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ORIGINAL ARTICLE

Optimal configuration of RC frames considering ultimate and serviceability limit state constraints

Configuração ótima de pórticos de concreto armado considerando restrições de estados limites últimos e de serviço

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Abstract: Most current structural design codes are based on the concept of limit states, that is, when a structure fails to meet one of its purposes, it is said that it has reached its limit state. In the design of reinforced concrete structures, the Ultimate Limit State (ULS) and the Serviceability Limit State (SLS) must be checked. Therefore, this paper presents an optimization scheme for reinforced concrete plane frames, in which the objective is to minimize the cost of structures for three cases of constraints: the first is related to ULS and SLS; the second refers only to the ULS; and the third is related only to the SLS. Computational routines for checking limit states of beams and columns are implemented in MATLAB, following the requirements of the Brazilian code. Structural analyses are performed by using the MASTAN2 software, taking into account geometric nonlinearities and a simplified physical nonlinearity method. The objective function considers the cost of concrete, reinforcement and formwork, and the optimization problems are solved by genetic algorithms. Two numerical examples of frames are presented. Regarding the optimal characteristics related to each type of limit state, it is noted that the beams and columns tend to have larger and more reinforced cross sections in the case of the ULS. Even so, optimal structures related to the ULS often do not satisfy SLS and vice versa, which indicates that the optimal characteristics related to each limit state may be different. In addition, it is observed that the SLS is less restrictive than ULS.

Keywords: optimization, reinforced concrete, limit states, genetic algorithms.

Resumo: A maioria das normas de projetos estruturais atuais se baseia no conceito de estados limites, ou seja, quando uma estrutura não atende a um de seus propósitos, diz-se que a mesma atingiu seu estado limite. No projeto de estruturas de concreto armado, deve-se verificar o Estado Limite Último (ELU) e o Estado Limite de Serviço (ELS). Portanto, este trabalho apresenta um esquema de otimização de pórticos planos de concreto armado, no qual o objetivo é minimizar o custo de estruturas para três casos de restrições: o primeiro está relacionado ao ELU e ao ELS; o segundo refere-se apenas ao ELU; e o terceiro está relacionado apenas ao ELS. São elaboradas rotinas de verificação dos estados limites de vigas e de pilares, em ambiente MATLAB, de acordo com normas brasileiras. As análises estruturais são realizadas com o uso do *software* MASTAN2, levando em consideração as não-linearidades geométricas e um método simplificado de não-linearidade física. A função objetivo considera o custo do concreto, da armadura e da forma, e são utilizados algoritmos genéticos para a otimização. São avaliados dois exemplos numéricos de pórticos. Quanto às características ótimas relacionadas a cada tipo de estado limite, nota-se que as vigas e os pilares tendem a apresentar seções transversais maiores e mais armadas no caso do ELU. Mesmo assim, muitas vezes a estrutura ótima relacionada ao ELU não satisfaz o ELS e vice-versa, o que indica que as características ótimas relacionadas a cada estado limite podem ser diferentes. Além disso, observa-se que o ELS é menos restritivo do que o ELU.

Palavras-chave: otimização, concreto armado, estados limites, algoritmos genéticos.

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1 INTRODUCTION

Due to the development of new technologies and the increase of market competitiveness, the search for more efficient and lower-cost designs has increased. At the same time, reinforced concrete has become a dominant structural material in engineering construction in many countries [1]. In this scenario, the importance of studies related to the design concept of reinforced concrete structures is valid.

To ensure the safety of a structure, the engineer must choose a design option which meets the requirements related to its purpose. However, due to the large number of variables usually involved in the design of reinforced concrete structures, there are several different configurations that can meet the required conditions, with different costs and performances. Many times, the choice of a configuration is not simple, which makes it difficult to obtain an optimal design using traditional methods. Thus, optimization techniques have been widely employed with this purpose [2]–[4].

As a result, several studies in the field of optimization of reinforced concrete structures have been developed in the last decades, with the objective of obtaining designs with optimal parameters, generally related to the minimum cost of the structures [5]–[15]. Many of these studies use genetic algorithms (GA) in the optimization [16]–[25]. In order to obtain optimal structures which can be used in practice, the requirements specified by standard designs may be applied as constraints within the optimization formulation.

The requirements presented by most current design codes, including the Brazilian code for design of concrete structures [26], are based on the concept of limit states. A limit state may be defined as the limit situation from which a structural element no longer meets one of its design goals, or in other words, when a structure fails to satisfy any of the purposes of its construction. Current Brazilian codes establish that the following limit states must be considered [26], [27]: Ultimate Limit State (ULS), related to the collapse, or to any other form of structural failure, which determines the interruption of the use of the structure; Serviceability Limit State (SLS), characterized by situations that, due to their occurrence, repetition or duration, generate structural effects that do not meet the conditions specified for the normal use of the structure, or indicate impairment of its durability.

Optimization of reinforced concrete structures considering standard design constraints have been already studied in some papers from the literature [9], [13], [15], [19], [20]. In the context of the Brazilian design standard, some studies have also been developed, for example, Bordignon and Kripka [28], Medeiros and Kripka [29], [30], Kripka et al. [31] and Correia et al. [32]. However, there is a lack in studies that discuss the characteristics of the optimal structures found and their relationships with the design constraints.

Following a research which was started by Juliani and Gomes [33], [34], the present paper proposes to analyze the optimal configuration of reinforced concrete plane frames, through the minimization of its costs, considering three cases of constraints: the first is related to ULS and SLS; the second refers only to the ULS; and the third is related only to the SLS. The design variables considered are the cross-section dimensions and the amount of longitudinal and transverse reinforcement of the structural elements. Also, for a proper representation of the real behavior of the structure, geometric nonlinearities and a simplified physical nonlinearity method are considered in the structural analysis. Although the Brazilian standard requires that designs satisfy both types of limit states simultaneously, the present paper considers them separately in some cases, in order to investigate and quantify the effects of each type on the optimal configuration. Thus, the main goal of the research is to improve the knowledge in structural design, indicating possible directions that lead to structures with minimal costs.

