefore, in the loads to which these structures are subjected, are factors that contribute to their poor performance. For a significant portion of Brazilian highway bridges, the need for interventions directed to the recovery, widening, or strengthening is substantial, under the risk of becoming critical points for the occurrence of accidents [4].

According to Barros and Vitório [5], the "C*onventional Widening Method with Reinforced Concrete*" is the most employed in the country. It entails the addition of reinforced concrete beams and slabs that are incorporated into the original deck. The solidarization between the original and the widened sections is usually accomplished through a slab cast in place, executed on the upper part of the deck. When using this method, the concrete in the widened section often has a higher characteristic compressive strength and, therefore, higher stiffness than the material of the original structure. Fontana [6] proposed to analyze the impact of variable stiffness in a widened straight cellular bridge, noting the significant influence of this parameter on the distribution of bending moments. Consequently, in curved bridges widened by this method, in addition to the curvature, another aspect that should considerably impact the load distribution would be the variation of stiffness along the cross section.

Thus, the main purpose of this research is to analyze the distribution of bending moments due to live loads in curved reinforced concrete bridges, considering the implementation of deck widening, to assess the effect of variable stiffness in widened sections. Furthermore, the study evaluates the influence of the curvature and the number of girders in these structures. The analysis was based primarily on the results obtained from numerical models, using the Finite Element Method, enabling the attainment of the Bending Moment Distribution Factors (BMDF) due to the live load. Additionally, the applicability of the approximate analytical method V-Load was analyzed by comparing its results with those of the FEM. In complement, the bending moments were analyzed according to an artifice proposed by Acosta and González [7], called the Modification Factor.

2 STRUCTURAL BEHAVIOR IN CURVED BRIDGES

The radius is the parameter that determines the deviation between the structural behavior of bridges with straight and curved layouts. In curved bridges, the curvature is responsible for setting an eccentricity between the center of gravity of the deck and the axis connecting the end supports. Thus, when the deck is subject to vertical loads, this eccentricity gives rise to torsional moments whose magnitude cannot be neglected as occurs in the analysis of straight bridges.

In a curved beam subjected to vertical loads, the actions of bending and torsional forces occur in a coupled way. Figure 1 shows an infinitesimal element of a curved beam loaded only in the direction normal to the horizontal plane by load p (dead load), as well as the internal forces generated by this loading. By balancing the forces on the Y-axis is possible to obtain Equation 1:

$$\frac{dV}{ds} = -p \tag{1}$$

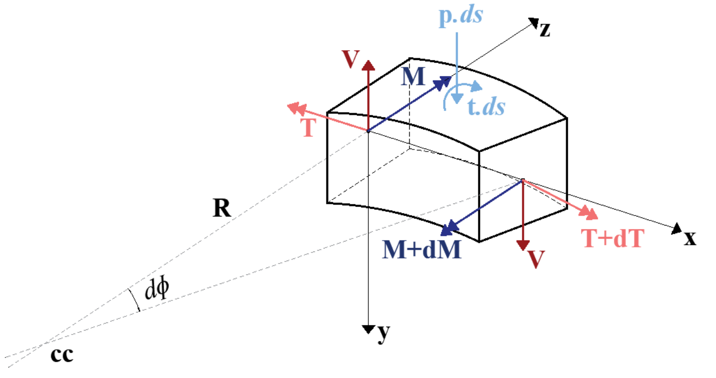

Figure 1. Internal forces in a curved beam element. Adapted from Barbosa [8].

Equation 1 shows that the shear force on an infinitesimal curved element does not depend on any geometric component of the beam. Therefore, one may conclude that there is no difference between a curved beam and a straight beam regarding the magnitude of the shear force, since the variation of the shear V along the segment *ds* results in the constant p.

From the balance of moments about the Z-axis (Figure 1) results Equation 2:

$$\frac{dM}{ds} = V - \frac{T}{R} \tag{2}$$

From the balance of moments about the X-axis results Equation 3:

$$\frac{dT}{ds} = t + \frac{M}{R} \tag{3}$$

Therefore, it is verified that there is an interaction between bending moment and torsional moment, since, according to Equations 2 and 3, the bending moment generates torsion, and the torsional moment causes longitudinal bending in the beam. It is important to highlight that both aforementioned equations are also functions of the geometric parameter radius of curvature (R). This correlation allows one to see that there is a variation of both bending moment and torsional moment along the segment *ds*, due to the radius of the curved segment.

3 APPROXIMATE ANALYSIS OF CURVED BRIDGES: V-LOAD METHOD

Fu and Wang [9] and AASHTO [10] classify structural analysis methods in refined and approximate. With the advance of technology, it has become much more practical and advantageous to employ refined methods, which comprises analyses in two (2D) or three dimensions (3D), instead of approximate methods, *i.e.*, a one-dimensional (1D) analysis. Although a wide variety of refined methods for structural analysis are available, among them FEM, approximate methods should not be disregarded.

Besides being simple to apply, approximate methods can be used in preliminary design, or they can serve as a parameter to validate results from more complex analysis methods. Furthermore, these methods often allow a better understanding of the structural behavior of the structures to which they are applied. For curved steel girder bridges, AASHTO [10] allows the use of the V-Load Method.

According to Fiechtl et al. [11], V-Loads result from the equilibrium, as a function of the radius of curvature (R), the bridge width (D), and the diaphragm spacing (d). Figure 2a shows the segment of curved bridge with two girders and five diaphragms spaced radially. Considering that the girders sections resist the bending moment entirely by longitudinal forces applied on the flanges, as shown in Figure 2b, the force on each flange of girder 1 is M_1/h_1, where h_1 is the distance between the points of application of the forces and M_1 is the bending moment.

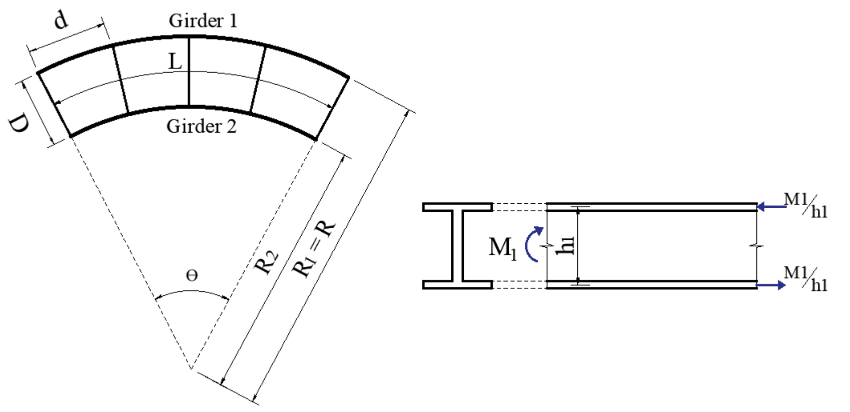

Figure 2. (a) Segment of a curved bridge with two girders; (b) Longitudinal bending and flange forces acting on girder 1. Adapted from Fiechtl et al. [11].

However, since the girders are curved, the longitudinal forces due to bending in different sections are not in equilibrium. Therefore, to ensure radial equilibrium on the flanges, there must be a force acting in the direction of the diaphragm, indicated by H_1. Similar forces must appear on the bottom flange.

The forces H_1 and H_2 generate a rotation tendency and, to ensure the stability of the diaphragm, vertical forces must appear, as indicated in Figure 3. These shear forces are called V-Loads, indicated by V. The force H_1 is determined by Equation 4, and the force H_2 can be obtained analogously.

$$H_1 = \frac{M_1 \cdot \theta}{h_1} \tag{4}$$

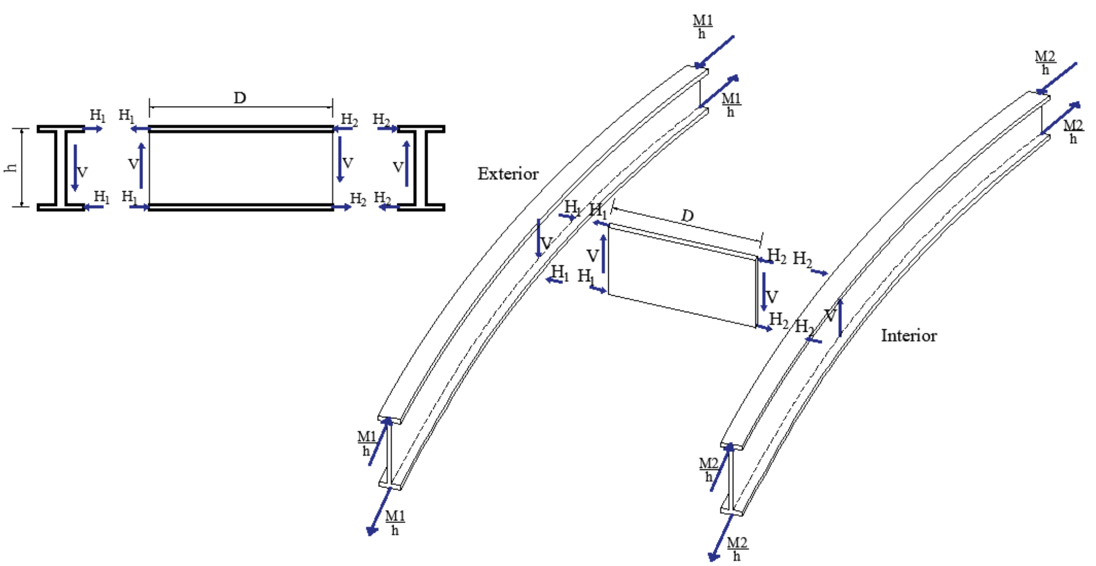

Figure 3. View of the bridge focusing on the diaphragms. Adapted from Fiechtl et al. [11].