This paper focuses on plane frames because there are many conventional structures that can be reduced to such models, as some kinds of buildings and industrial sheds. Furthermore, the analysis of plane frames requires consideration of the global structural behavior and the interaction between the two different types of structural elements involved: beams and columns. However, this model does not take into account torsional forces, for example, restricting the analysis to axial and shear forces and bending moments, in the plane. Although limited, plane frames allow a significant reduction in the computational effort of the optimization process, when compared to spatial models. This is important because the optimization process requires many structural analysis evaluations.

The paper is organized as follows: section 2 introduces the formulation and implementation of the problem that is addressed in this paper; the optimization method used herein is described in section 3; section 4 presents two numerical examples and some conclusions drawn from the results are presented in section 5.

2 OPTIMIZATION FORMULATION AND IMPLEMENTATION

The optimization problem is usually defined by some design variables, one or more objective functions and some constraints, as described in the following.

2.1 Design variables

The cross-sections of beams and columns are assumed to be rectangular. The design variables to be determined in the optimization process are adopted as discrete and illustrated in Figure 1, where: b and h are the cross section width and height, respectively, and the height is parallel to the plane of the frame; n_s is the number of longitudinal reinforcement bars, whose diameter is represented by ϕ_s; n_{sw} is the number of transverse reinforcement bars, whose diameter is represented by ϕ_{sw}.

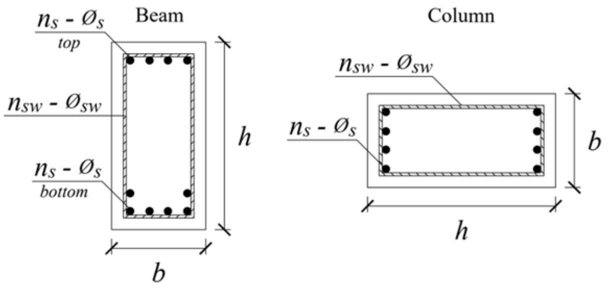

Figure 1. Typical section of a beam and a column.

In order to vary the amount of reinforcement along the beam, the structural element is discretized in a predefined number of segments. Then, for each segment, values of n_s^{top}, n_s^{bottom} and n_{sw} are determined, based on the maximum values of bending moment and shear force on the segment. The bars extend over the entire length of the segment and the anchoring of longitudinal bars is not considered. For simplification, the values of n_s^{top} and n_s^{bottom} are obtained considering the respective bars as tension reinforcement. The values of the other variables are the same over the entire length of the beam.

Since this paper deals with plane frames, the cross sections of the columns are longitudinally reinforced only on two faces, in a symmetrical way, assuming that the direction of the acting bending moment may change. Each column is discretized with one segment, so that the structural element has the same cross section throughout its length, and anchorage of the reinforcement is disregarded.

2.2 Objective function

The objective function employed corresponds to the cost of the structure, based on the cost of concrete volume, longitudinal and transverse reinforcement mass and formwork area, as described by Equation 1, where: $\mathbf{x}$ is the vector of design variables; V_c is the concrete volume; M_s and M_{sw} are the mass of the longitudinal and transverse reinforcement, respectively; A_f is the formwork area; C_c, C_s, C_{sw} and C_f represent, respectively, the unit cost of concrete, longitudinal and transverse reinforcement, and formwork; n_{el} is the number of structural elements of the frame (beams and columns). The formwork areas are given by Equation 2, where l is the length of the structural element.

$$f(\mathbf{x}) = \sum_{i=1}^{n_{el}} \left(V_{c_i} C_c + M_{s_i} C_s + M_{sw_i} C_{sw} + A_{f_i} C_f \right) \tag{1}$$

$$\begin{cases} A_f^{beam} = (2h+b)l \\ A_f^{column} = (h+b)2l \end{cases} \tag{2}$$

2.3 Constraints

The optimization constraints considered herein are based on the requirements of NBR 6118 [26], represented by the vector $\mathbf{g}$, and divided into constraints of the ultimate and serviceability limit states, as shown in Figure 2. In addition to the constraints related to the limit states, in all situations, constructive constraints are also considered; for example: maximum and minimum limits of dimensions of cross-sections, reinforcement rates and space between reinforcement bars; ductility conditions for beams; and others. All these constraints are implemented in MATLAB (MathWorks [35]), to be included in the optimization. It should be noted that constraints related to the lateral instability of the beams are not considered. The consideration of this effect can be important, especially in cases of slender beams and with insufficient lateral locking; however, this topic will not be addressed herein for simplification purposes, remaining a topic for future investigations.

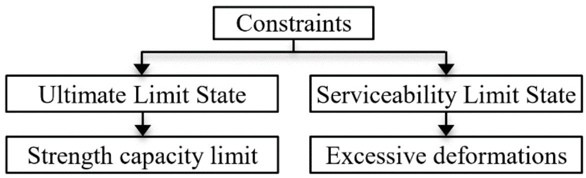

Figure 2. Constraint scheme of the problem.

Verification of the constraints requires determination of internal forces and deflections of the structure, which can be achieved by structural analysis. For this purpose, MASTAN2 software [36] is employed. In all cases, geometric nonlinearities are considered using the software formulation [37], whereas physical nonlinearities are considered in a simplified manner, based on stiffness reductions, as indicated by NBR 6118 [26]: the stiffnesses of the beams and columns are considered equal to $40\,\%$ and $80\,\%$ of the total stiffness of the concrete section, respectively. For more details about nonlinearities the readers are referred to Bathe [38] and Belytschko et al. [39]. As the present paper deals with plane frames, beams are considered subjected to the simple bending and columns to uniaxial bending.

2.3.1 Ultimate limit state constraints

Considering the behavior of the analyzed structure, the beams must withstand the design bending moment (M_{Sd}) and the design shear force (V_{Sd}).

The section of the beam is safe with respect to the bending moment if it satisfies the constraint given by Equation 3, where M_{Rd} is the design bending strength.

$$g_1(\mathbf{x}) = M_{Sd} - M_{Rd} \leq 0 \tag{3}$$

M_{Rd} is calculated from Equation 4, obtained from the equilibrium of moments in the cross section, where A_s is the cross-sectional area of longitudinal tension reinforcement, f_{yd} is the design yield strength of longitudinal steel reinforcement, d is the effective depth, λ is a parameter depending on the characteristic compressive strength of concrete (f_{ck}) and z defines the position of the neutral axis.

$$M_{Rd} = A_s f_{yd}\left(d - \frac{\lambda z}{2}\right) \tag{4}$$

The value of z is obtained from the equilibrium of forces in the cross section, according to Equation 5, where α_c is a parameter of reduction of the compressive strength of concrete and f_{cd} is the design compressive strength of concrete.

$$z = \frac{A_s f_{yd}}{\lambda \alpha_c f_{cd} b} \tag{5}$$

The strength of the section with respect to the shear force is guaranteed if the constraints given by Equations 6 and 7 are checked, where V_{Rd2} and V_{Rd3} are, respectively, the design shear strength of concrete compressive diagonals and the design shear strength of tension diagonals (supplied by concrete, $V_{\bar{c}}$, and transverse reinforcement, V_{sw}). Model II of the code (NBR 6118 [26]) is used to obtain shear strengths.

$$g_2(\mathbf{x}) = V_{Sd} - V_{Rd2} \leq 0 \tag{6}$$

$$g_3(\mathbf{x}) = V_{Sd} - V_{Rd3} \leq 0 \tag{7}$$

Equation 8 defines the value of V_{Rd2}, where α_{v2} is a parameter which depends on the f_{ck}, α is the angle of inclination of the transverse reinforcement, adopted as 90º, and θ is the angle of inclination of compressive diagonals, adopted as 30º.

$$V_{Rd2} = 0.54\alpha_{v2} f_{cd} bd\left(\cot\alpha + \cot\theta\right) sin^2\theta \tag{8}$$

V_{Rd3} is obtained from Equation 9, where A_{sw} is the cross-sectional area of transverse reinforcement bar, s is the spacing between the transverse reinforcements, f_{ywd} is the design tension in the transverse reinforcement and f_{ctd} is the design tensile strength of concrete.

$$\begin{aligned} &V_{Rd3} = V_{sw} + V_{\bar{c}} \\ &\Rightarrow V_{sw} = \frac{A_{sw}}{s} 0.9 d f_{ywd}\left(\cot\alpha + \cot\theta\right)\sin\alpha; \\ &\Rightarrow V_{\bar{c}} = \begin{cases} 0.6 f_{ctd} bd & \text{if } V_{Sd} \leq 0.6 f_{ctd} bd; \\ 0 & \text{if } V_{Sd} = V_{Rd2}\text{, interpolating linearly to intermediate values.} \end{cases} \end{aligned} \tag{9}$$

The columns must have sufficient structural capacity to withstand combined effects of axial load and bending moment, that is, M_{Rd} must be greater than M_{Sd} at the same time as N_{Rd} (design axial strength) must be greater than N_{Sd} (design axial force). To guarantee this requirement, a load-moment interaction diagram M_{Rd} x N_{Rd} is constructed for each column, which is a curve that delimits the actions that can act in the section safely. If the combination of M_{Sd} and N_{Sd} is in the safe region of the diagram, the capacity of the designed column is adequate. Figure 3 shows an example of a diagram constructed for a column.

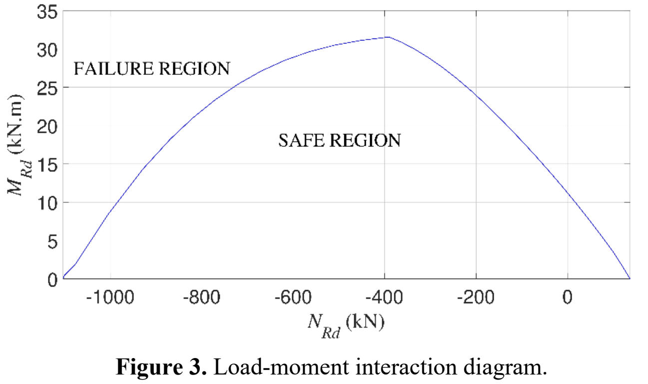

Figure 3. Load-moment interaction diagram.

During the optimization process, after the construction of the diagram, the developed algorithm searches for the M_{Rd} associated with the N_{Sd} , and if that moment value is greater than M_{Sd} , the section is considered safe (Equation 10).

$$g_4(\mathbf{x}) = M_{Sd} - M_{Rd} \leq 0 \tag{10}$$

The design solicitations (M_{Sd} , V_{Sd} and N_{Sd}) in the ULS are obtained from the normal ultimate combination of applied loads, according to NBR 6118 [26].

2.3.2 Serviceability limit state constraints

In the serviceability limit state of excessive deformations, the vertical deflection a_v of the beams are restricted by the vertical deflection limit a_v^{lim} allowed by the code, as shown in Equation 11, where the limit is given by $\frac{l}{250}$. a_v is obtained by the direct stiffness method, using the quasi-permanent load combination [26], and adding a deflection portion related to the creep of the concrete.

$$g_5(\mathbf{x}) = a_v - a_v^{lim} \leq 0 \tag{11}$$

The frame is also checked for horizontal displacements. The horizontal displacement between two consecutive story a_h must comply with the horizontal displacement limit a_h^{lim} defined by the code, where the limit is given by $\frac{l}{850}$. Equation 12 represents this constraint.

$$g_6(\mathbf{x}) = a_h - a_h^{lim} \leq 0 \tag{12}$$

The constraint given by Equation 13 determines that the horizontal displacement at the top of the frame a_h^t must meet the limit established by the code $a_h^{t_{lim}}$, adopted equal to $\frac{l_{total}}{1700}$, where l_{total} is the height of the frame.

$$g_7(\mathbf{x}) = a_h^t - a_h^{t_{lim}} \leq 0 \tag{13}$$

The horizontal displacements are obtained by the direct stiffness method, using the frequent load combination [26].