In Equation 4, θ is the angle between adjacent diaphragms, which is assumed to be small. Thus, considering the arc length and substituting the value of θ, H_1 can be given by:

$$H_1 = \frac{M_1 \cdot d_1}{h_1 \cdot R_1} \tag{5}$$

To maintain the equilibrium of moments in the diaphragm, the vertical force must be:

$$V = (H_1 + H_2) \cdot \frac{h}{D} \tag{6}$$

In Equation 6, the term h refers to the diaphragm height. Considering the two girders and the diaphragm with the same height, so that $h = h_1 = h_2$, we obtain:

$$V = \frac{M_1 \cdot \frac{d_1}{R_1} + M_2 \cdot \frac{d_2}{R_2}}{D} \tag{7}$$

Since $d_1/R_1 = d_2/R_2 = d/R$, the shear force on the diaphragm is:

$$V = \frac{M_1 + M_2}{R \cdot D/d} \tag{8}$$

The bending moments due to external loads (those that were applied to the bridge) are called "primary moments" and will be identified by the index "p". The additional bending moments due to the curvature, represented by the vertical forces (V-Loads), will be denoted by the index "v". Thus, the total bending moment is given by:

$$M_1 = M_{1p} + M_{1v} \tag{9}$$

According to Monzon et al. [12], the same procedure can be followed for bridges with three or more girders. However, in these cases, beyond summing the bending moments due to external loads, a coefficient that depends on the number of girders must be applied. Therefore, the V-Load is calculated by:

$$V = \frac{\sum_{i=1}^{N_g} M_{ip}}{C \cdot R \cdot D} d \tag{10}$$

Where M_{ip} is the primary moment on the girder "i"; D is the distance between the axis of the inner girder and that of the outer girder; R is the radius of curvature; d is the spacing between the bracing or diaphragms; C is the coefficient that considers the linear distribution of the V-Loads according to the number of N_g stringers in the cross section.

4 BENDING MOMENT DISTRIBUTION FACTOR AND MODIFICATION FACTOR

Brockenbrough [13] was one of the pioneers in the study of live load distribution factors in curved bridges. In his research, the author proposes to obtain the distribution factors through the ratio between the bending moment obtained by a refined three-dimensional model and those resulting from a simplified girder model. Based on this concept, in this paper, the distribution factors are determined through the ratio between the moment at the most solicited section (mid-span section) for each girder, obtained from 3D models, and the bending moment for the whole bridge, obtained by modeling the bridge as a beam (1D). Thus, the bending moment distribution factors are attained through Equation 11.

$$BMDF = \frac{M^{MEF,3D}{}_{girder}}{M^{1D}{}_{bridge}} \tag{11}$$

In addition to bending moment distribution factors, this study also had its analyses based on the Modification Factor (MF), developed by Acosta and González [7], obtained by the ratio between the maximum bending moment on the girders of a curved bridge (CB) and the maximum bending moment on the equivalent girders of a straight bridge (SB) (Equation 12). According to the authors, the purpose of Modification Factors is to serve as a reference in design situations, as it allows to estimate the internal forces in a curved bridge from the results of a straight bridge with equivalent dimensions, *i.e.*, same span length and cross-section.

$$MF = \frac{Max.girder\ bending\ moment\ (curved\ bridge)}{Max.girder\ bending\ moment\ (\ straight\ bridge)} = \frac{M_{CB}}{M_{SB}} \tag{12}$$

5 MODELS

The preexisting bridge has a total width of 8.30 m, supported by three girders. After the widening, the deck will be 12.80 m wide. The analysis will consider two different scenarios:

— Scenario: 1: Widening performed through symmetric addition of two girders (one on each side), amounting to five girders;

— Scenario 2: Widening performed through symmetric addition of four girders (two on each side), amounting to seven girders;

The preexisting structure was generated admitting concrete compressive characteristic strength (f_{ck}) equal to 25 MPa. In regards to the widenings, models with uniform and variable stiffness along the cross section were developed. The models with uniform stiffness had their widenings modeled with the same f_{ck} of the original structure and served for comparison with the models with variable stiffness. In these models, the widened regions were modeled either considering a concrete compressive strength of 40 MPa or of 60 MPa.

In curved bridges, the curvature is usually established by fixating the radius or central angle along the entire length of the bridge. In terms of curvature, the models were classified into three groups:

— Group 01 (G1): Infinite radius of curvature (straight bridge);

— Group 02 (G2): Radius of 150 meters (central angle equals to 12.25°);

— Grupo 03 (G3): Radius of 50 meters (central angle equals to 36.6°).

A total of 18 models were analyzed, as shown in Table 1. The models were identified according to the number of girders, radius, and concrete compressive strength in the widened section. Thus, model 7LG2-40R, for example, consists of a 7-girder bridge with a radius of 150 meters and concrete compressive strength in the widened section of 40 MPa.

Table 1. Summary of developed models

Number of girders	Group	Compressive strength of concrete in the widening region	Model identification
5	G1	25	5LG1-25R
		40	5LG1-40R
		60	5LG1-60R
	G2	25	5LG2-25R
		40	5LG2-40R
		60	5LG2-60R
	G3	25	5LG3-25R
		40	5LG3-40R
		60	5LG3-60R
7	G1	25	7LG1-25R
		40	7LG1-40R
		60	7LG1-60R
	G2	25	7LG2-25R
		40	7LG2-40R
		60	7LG2-60R
	G3	25	7LG3-25R
		40	7LG3-40R
		60	7LG3-60R

5.1 Geometry

Figure 4 shows the cross sections adopted, considering the widening performed by adding two (Figure 4a) and four girders (Figure 4b).

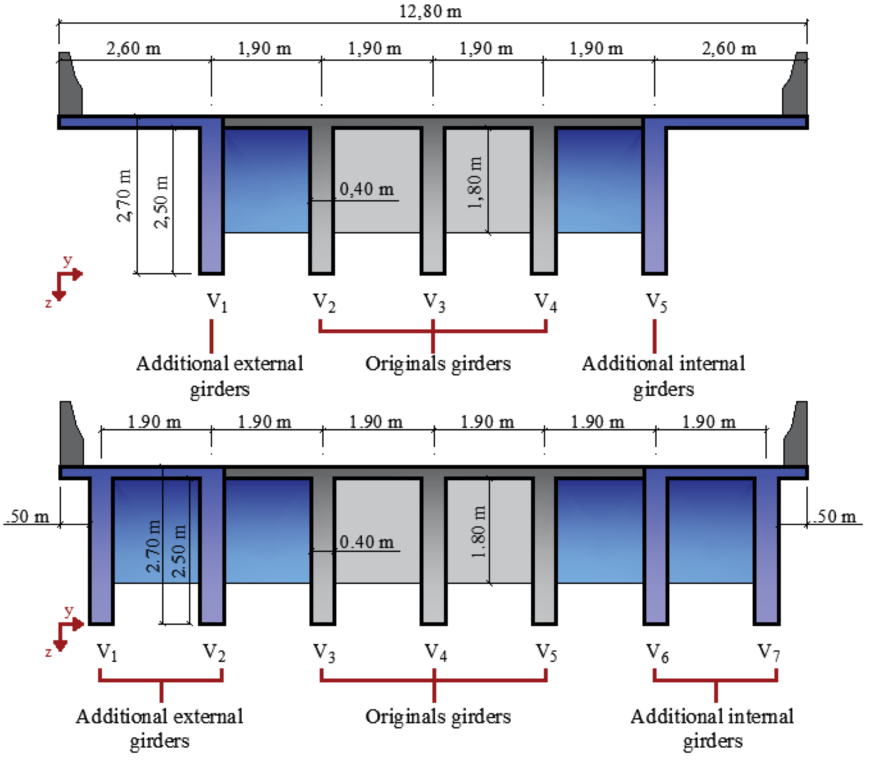

Figure 4. Cross sections after widening: (a) 5-girder model; (b) 7-girder model.

5.2 Materials

In all models, the material properties were determined following ABNT NBR 6118 [14]. To assess the influence exerted by the stiffness variation in the distribution of bending moments, the elasticity modulus (E_{cs}), calculated using the characteristic compressive strength of concrete, will be admitted. The elasticity moduli for the three types of concrete considered in the research are presented in Table 2. Considering that the expressions used in determining the elasticity modulus are related to the type of aggregate used in the concrete, it is relevant to emphasize that the values shown were determined admitting the use of granite aggregate.

Table 2. Material properties

f_{ck} (MPa)	E_{cs} (MPa)
25	24150
40	31875
60	41208

5.3 Load cases and boundary conditions

This research only considers the live loads of the design truck TB-450 as defined by the ABNT NBR 7188 [15]. As established by this code, the TB-450 is a three-axle vehicle that weighs 450 kN, and occupies an area of 18 m^2, as shown in Figure 5. Outside the region occupied by the design truck, a 5.0 kN/m^2 uniformly distributed load is applied.