3 GENETIC ALGORITHMS

Genetic algorithms are zero-order stochastic optimization methods, developed by Holland [40] in the 1970s. They are based on the theory of evolution of species, declared by Charles Darwin in the XIX century [41]. The algorithm can be applied to solve problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, nondifferentiable or highly nonlinear. These algorithms use some terms in analogy to natural genetics: the individual is a solution, which may or may not be viable; the population is the set of solutions; the chromosome is the coding that represents the individual; the gene is the coding that represents the variable.

In general, the genetic algorithm method creates a random initial population and then performs the following steps: initially, the algorithm evaluates the fitness of each individual of the current population; then selects some of these individuals based on the value of their fitness, naming them as parents; later, children are produced from changes in the characteristics of a single parent (mutation) or by combining characteristics of a parent pair (crossover); after that, individuals with the best fitness of the current population are chosen as “elite members” (elitism); finally, the current population is replaced by the individuals generated during the mutation, the crossover and the elitism phases, forming the new generation of the population. The steps are repeated until some stop criteria are satisfied. Figure 4 presents a flowchart of the GA. For more details about optimization and genetic algorithm see Arora [3] and Sivanandam and Deepa [42].

In this paper, a genetic algorithm routine available in MATLAB, is used (MathWorks [43]). In this routine, the constraints of the optimization problem are considered by penalizing the fitness of an individual when it does not meet the constraints. The fitness of the *i*-th individual of the population can be described by Equation 14.

$$Fitness\left(\mathbf{x}^{(i)}\right)=\begin{cases} f\left(\mathbf{x}^{(i)}\right), & \text{if } \mathbf{x}^{(i)} \text{ is feasible;} \\ f_{worst}+\sum_{j=1}^{\mu}|\,\bar{g}_j\left(\mathbf{x}^{(i)}\right)|, & \text{otherwise.} \end{cases} \tag{14}$$

In this way, if the individual is feasible, the fitness function is the objective function. If the individual is infeasible, the fitness function is the value of the objective function of the worst feasible solution currently available in the population (f_{worst}), plus a sum of constraint violations, where $\bar{g}$ are the constraints violated and μ is the number of these constraints [44].

The genetic algorithm was chosen because it is an established method and widely used in similar studies. Other optimization methods could be used, which could achieve the optimal results more or less efficiently than the genetic algorithm. The focus of this work, however, was on optimization results.

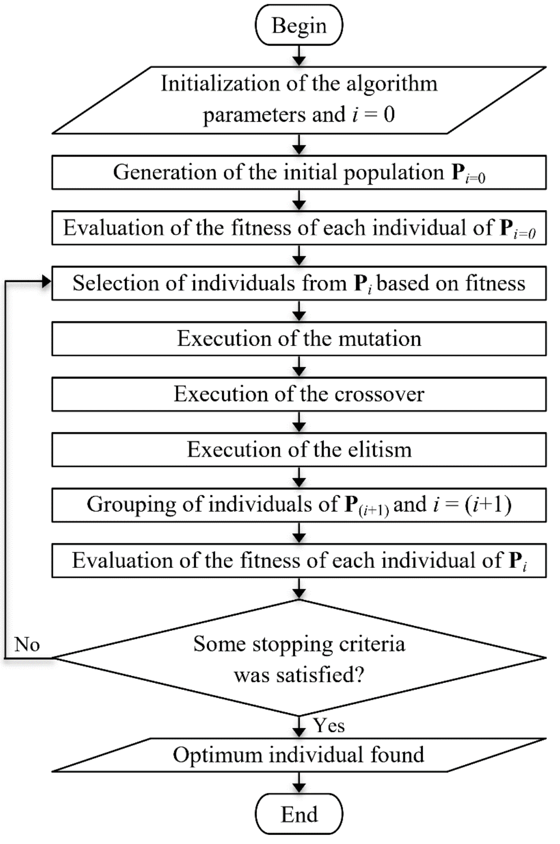

Figure 4. Basic scheme of a genetic algorithm.

4 NUMERICAL EXAMPLES

This section presents two numerical examples: a one-bay two-story frame and a two-bay six-story frame. Table 1 shows some of the input data used, common to both examples. The unit costs adopted were obtained from SINAPI [45], the Brazilian system of costs survey and indexes of construction, and include the costs of materials and labor.

Table 1. Input data of examples I and II.

	Data	Value	Unit
f_{yk}	Characteristic yield strength of longitudinal and transverse steel reinforcement	500	MPa
E_s	Modulus of elasticity of longitudinal and transverse steel reinforcement	210000	MPa
ρ_s	Unit mass of steel of the longitudinal and transverse reinforcement	7850	kg/m³
f_{ck}	Characteristic compressive strength of concrete	25	MPa
E_{ci}	Modulus of initial elasticity of concrete	28000	MPa
ρ_c	Unit mass of reinforced concrete	2500	kg/m³
c	Cover to reinforcement	2.5	cm
C_c	Unit cost of concrete for beams - C25	336.02	R$/m³
	Unit cost of concrete for columns - C25	340.94	R$/m³
C_s	Unit cost of longitudinal reinforcement - ϕ_{10}	5.84	R$/kg
	Unit cost of longitudinal reinforcement - $\phi_{12.5}$	5.21	R$/kg
C_{sw}	Unit cost of transverse reinforcement - $\phi_{6.3}$	7.44	R$/kg
C_f	Unit cost of formwork for beams	57.32	R$/m²
	Unit cost of formwork for columns	47.06	R$/m²

The cases of constraints applied in each example are: Case I (ULS + SLS); Case II (ULS); Case III (SLS). In relation to the optimization algorithm, the initial population of case I includes a feasible pre-defined design $\mathbf{x}^{(1)}$. Cases II and III use the optimal result of case I as the design $\mathbf{x}^{(1)}$ included in the initial population. Preliminary tests indicated a high probability of convergence to local minima. For this reason, each case is run 10 times, considering different initial populations, related to different seeds of the pseudo-random generator. The best result obtained is taken as the final of the optimization process. Based on adjustments performed in these preliminary testes, in order to achieve satisfactory results in both examples, the following parameters are adopted in the optimization algorithm: population size equal to 50 individuals; limit of generations equal to 10000; stall generation of 500 generations. The computational times presented refer to an Intel Core i7-5820K processor.

4.1 Example I: one-bay two-story frame

The example consists of a reinforced concrete plane frame, whose geometry was presented by Adamu and Karihaloo [46]. Figure 5 shows the structure and Table 2 the characteristic loads applied in addition to the weight of the structure.

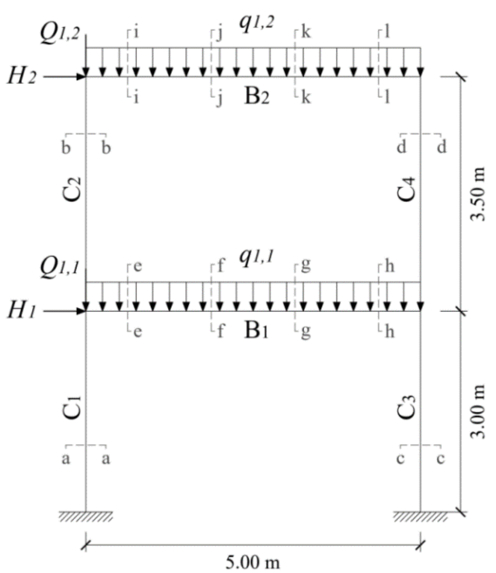

Figure 5. Plane frame - Example I.

Table 2. Characteristic loads - Example I.

Characteristic loads	Permanent loads	Variables loads
H_1	-	11.9 kN
$Q_{1,1}$	21.4 kN	14.3 kN
$q_{1,1}$	25.7 kN/m	17.1 kN/m
H_2	-	6.0 kN
$Q_{1,2}$	12.9 kN	8.6 kN
$q_{1,2}$	17.1 kN/m	11.4 kN/m

Regarding the variables, the beams B_1 and B_2 are discretized in four segments of equal length, so that each beam has 17 variables: b, h, ϕ_s^{bottom}, ϕ_s^{top}, ϕ_{sw}, $n_{s_1}^{bottom}$, $n_{s_1}^{top}$, n_{sw_1}, $n_{s_2}^{bottom}$, $n_{s_2}^{top}$, n_{sw_2}, $n_{s_3}^{bottom}$, $n_{s_3}^{top}$, n_{sw_3}, $n_{s_4}^{bottom}$, $n_{s_4}^{top}$ and n_{sw_4}, in which the numerical sub-index indicates the segment of the beam. Also, some columns are assumed to be equal, C_1=C_2 and C_3=C_4, except for the amounts of reinforcement. In this way, each pair of columns (C_1+C_2 and C_3+C_4) has 8 variables; for example, for the pair of columns C_1 and C_2 the design variables are: b, h, ϕ_s, ϕ_{sw}, $n_s^{C_1}$, $n_{sw}^{C_1}$, $n_s^{C_2}$ e $n_{sw}^{C_2}$. Thus, the problem has 50 variables in total. For the variables related to the beams, the following possible values are adopted: $b =$ [12, 26] cm and $h =$ [30, 60] cm, in increments of 2; $n_s^{bottom} =$ [2, 10], $n_s^{top} =$ [2, 10] and $n_{sw} =$ [6, 13], in increments of 1. For the variables related to the columns, the following possible values are adopted: $b =$ [19, 31] cm, $h =$ [19, 51] cm and $n_{sw} =$ [20, 36], in increments of 2; $n_s =$ [2, 8] in increments of 1. Diameter of 10 mm is adopted for the longitudinal reinforcement and 6.3 mm for the transverse reinforcement.

Figures 6 and 7 show the result obtained for case I. The total cost of the structure is R$ 2853.50, with 46.24% corresponding to the formwork, 35.88% related to the reinforcement and 17.88% related to the concrete. 1827 generations and 91400 evaluations of the objective function were required by the optimization procedure, with a computational time of 8.09 hours. For case II, the optimal sections found are the same as those obtained in case I, with a total of 501 generations and 25100 evaluations of the objective function and a computational time of 3.75 hours. Finally, the optimal sections for case III are illustrated in Figure 8. The total cost of the structure in this case is R$ 2080.21, where 53.88%, 26.62% and 19.50% correspond to the cost of formwork, reinforcement and concrete, respectively. 44800 evaluations of the objective function over 895 generations were required in this case, which corresponds to a computational time of about 4.2 hours.

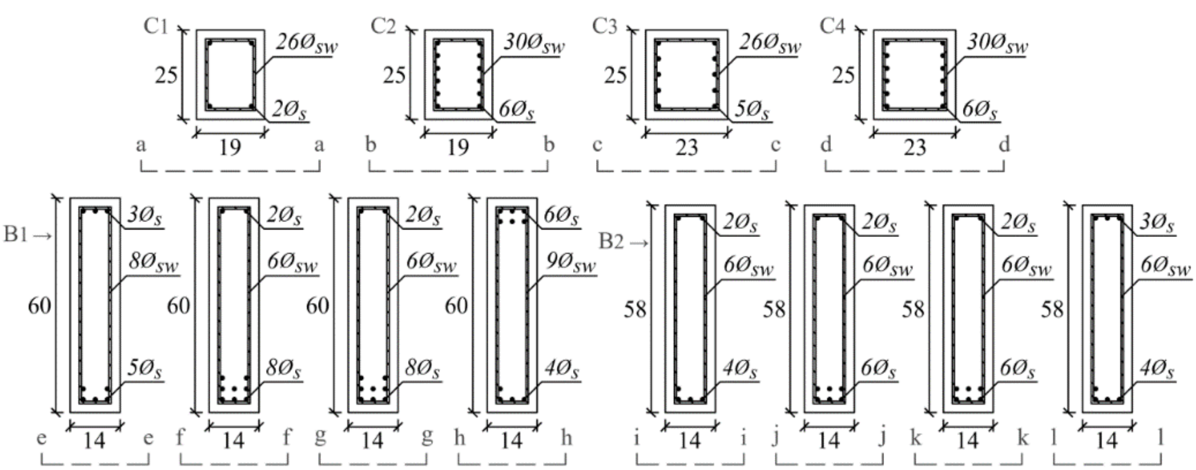

Figure 6. Detailing the optimal sections (dimensions in cm) - Example I: case I.