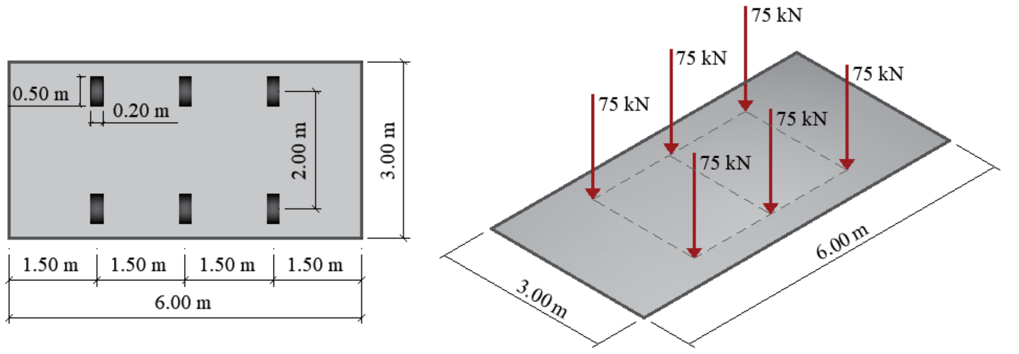

Figure 5. Vehicle type TB-450. Adapted from ABNT NBR 7188 [15].

Each girder absorbs a percentage of the total bending moment and to determine accurately this effect, the design vehicle was fixated in strategic transversal positions, moving only along the length of the bridge. They were called load cases 01 and 02. In load case 01, the design vehicle is fixed at the outermost position of the deck, while in load case 02 it is positioned on the centerline of the deck (Figure 6).

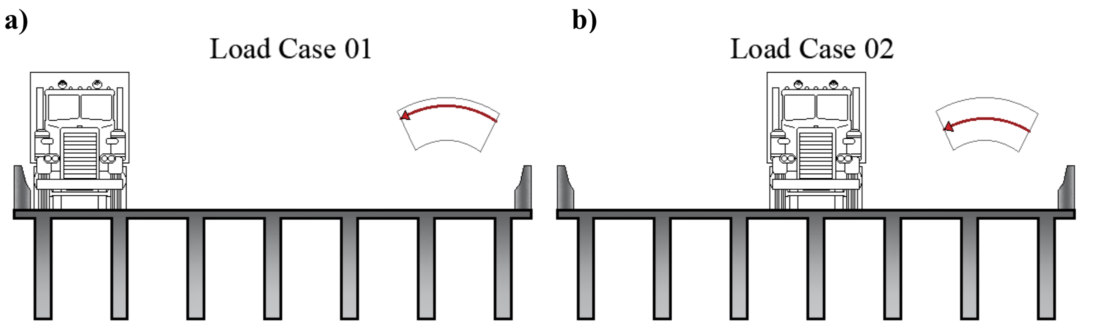

Figure 6. (a) Load case 01 – vehicle positioned at the outside of the curve. (b) Load case 02 – vehicle positioned at the curve centerline.

The boundary conditions, in turn, were defined following what was proposed by Samaan et al. [16]. The authors identified that the *tangential method of restriction*, in which the translations are restrained only in the directions tangential or radial to the curve, produces results consistent with experimental data, besides being easier to execute. In all models, the girders are directly supported on the abutments, restraining vertical translations in all supports. However, to ensure the global equilibrium of the structure, in one of the girders, the boundary conditions are slightly different. In addition to the vertical constraint, the support at one end also constrains the tangential displacement, while the other extremity constrains tangential and radial displacements. Rebouças et al. [17] successfully applied these conditions in order to perform a similar analysis.

6 3D MEF MODELS

The numerical modeling was performed through the software CSIBridge® (version 21), which utilizes FEM. Fu and Wang [9] state that the level of accuracy of bridge modeling depends on the desired results and recommend the generation of three-dimensional models with two-dimensional elements. Studies such as Kim et al. [1] and Nevling et al. [18] show that this methodology provides highly satisfactory results, similar to data obtained in the field.

This research developed three-dimensional models, with four nodes shell elements representing the slabs, girders, abutment, and diaphragms (Figures 7 and 8). In the development of the models, through consideration of boundary conditions and finite element analysis, it was decided to fixate the element dimension along the length of the bridge to 1.00 meter. Furthermore, the aspect ratio was limited to 2.5, since the use of smaller elements required too much time in computational work for little gain in precision. It is worth noting that according to Logan [19], and with the corroboration of Fu and Wang [9] and Fatemi et al. [20], aspect ratios over 4 generate processing errors higher than 15%.

The determination of the bending moment distribution factors from the numerical models developed in CSIBridge® was done by the ratio between the maximum bending moment of each girder and the total bending moment acting on the bridge.

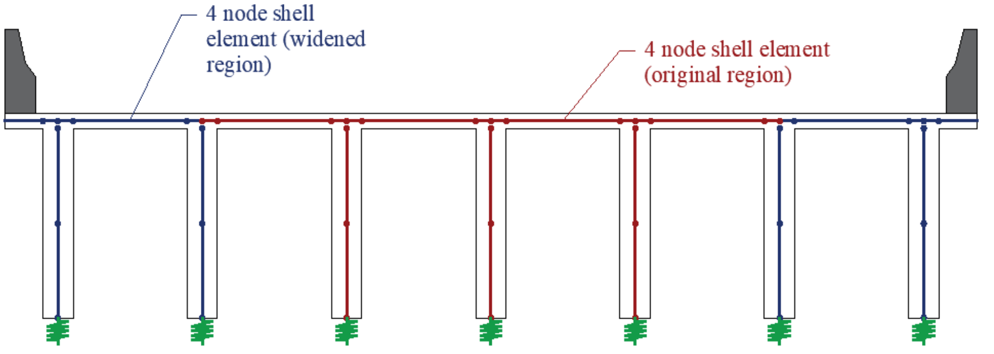

Figure 7. Discretization of bridge cross section in numerical model.

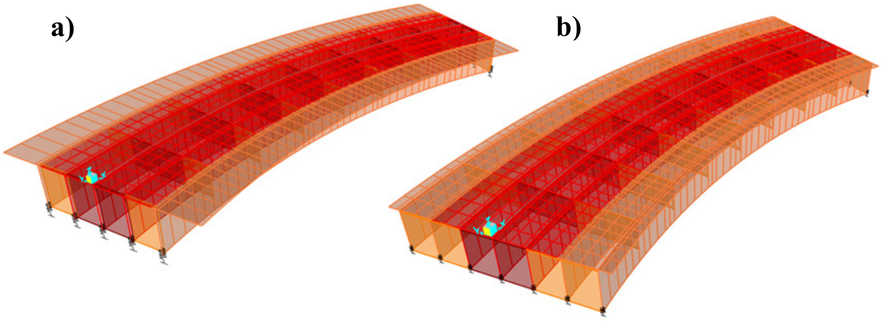

Figure 8. Three-dimensional finite element bridge model adopted in the analysis: (a) 5-girder model; (b) 7-girder model.

7 ANALYSES OF RESULTS

To assess the behavior of the bending moment distribution factors, the percentage variations between the BMDF's were calculated through two separate approaches: Global Analysis and Local Analysis. In the Global Analysis, the percentage difference captures the variation of the bending moment distribution factors (due to live load) that reaches the girders ($V_i, V_{i+1}, V_{i+2}, V_{i+3}$) compared to the factor in the reference girder within the same model (Figure 9). This approach provides a better understanding of the general behavior of bending moment distribution per model since the comparison occurs between girders. In all cases, the percentages have the innermost girder (right side of the deck) as reference.

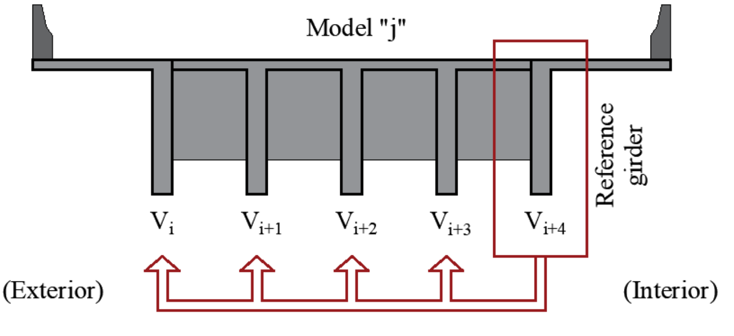

Figure 9. Methodology for Global Analysis.

In Local Analysis, the percentage reflects the magnitude of increase or decrease in the BMDF's of a girder ${V_i}^j$ compared to the factor of a girder of the same local position (${V_i}^{j+1}$, ${V_i}^{j+2}$) but of a different model (Figure 10). Thus, greater emphasis will be given to this approach since it enables the direct analysis of the influence of the considered parameters. In the terms that identify the girders, shown in Figures 9 and 10, the index "i" specifies the girder while the index "j" refers to the model.