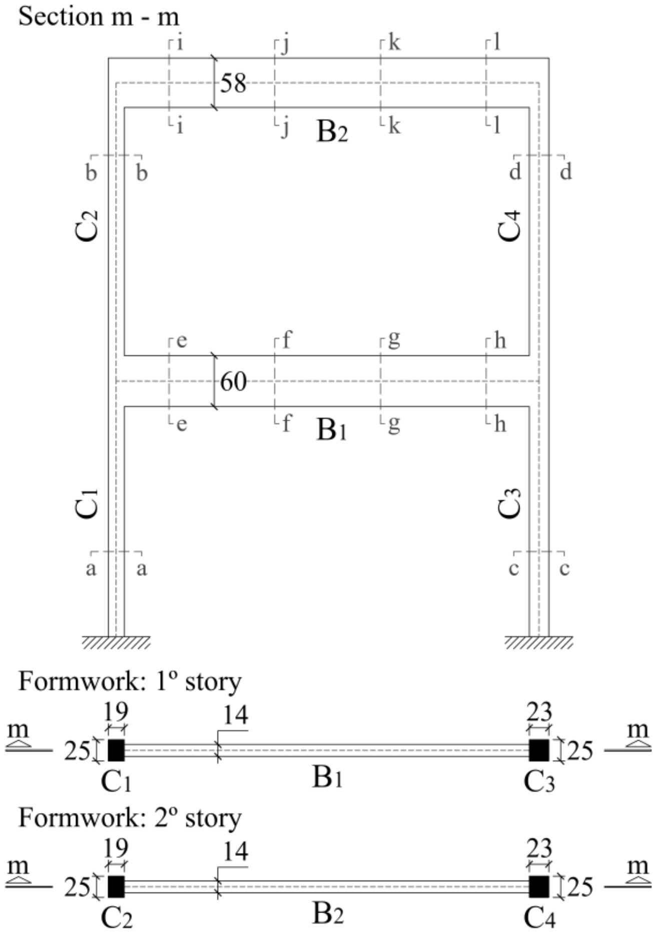

Figure 7. Optimal structure (dimensions in cm) - Example I: case I.

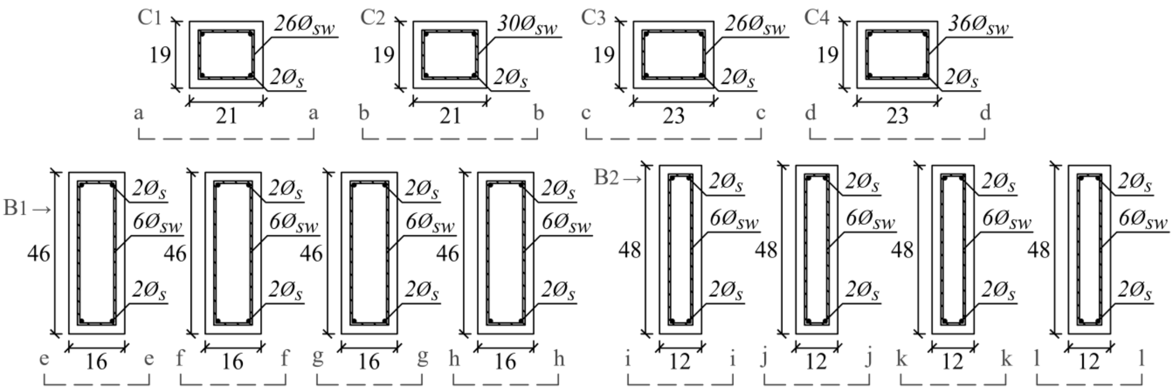

Figure 8. Detailing the optimal sections (dimensions in cm) - Example I: case III.

4.2 Example II: two-bay six-story frame

The second example consists of a structure previously studied by some authors [47], [20], [9], [12], from which the geometry was defined. The frame is illustrated in Figure 9 and the characteristic loads are described in Table 3. The weight of the structure is also considered.

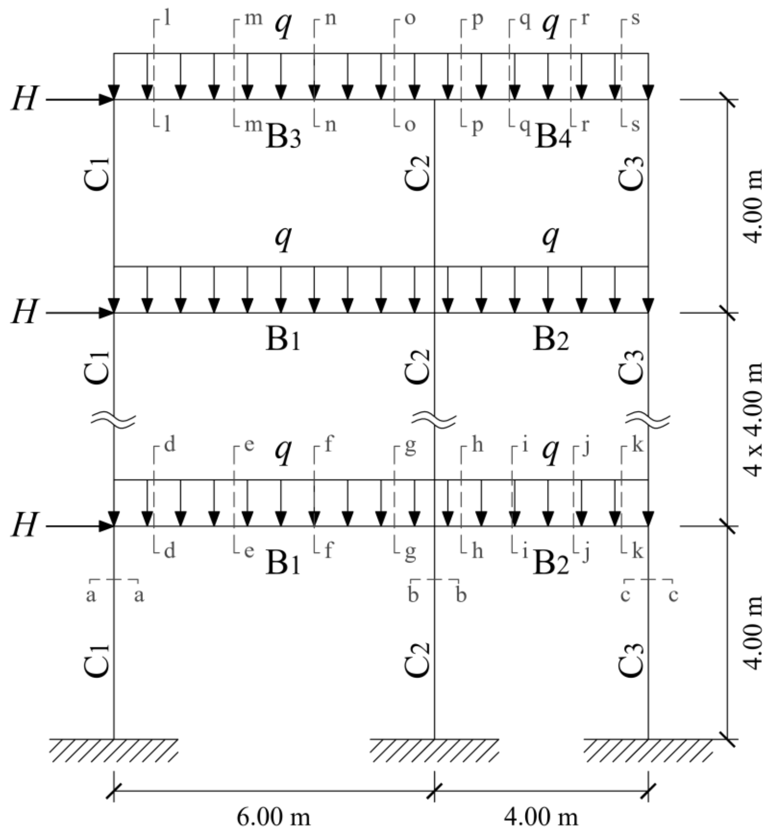

Figure 9. Plane frame - Example II.

Table 3. Characteristic loads - Example II.

Characteristic loads	Permanent loads	Variables loads
H	-	11.9 kN
q	12.9 kN/m	8.6 kN/m

Four different types of beams (B_1, B_2, B_3 and B_4) are adopted and discretized into four segments each. Therefore, each beam has 17 variables, as in Example I. Also, three different types of columns (C_1, C_2 and C_3) are adopted. Thus, each column has 6 variables: b , h , ϕ_s , ϕ_{sw} , n_s and n_{sw} . Therefore, the problem has 86 variables in total. For the variables of the beams, the following possible values are adopted: $b =$ [12, 30] cm and $h =$ [30, 60] cm, in increments of 2; $n_s^{bottom} =$ [2, 10], $n_s^{top} =$ [2, 10] and $n_{sw} =$ [5, 15], in increments of 1. While, for the columns: $b =$ [19, 39] cm, $h =$ [19, 55] cm and $n_{sw} =$ [28, 40], in increments of 2; $n_s =$ [2, 9] in increments of 1. Diameter of 12.5 mm is adopted for the longitudinal reinforcement and 6.3 mm for the transverse reinforcement.

The optimal structure for case I is presented in Figures 10 and 11. The total cost of the frame is R$ 16801.08, where 50.49%, 28.35% and 21.16% correspond to formwork, reinforcement, and concrete, respectively. Taking the cost of the design $\mathbf{x}^{(1)}$ as a reference, the convergence history of the relative cost over the generations is illustrated in Figure 12, where it is possible to observe that the optimization leads to a design configuration which is approximately 26% less expensive than the initial design. In this situation, 2646 generations and 132350 evaluations of the objective function were required, so that the computational time was about 45 hours. Figure 13 shows the result for case II, which corresponds to a total cost of R$ 16433.20, where 50.89% corresponds to the formwork, 28.70% is relative to the reinforcement and 20.41% is related to the concrete. 53900 evaluations of the objective function and 1077 generations were required, leading to a computational time of 17.85 hours. In the third case, illustrated in Figure 14, the computational time was 19.41 hours, with 1155 generations and 57800 evaluations of the objective function. The total cost of the structure is R$ 15776.73, with 52.60% corresponding to the formwork, 25.97% relative to the reinforcement and 21.44% related to the concrete.

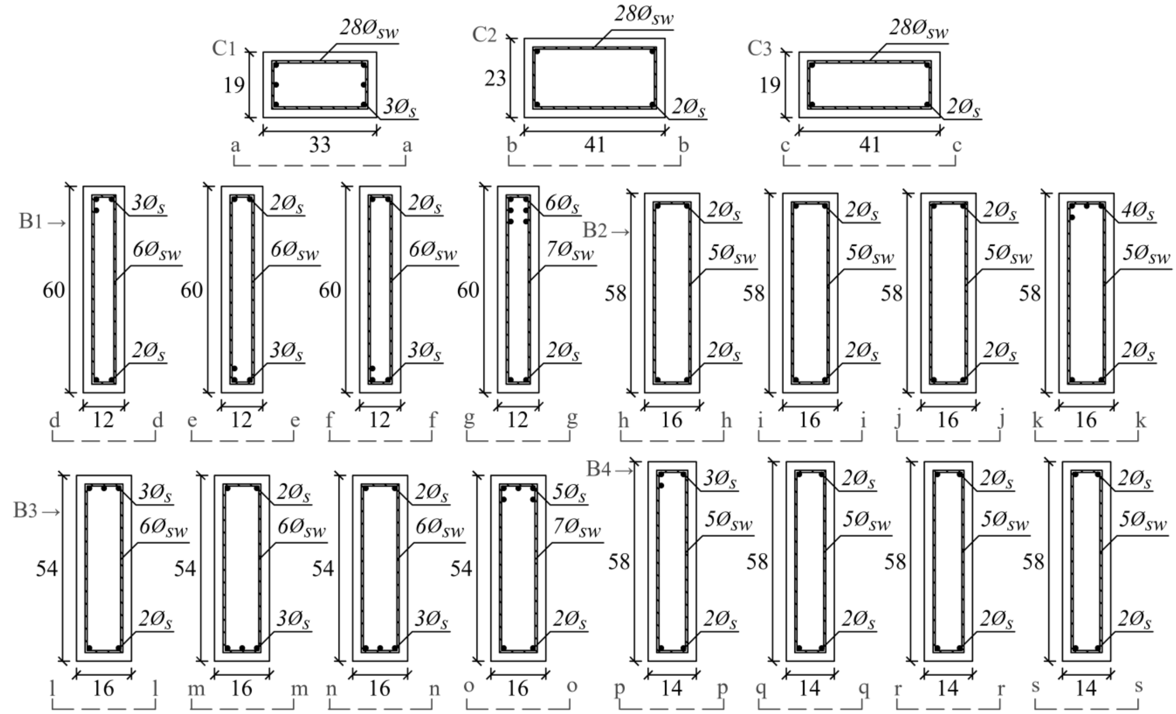

Figure 10. Detailing the optimal sections (dimensions in cm) - Example II: case I.