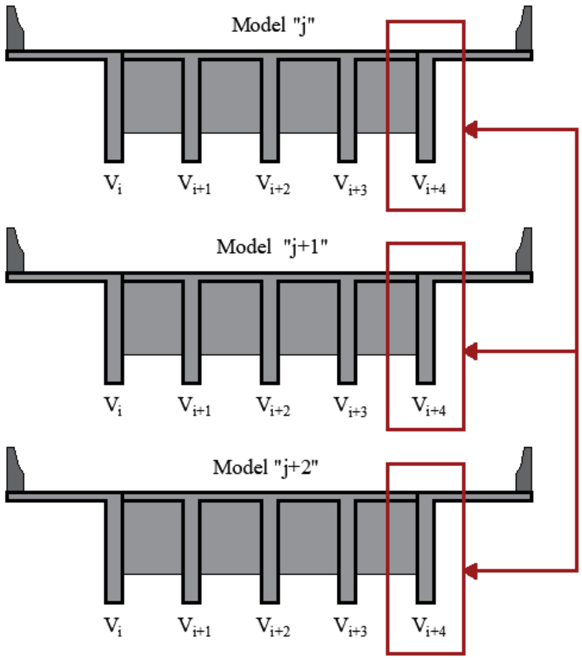

Figure 10. Methodology for Local Analysis

7.1 Bending Moment Distribution Factors

Figure 11 presents the percentual differences for global analysis obtained for the BMDF's from group G3 models (R = 50 m), considering the load cases 01 and 02. The reference girder is always the innermost one (V5 in 5-girder models, and V7 in 7-girder models). First, it is verified that the maximum percentages occur in the outermost girder (V1) for both load cases, regardless of the stiffness of the widened regions. That indicates that said girder absorbs the majority of the total bending moment.

According to the Precast Concrete Institute (PCI) [21], the bending stresses in the outermost girder tend to be substantially higher than in the other girders. One of the reasons for this behavior is that the arc length in the axis of the outermost girder is longer than the centerline of the bridge. This increases the bending moments in the external girder by approximately the square of the ratio between the arch lengths. Additionally, there is the fact that the girders transfer a portion of their torsional moments to the adjacent beam. This transfer develops to the external direction of the curve (from inside to outside). Thus, the outermost girder receives the contribution of the adjacent beam, but without redistributing a portion of these internal forces. In general, the direction of growth of the BMDF's is from inside to outside. Since all the percentage variations obtained are positive, one may conclude that the BMDF's are minimum in the innermost girder, that is, V5 in 5-girder models and V7 in 7-girder models.

Still considering the values obtained through load case 01, regarding the additional external girders, the rise in widening stiffness amplifies the global percentage. Thus, it is possible to deduce that the increase in stiffness heightens the differences between the BMDF's of the additional external girders and the reference girder. For example, the percentage increase of the BMDF for girder V1 compared to V5, considering the widened section with f_{ck} of 60 MPa, is 509.2%, while in the model with uniform stiffness, this variation is 418.6% (Figure 11a).

In the original girders, the behavior is the opposite. In the 5-girder model (Figure 11-a), for girders V2 and V3, the increase of stiffness in the widening reduces the percentage differences. In the girder V4, increases in the percentages are registered, although the variation rate is low. In the 7-girder model (Figure 11b), the original girders (V3, V4, and V5) behave consistently, presenting reductions with increasing stiffness.

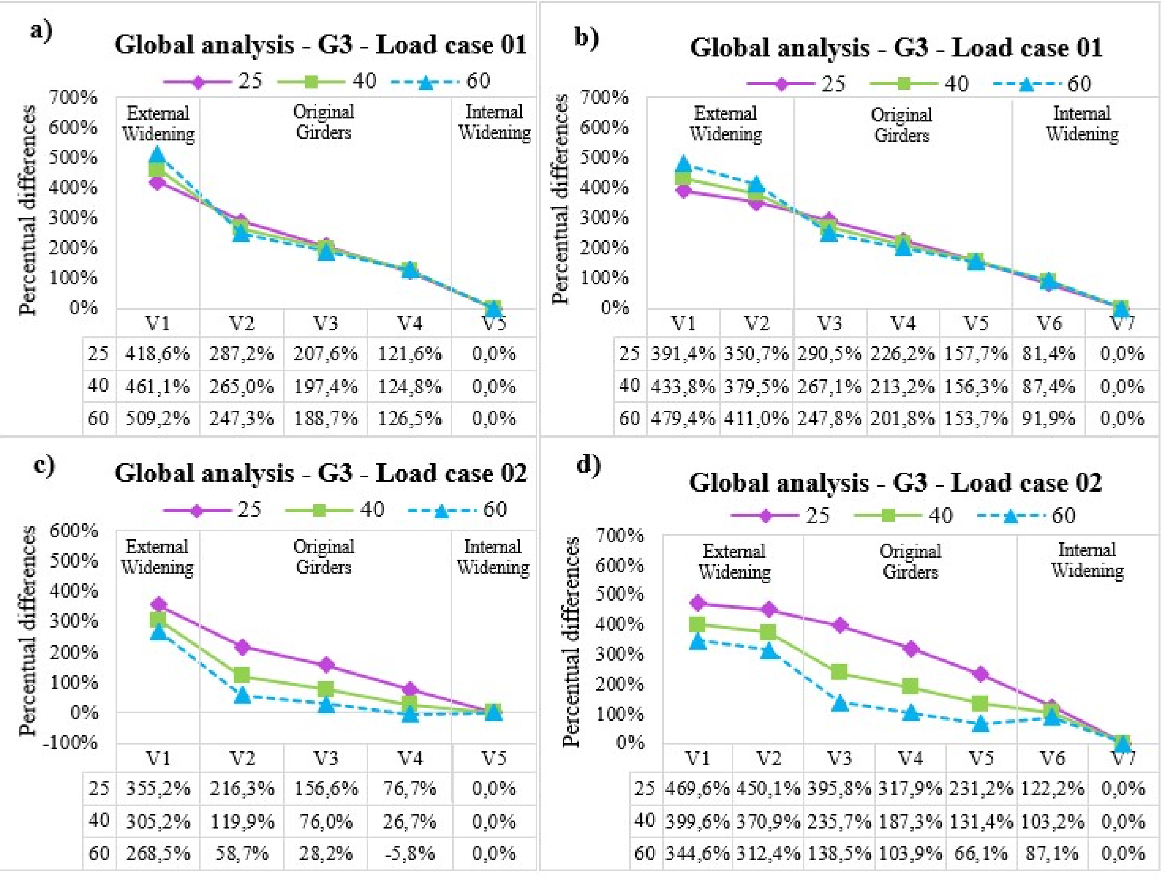

Figure 11. Global analysis of the BMDF's for the models with 25, 40, and 60 MPa widening, in comparison to the innermost girder: (a) 5L-G3 (Load case 01); (b) 7L-G3 (Load case 01); (c) 5L-G3 (Load case 02); (d) 7L-G3 (Load case 02).

Moving on to the analysis of the percentages resulting from load case 02, in the case of the 5-girder models (Figure 11c), once again, the BMDF's are minimal in the reference girder (V5). The only exception is the model in which the widening has f_{ck} of 60 MPa, in which the minimum BMDF occurs in the girder V4, although the percentage is of little importance (-5.8%). Analogously, in the 7-girder model (Figure 11d), all percentages are positive, indicating that the minimum BMDF's occur in the girder V7. It is important to note that the percentage variations related to load case 01 are higher than those obtained by load case 02. This behavior stem from the fact that, despite the asymmetry inherent to curved bridges, positioning the type-vehicle on the centerline of the deck, associated with the presence of diaphragms, contributes to a more homogeneous distribution. Furthermore, unlike what was verified for load case 01, the global percentages from load case 02 tend to reduce with the stiffening of the widening areas.

As for the influence of stiffness, generally, the variations tend to be smaller in models with stiffer widenings. According to the data presented in Figures 11c,d, the global percentages behavior is approximately linear in models with uniform stiffness. However, in models with variable stiffness, the reduction rate of the global percentages caused by raising the stiffness is higher in the original girders, being more subtle in the widened sections.

7.2 Analysis of the influence of stiffness in the widenings by means of concrete compressive strength

Figure 12 summarizes the local percentage differences calculated for the group G3 models (R = 50 m), considering load cases 01 and 02. These results will be used to assess the effects induced in each girder by raising the widening stiffness.

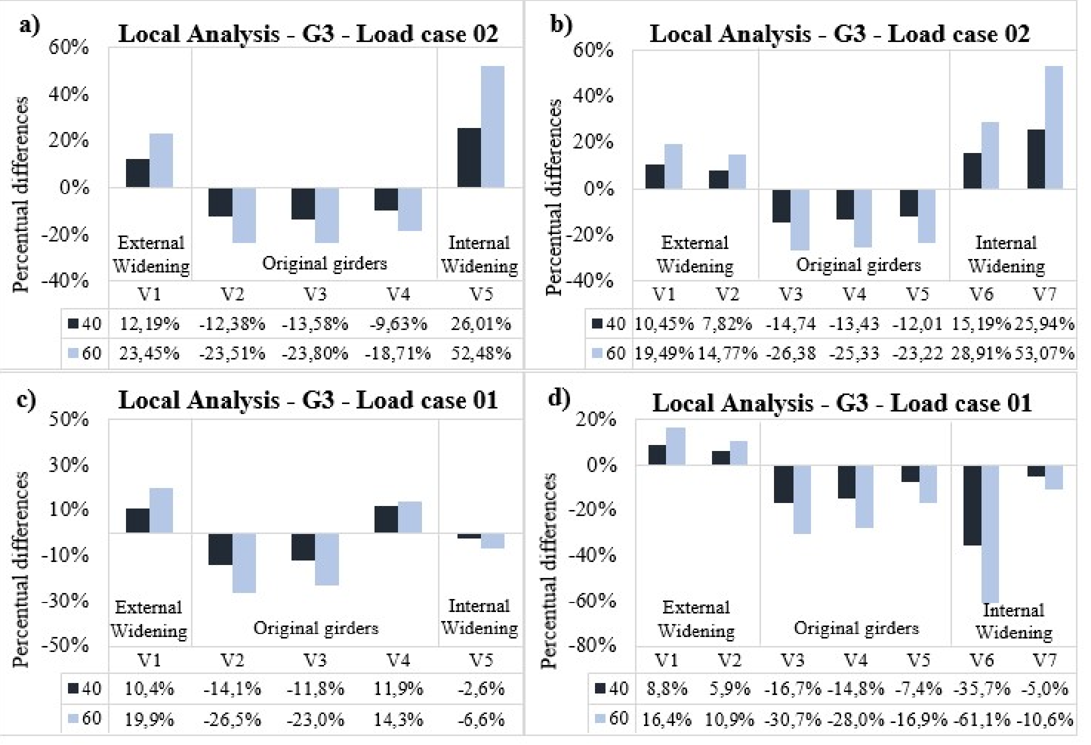

Figure 12. Local analysis of the BMDF's for models with 40 and 60 MPa widening, compared to the models with uniform stiffness.: (a) 5L-G3 (Load case 02); (b) 7L-G3 (Load case 02); (c) 5L-G3 (Load case 01); (d) 7L-G3 (Load case 01).

When studying the effect of widening in straight box bridges, Fontana [6] observed that the stiffer widenings began to support a higher percentage of loading, which led to an increase in the distribution factors in these areas. As a result, the distribution factors in the girder corresponding to the original deck suffered reductions. The author also verified that the loading decreases registered in the original girders, as well as the loading increases in the additional girders, tend to be higher the stiffer is the widening. Accordingly, this was the behavior verified in the values for load case 02 (Figure 12a,b).

In all cases, the percentages are higher for the models where the widening performed with characteristic compressive strength of 60 MPa. For the 5-girder models (Figure 12a), the increments in the additional girder V5 are higher than in the girder V1. Similarly, in the 7-girder model, the percentages attained for the additional internal girders (V6 and V7) are higher than those of the external girders V1 and V2. For both cases, in the original girders, the percentages decrease, in absolute value, towards the inner side of the curve, although the observed variation is low.

When it comes to curved bridges, the position of the load has a significant influence on the load distribution. In Figures 12c,d, related to load case 01, the additional external girders (V1 in the 5-girder model, V1, and V2 in the 7-girder model) experiences loading increases. For the 5-girder model, girders V2 and V3 show reductions, while in the 7-girder model, decreases occur in girders V3, V4, and V5. So far, the observations are within the expected behavior considering only the effect of stiffening the widening. However, in the 5-girder model, the original girder V4 experiences an increase in loading, while in the additional girder V5 occurs a reduction. In the 7-girder model, the additional internal girders (V6 and V7) present reductions, particularly the girder V6, in which the decrease reaches -61.1%.

Executing widenings with a higher stiffness than that of the original deck should relieve the pre-existing girders, not overload them. The behavior observed in the BMDF's attained from load case 01 is highly problematic since it has original girders receiving additional loading while widening girders, with concrete of higher compressive strength, receive less load than they would in a deck with uniform stiffness. This inconvenience is mitigated by the fact that the BMDF's on the girders positioned in the inner side of the deck (right side) tend to be smaller.

7.3 Analysis of the influence of curvature

Kim et al. [1] and Zhang et al. [2] presented the curvature as one of the most influential parameters in the distribution of bending moments in curved bridges. For this reason, it is relevant to evaluate the effect that varying the radius has on the distribution of bending moments in the widened deck.

In the previous subsection, the percentages resulted from the comparison of the BMDF's of the models of variable stiffness in relation to those of uniform stiffness, considering the models of group G3 (Figure 12). Deepening this analysis, Figure 13 introduces the percentages obtained from a similar analysis applied to the models of groups G1 and G2. It should be noted that, although Figure 13 shows the local percentage differences only for the 7-girder models, the behavior verified in the 5-girder models was analogous.

The first noteworthy aspect is that the variation in curvature does not significantly affect the local percentages. This allows for the conclusion that the effects of stiffness variation in the widened sections are independent of the bridge radius of curvature.

Preliminarily, one can see that the increases and decreases fit the observations made in the analysis of the percentages obtained for group G3 (Figure 12). For load case 01 (Figure 13a,c), as expected, loading increases happen in the additional external girders V1 and V2, while the original girders experience decreases. In the additional internal girder V7, contrary to expectations, reductions are verified.

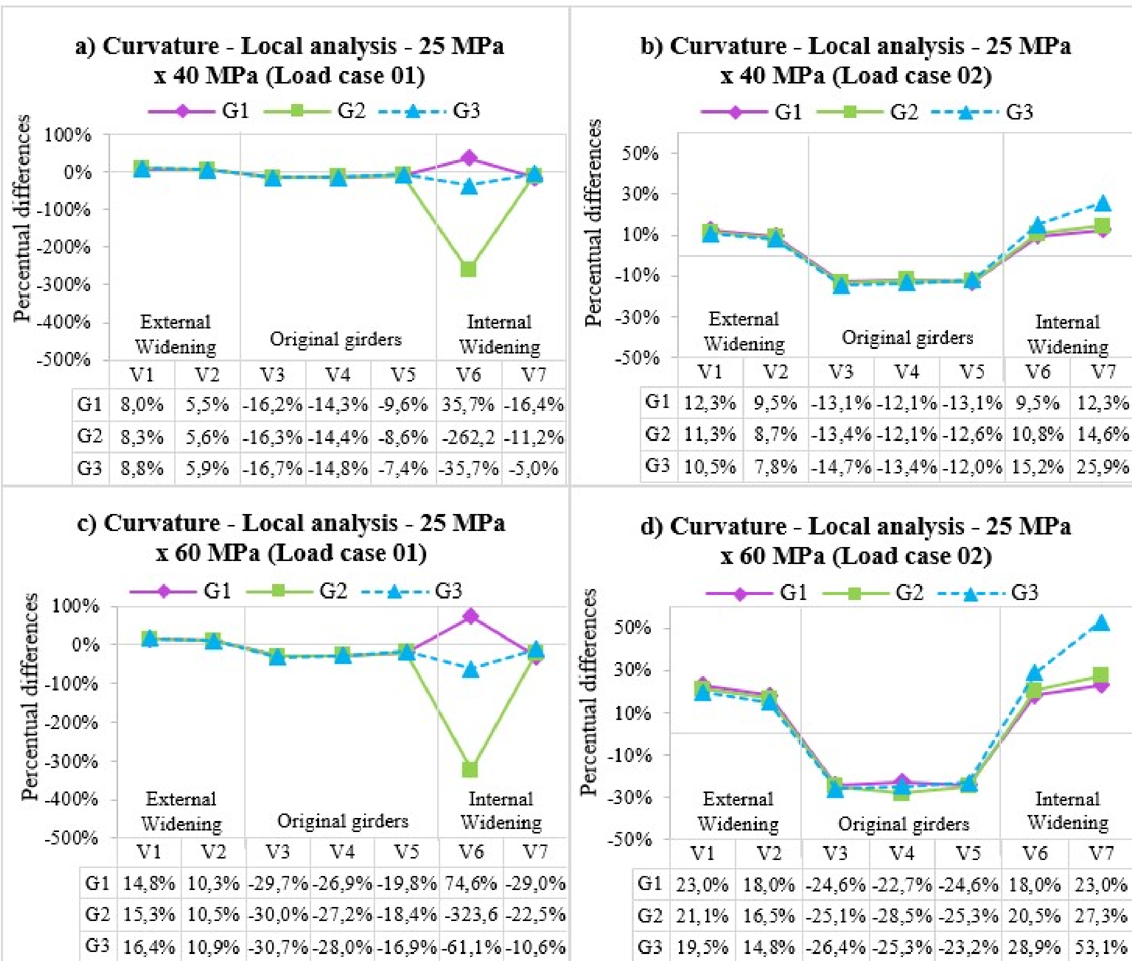

Figure 13. Local analysis of the BMDF's as a function of the radius for the 7-girder models: (a) widening of 40 MPa (Load case 01); (b) widening of 40 MPa (Load case 02); (c) widening of 60 MPa (Load case 01); (d) widening of 60 MPa (Load case 02);

The girder V6, however, experiences increases in the straight model (G1) and decreases in the curved models (G2 and G3). These reductions are notably high for the models with the greater radius (G2), and these discrepancies occur mainly due to two reasons. Firstly, for load case 01, the bending moments tended to concentrate on the outermost girders. The values of the BMDF's attained for the innermost girders (the girders V6 and V7 in particular) were in general very low. Thus, the percentages calculated by dividing the BMDF variation by the corresponding distribution factor in the model with uniform stiffness (reference model) contributed to producing high values. In addition to that, in the model with uniform stiffness, contrary to the expectations, the bending moment direction caused tension on the upper fibers of the girder. However, when analyzing the models with variable stiffness, the influence exerted by the widening stiffening caused the inversion of the bending moment direction, which, as in the other girders, tensioned the lower fibers of the girder V6. Since the obtained data considered the difference between the load factor of the model with variable stiffness and the load factor on the reference model, the percentages in question reflected the total amplitude of the variation that led to the inversion of bending moment direction and not the occurrence of expressive variations in load distribution.

It is worth noting that the phenomenon of reversal of moment direction only occurred in the girder V6 of the group G2 models. Although after the inversion, the modulus of the acting moment became lower, the inversion on its own represents an issue since, aside from causing relief of the original reinforcement, it creates a demand for new reinforcement in a different region.

For the percentages obtained through load case 02 (Figure 13b,d), the effect of the curvature variation also proves to be negligible, and the highest impact is registered on the innermost girder (V7). In these girders, the increments rise with the reduction of the radius, being maximum for the group G3 models.

7.4 Analysis of the influence of the number of girders

Works such as Zhang et al. [2] highlight the number of girders as a factor of high impact on the load distribution in curved bridges. According to this study, for bridges with more than three girders, the addition of girders causes a significant reduction in the load distribution factors. In this sense, this analysis examines the effect of adding girders, comparing the results obtained from 5-girder models with those from 7-girder models. To achieve that, this assessment compares the sum of the factors in the original girders (O.G.), additional external (Ext. G.), and internal (Int. G.) between the 5-girder and 7-girder models. To better explain, the sum of the BMDF's from girders V2, V3, and V4 (5-girder model) will be compared to the sum of the factors from girders V3, V4, and V5 (the 7-girder model). Similarly, the BMDF of girder V1 will be compared to the sum of the factors from girders V1 and V2, while the factor from girder V5 will be compared to the sum of BMDF's from girders V6 and V7. These comparisons resulted in local percentage variations, as shown in Figure 14, considering the group G2 models (R = 150 m).

For both load cases, in response to the increase in the number of girders, increments are observed in the quantity of bending moment absorbed by the external widened section (Ext. G.). Although these values indicate that the total moment absorbed in this section is higher in models with 7 girders, it should be noted that this section includes two girders, which share this loading and, therefore, are relieved. With the stiffening of the widening, these increments drop. In the original section, the portion of bending moment absorbed is smaller in the 7-girder model, indicating that the increase in the number of girders relieved the pre-existing girders, and said relief is more expressive the higher the stiffness of the widening.

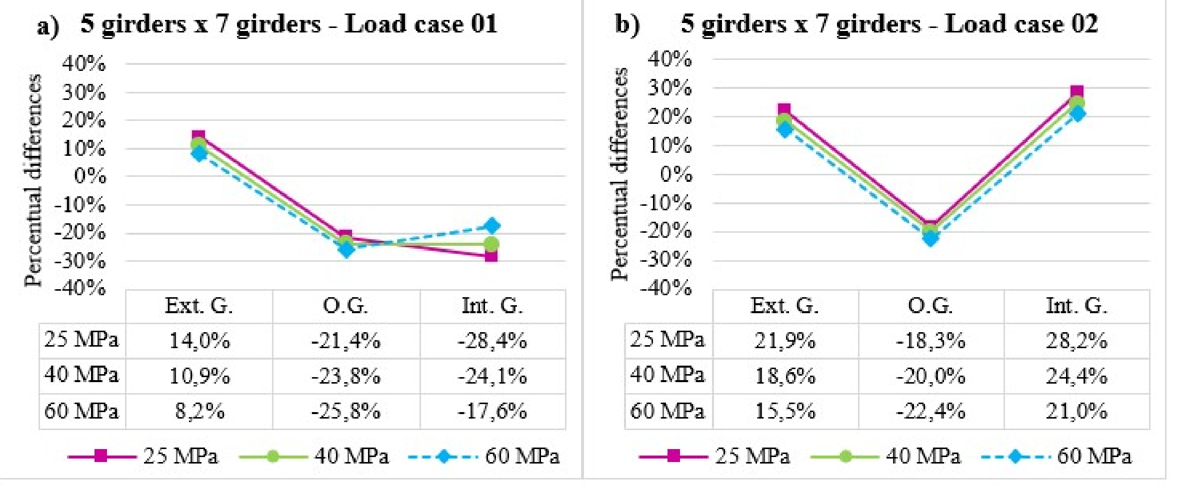

Figure 14. Local analysis between the BMDF's of the 5-girder and 7-girder models: (a) Load case 01; (b) Load case 02.

In the internal widened section, reductions are registered for load case 01 (Figure 14a), while increases occur for load case 02 (Figure 14b). However, for both load cases, in response to the widening stiffening, the differences between the total bending moment absorbed by the widenings reduce.

7.5 Comparison between numerical models and the V-Load method

In order to assess its applicability, the results obtained using the V-Load Method were compared with those generated through FEM. The comparison is based on percentage differences, obtained through local analysis. Therefore, the percentage reflects the variation between the BMDF calculated by the V-Load method compared to the value generated by FEM. This methodology was applied to models with uniform stiffness, as well as to those with variable stiffness along the cross-section. Figure 15 presents the data generated through this analysis.

Figures 16a,b show the local percentage variations for the group G2 models (R = 150 m). For the girders included in the widened outermost section (V1 for the 5-girder model, V1 and V2 for the 7-girder model), decreases are registered, indicating that the BMDF's obtained by the V-Load Method in these girders are lower than those resulting from the FEM. In the central girders on both 5 and 7-girder models, reductions are also verified. In the 5-girder model, the percentages are higher, considering absolute value, in the girder V5. Similarly, the variations for the 7-girder model tend to be higher in the innermost girders and are maximum in the girder V7.

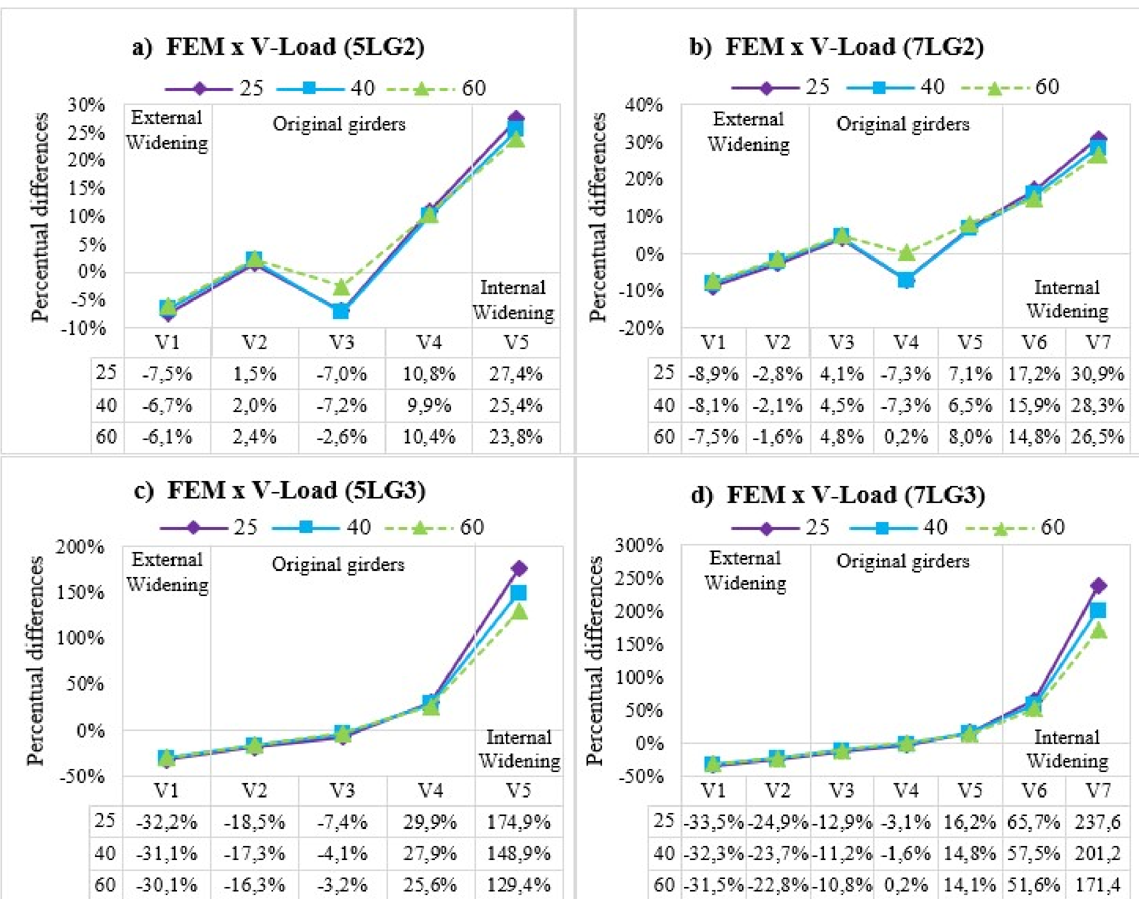

Figure 15. Local analysis between the FEM and the V-Load Method: (a) 5L-G2; (b) 7L-G2; (c) 5L-G3; (d) 7L-G3.

For the models belonging to group G3 (Figure 15c,d), regardless of the number of girders, reductions are observed from the outermost (V1) to the central girder. The main difference between the percentage's behavior from group G2 and G3 models is that, for the latter, the variations at the girder adjacent to the left widened section (V2 for the 5-girder model, V3 for the 7-girder model) is negative. Furthermore, the percentages are, in general, higher for the G3 group models. According to Monzon et al. [12], the V-Load method is better suited to bridges with larger radii. As observed for group G2, the percentages in the girders on the inner side of the bridge tend to be higher, being maximum in the innermost girders V5 and V7. Ribeiro [22] also observed the incremental effect exerted by the radius on the divergences between the FEM and the V-Load method and, corroborating what was verified in these assessments, the author found that the highest discrepancies tend to occur in the innermost girders.

Regarding the effect of stiffness variation in the widened regions, in general, for the external girders, the higher the stiffness, the smaller the discrepancies compared to the FEM results. For the internal girders, the same behavior is observed. For the external additional and the original girders, the stiffness variation in the widened regions does not significantly affect the differences between analytical and numerical models. It is valid to point out that the V-Load method was conceived for bridges in which all girders have the same bending stiffness [11].

7.6 Modification Factors Analysis

In order to complement the analyses presented so far, this item presents the Modification Factors obtained for the curved models, considering the variation of the radius and widening stiffness.

According to the concepts previously discussed, the MF result from the ratio between the maximum value of the bending moment obtained for a curved bridge and a straight bridge, considering the same girder. To limit the data volume, for both 5-girder and the 7-girder models, the MF analysis only involved the external, central, and internal girders, as shown in Figure 16.

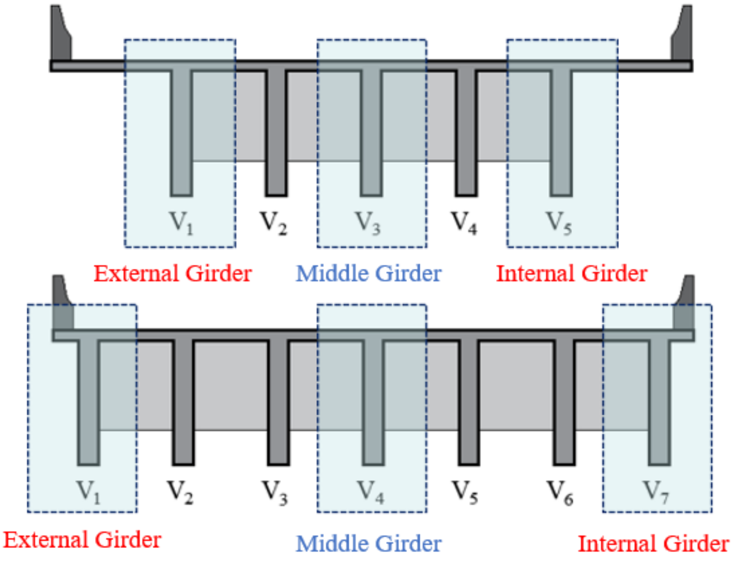

Figure 16. Girders considered for the attainment of Modification Factors.

Figure 17 shows the MF's obtained by applying load case 01 for the 5-girder and 7-girder models, each graphic as a function of widening stiffness. In general, for this load case, the factors are similar for the external and central girders while increasing noticeably for the internal girders, especially in the MF obtained for group G3 models (Figure 17c,d).

The comparison between the results for groups G2 and G3 shows that the MF's are higher for models with greater curvature, and this disparity is particularly relevant considering the factors of the internal girders. These behavioral aspects of the MF's indicate that, concerning load case 01, inserting a curvature affects the internal stringers more expressively, and its influence is more intense the higher the radius. When considering the effect of the compressive strength variation in the widened sections, the data shows that the most relevant repercussions are felt only in the internal girders, especially in the group G3 models, with no significant variations for the other girders.

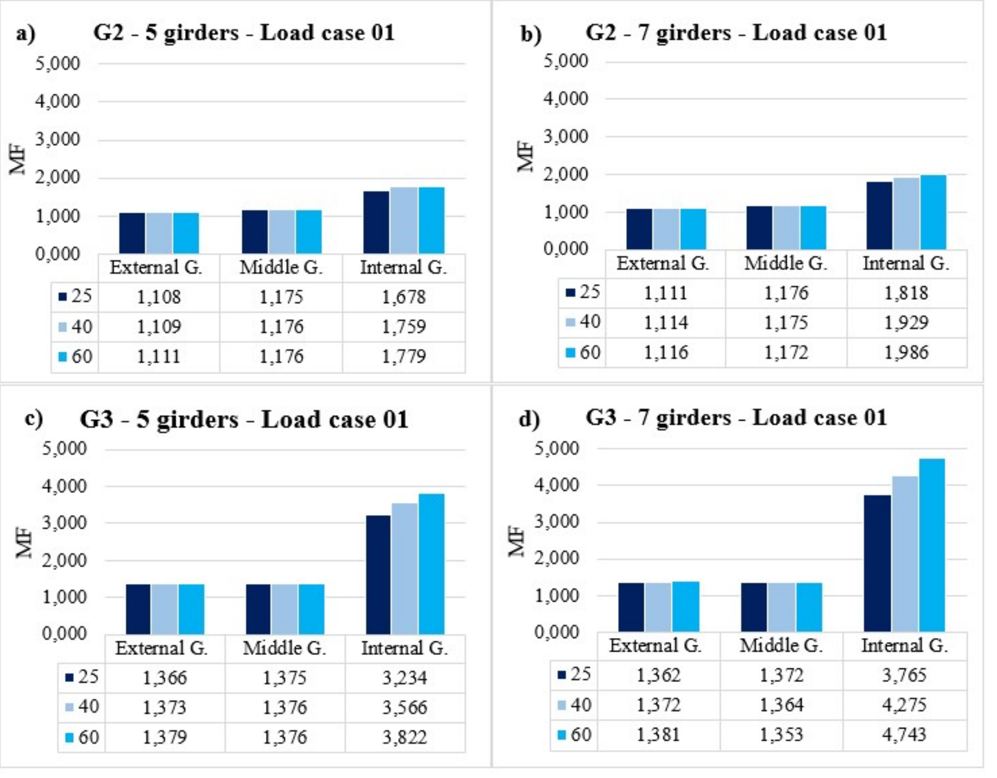

Figure 17. Modifications Factors for load case 01: (a) 5L-G2; (b) 7L-G2; (c) 5L-G3; (d) 7L-G3.

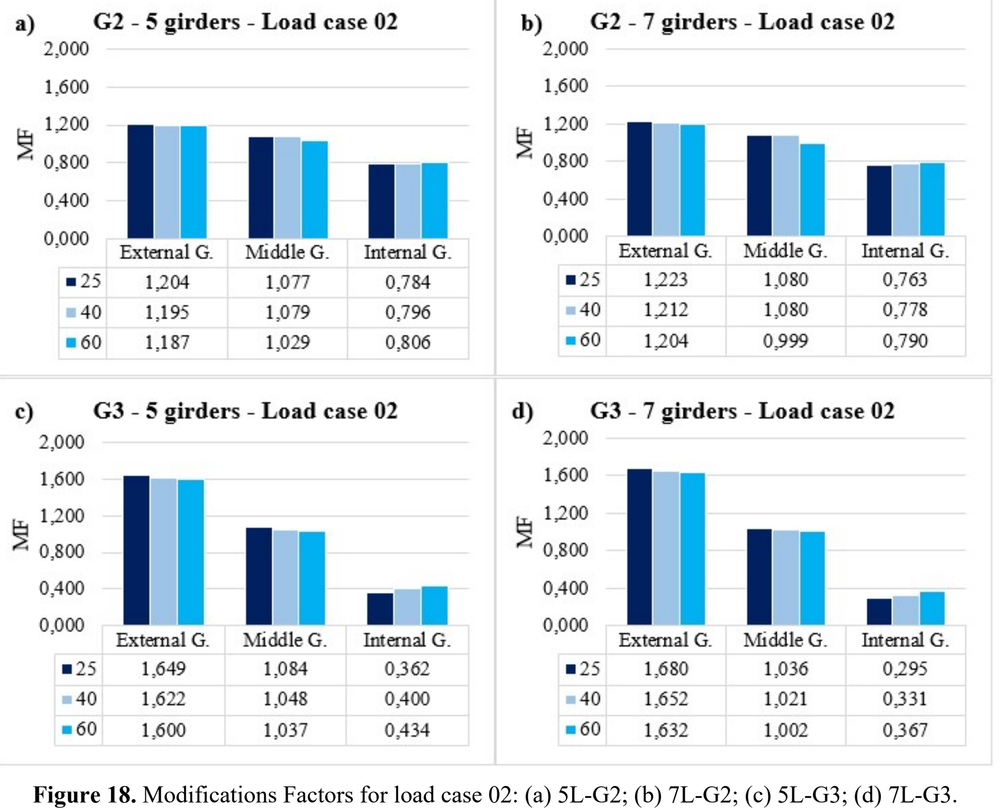

Figure 18. Modifications Factors for load case 02: (a) 5L-G2; (b) 7L-G2; (c) 5L-G3; (d) 7L-G3.

Figure 18, in turn, introduces the MF's obtained from the bending moments resulting from load case 02. Firstly, the growth tendency of the MF's, in this case, is opposed to the behavior exhibited by the values pertaining the load case 01 since they reduce towards the internal girder. Another aspect identified is that, compared to the results of load case 01, the MF's tend to be smaller for load case 02, except for the external girders. It is interesting to note that, regardless of the number of girders or the radius, the MF's attained for the central girders are very close to the unity, indicating that there is no significant divergence between the bending moments obtained in the straight and curved models for the girder in question. Contrary to what occurred in the analysis of factors from load case 01, the internal girders have the lowest MF's, being, in all cases, lesser than 1.0.

Considering the influence of curvature, although the MF's in the central girders are not affected by the change of radius, the factors in the external girders increase while decreasing in the internal girders. This behavior is compatible with what was verified by Acosta and González [7] and Rebouças et al. [17]. Furthermore, regarding the widening stiffness, there are no relevant changes due to the variation of this parameter.

8 CONCLUSIONS

This paper sought to analyze the transverse distribution of bending moments in curved bridges considering the execution of widenings with concrete of variable compressive strength. The analysis shows that the effects of this parameter depend not only on the magnitude of the stiffness but also on the live load positioning. In all cases, the percentage variations are greater the higher the stiffness of the widenings. For load case 02, consistently, the bending moment distribution factors in the widened regions tend to grow with the stiffening. As a consequence, the distribution factors from the original girders reduce. For load case 01, while the external widened sections experience increases, and the original girders are relieved, this relief also occurs in the internal widened regions.

Regarding the curvature effect, the radius variation did not significantly alter the percentages calculated for bending moment distribution. This behavior allows deducing that, although the curvature is a key parameter for load distribution in curved bridges, changing the radius does not change the effect of load redistribution induced by the widening stiffness variation.

Comparing the models with 5 and 7 girders, for load case 02, the load fraction absorbed by the original section of the deck with 7 girders reduced, while the widened regions experienced increases. However, for load case 01, not only the original girders were relieved, but the bending moments of the internal widened section also suffered reductions. Therefore, the addition of girders produced a similar effect to that induced by the stiffening of the widenings. This similarity is probably related to the fact that the addition of the girders, even when maintaining the concrete compressive strength, reduces the overhang length and stiffens the widening. Furthermore, the analysis shows that the higher the stiffness in the widened sections, the greater the percentage reduction, *i.e*., the higher the relief experienced by the original region of a 7-girder model compared to the loading in the same area on a 5-girder model. However, for the models with variable stiffness, the discrepancy between the loading in the widened sections of the 5 and 7-girder models tends to decrease.

The comparison of bending moments obtained from FEM and those calculated by the approximate analytical V-Load method shows that the percentage differences are higher in the innermost girders, which are even more significant for the group G3 models. Moreover, although the V-Load method was idealized considering the hypothesis of girders with uniform stiffness, the percentages for models with variable stiffness did not diverge significantly from those obtained in models with uniform stiffness.

Finally, given the analyses of the Modification Factors, regarding the effect of the increase in curvature, the factor attained with load case 01 increased in general, particularly in the internal girders. However, for load case 02, while the central girders were hardly affected, the MF's of the external and internal girders increased and decreased, respectively. Compared to the curvature effect, the variation of widening stiffness only produced relevant changes for the internal girders of the group G3 models, considering load case 01, while being negligible in the other cases.

9 REFERENCES

[1] W. S. Kim, J. A. Laman, and D. G. Linzell, "Live load radial moment distribution for horizontally curved bridges," *J. Bridge Eng.*, vol. 12, no. 6, pp. 727–736, Nov 2007, http://dx.doi.org/10.1061/(ASCE)1084-0702(2007)12:6(727).

[2] H. Zhang, D. Huang, and T. Wang, "Lateral load distribution in curved Steel I-girder bridges," *J. Bridge Eng.*, vol. 10, no. 3, pp. 281–290, 2005, http://dx.doi.org/10.1061/(ASCE)1084-0702(2005)10:3(281).

[3] Departamento Nacional de Infraestrutura de Transportes, *Manual de Inspeção de Pontes Rodoviárias*, 2ª ed. Rio de Janeiro: IPR, 2004.

[4] J. A. P. Vitório, “Um estudo comparativo sobre métodos de alargamento de pontes rodoviárias de concreto armado com a utilização das normas brasileiras e eurocódigos,” Ph.D. dissertation, FEUP, Porto, 2013.

[5] R. M. C. Barros, J. A. P. Vitório, “Análise paramétrica de projetos de alargamento e reforço de pontes rodoviárias de concreto armado,” in *Jorn. Sul Am. Eng. Estrut.*, Rio de Janeiro, set. 2012.

[6] P. Fontana, “Análise de modelos de distribuição transversal de cargas em superestruturas de viga em seção caixão de pontes e viadutos alargados,” M.S. thesis, PPGEC, UFTPR, Curitiba, 2016.

[7] R. Acosta and J. V. González, "Structural response of plan-curved steel i-girder bridges from an equivalent straight bridge analysis," *J. Bridge Eng.*, vol. 23, no. 3, Mar 2018, http://dx.doi.org/10.1061/(ASCE)BE.1943-5592.0001187.

[8] R. L. Barbosa, “Pontes curvas unicelulares em regime elástico,” M.S. thesis, PEF, USP, São Paulo, 1997.

[9] C. C. Fu and S. Wang, *Computational Analysis and Design of Bridge Structures*, 1st ed. Boca Raton, USA: CRC Press, 2015.

[10] American Association of State Highway and Transportation Officials, *AASHTO LRFD Bridge Design Specifications*, 7th ed. Washington, DC, USA: AASHTO, 2014.

[11] A. L. Fiechtl, G. L. Fenves, and K. H. Frank, *Approximate Analysis of Horizontally Curved Girder Bridges* (Rep. 360-2F), UT, Austin, TX, USA, 1987.

[12] E. V. Monzon, A. M. Itani, and M. L. Reno, “Horizontally curved girder bridges,” in *Bridge Engineering Handbook*, 2nd ed. Boca Raton, USA: CRC Press, 2014, ch. 6, pp. 259-281.

[13] R. L. Brockenbrough, "Distribution Factors for Curved I-Girder Bridges," *J. Struct. Eng.*, vol. 112, no. 10, pp. 2200–2215, Oct 1986, http://dx.doi.org/10.1061/(ASCE)0733-9445(1986)112:10(2200).

[14] Associação Brasileira de Normas Técnicas, *Projeto de Estruturas de Concreto – Procedimento*, NBR 6118, 2014.

[15] Associação Brasileira de Normas Técnicas, *Carga Móvel Rodoviária e de Pedestres em Pontes, Viadutos, Passarelas e Outras Estruturas*, NBR 7188, 2013.

[16] M. Samaan, K. Sennah, and J. B. Kennedy, "Positioning of bearings for curved continuous spread-box girder bridges," *Can. J. Civ. Eng.*, vol. 29, no. 5, pp. 641–652, Oct 2002, http://dx.doi.org/10.1139/l02-049.

[17] A. Rebouças, J. Silva Fo, R. Barros, Y. Nascimento, and P. Coutinho, "Curved bridges live load bending moment distribution using straight and curved girders," *Rev. IBRACON Estrut. Mater.*, vol. 15, no. 2, e15208, Apr 2022, http://dx.doi.org/10.1590/s1983-41952022000200008.

[18] D. Nevling, D. Linzell, and J. Laman, "Examination of level of analysis accuracy for curved I-girder bridges through comparisons to field data," *J. Bridge Eng.*, vol. 11, no. 2, pp. 160–168, Mar 2006, http://dx.doi.org/10.1061/(ASCE)1084-0702(2006)11:2(160).

[19] D. L. Logan, *A First Course in the Finite Element Method*, 5th ed. Stamford, CT, USA: Cengage Learn., 2012.

[20] S. J. Fatemi, M. S. Mohamed Ali, and A. H. Sheikh, "Load distribution for composite steel-concrete horizontally curved box girder bridge," *J. Construct. Steel Res.*, vol. 116, pp. 19–28, Jan 2016, http://dx.doi.org/10.1016/j.jcsr.2015.08.042.

[21] Precast Concrete Institute, *Bridge Design Manual*, 3th ed. Chicago, IL, USA: Precast/Prestressed Concr. Inst., 2011.

[22] V. A. Ribeiro, “Avaliação da influência das curvaturas e das transversinas na distribuição do momento fletor devido à carga móvel em pontes curvas de concreto armado,” M.S. thesis, PEC, UFRN, Natal, 2019.

Author contributions: YN: conceptualization, methodology, data curation, software, formal analysis, writing; JN: formal analysis, project administration, supervision, writing-review and editing; AR: data curation, writing-review and editing; RB: conceptualization, supervision, visualization; JN: conceptualization, supervision.

Editors: Mauro de Vasconcellos Real, Guilherme Aris Parsekian.