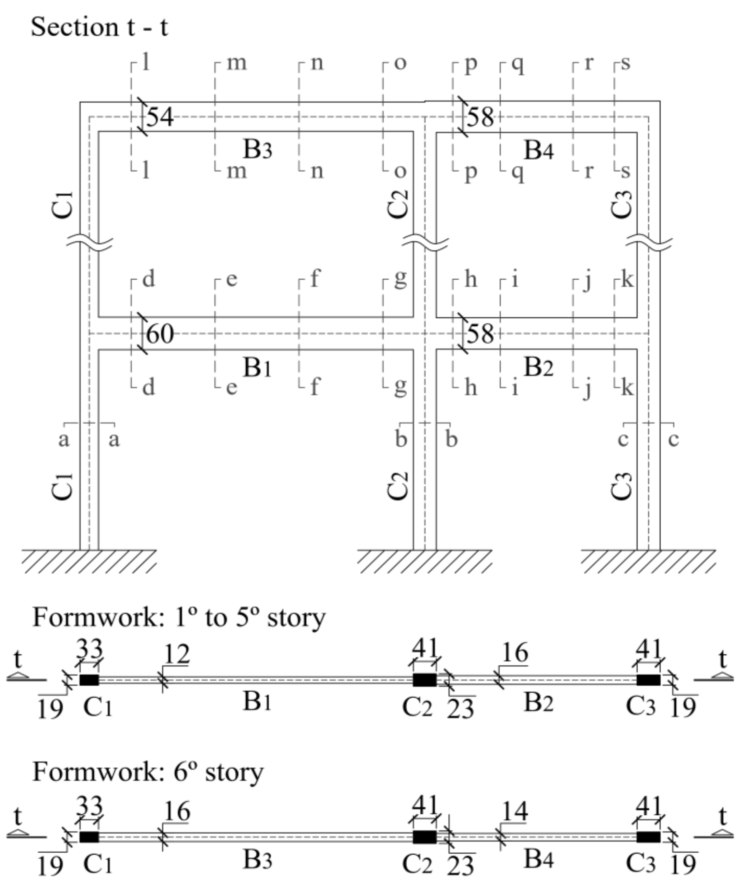

Figure 11. Optimal structure (dimensions in cm) - Example II: case I.

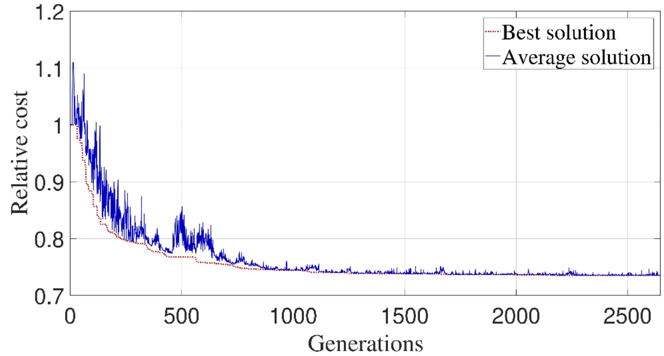

Figure 12. Convergence history - Example II: case I.

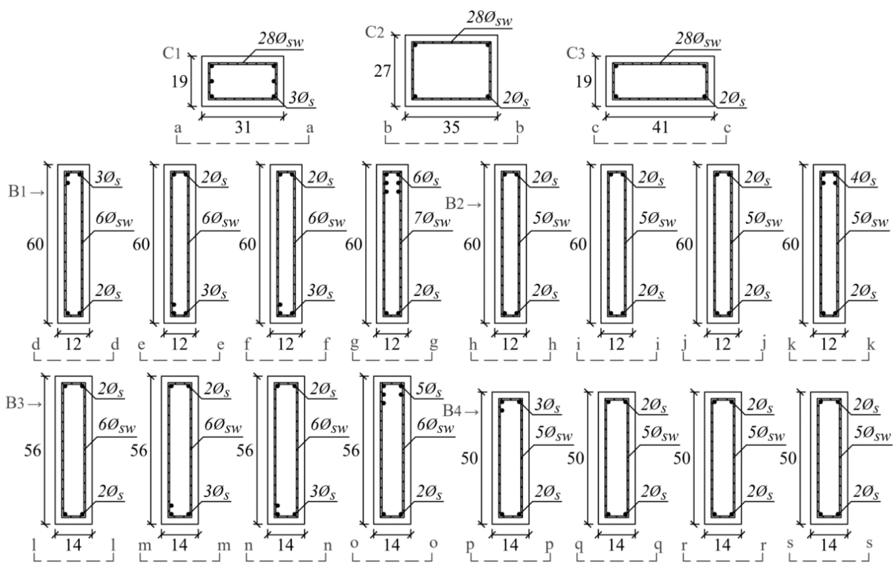

Figure 13. Detailing the optimal sections (dimensions in cm) - Example II: case II.

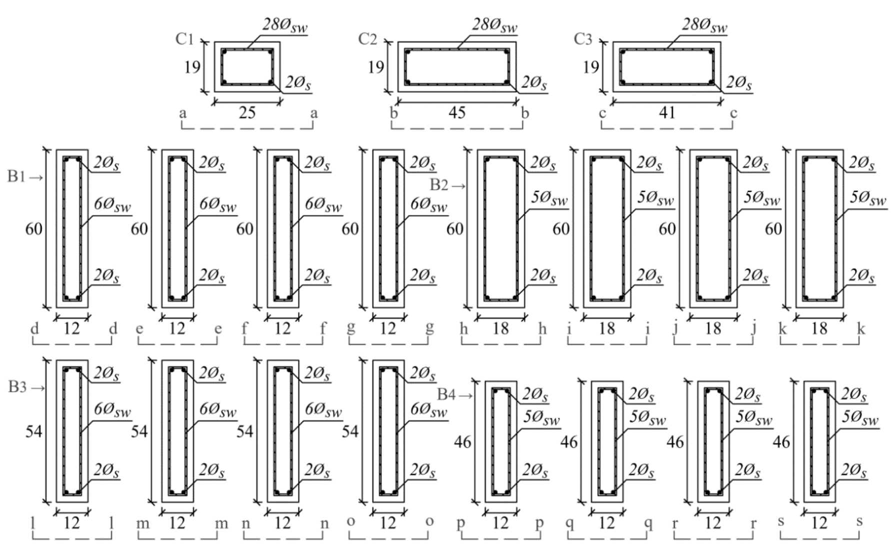

Figure 14. Detailing the optimal sections (dimensions in cm) - Example II: case III.

4.3 Discussion of results

In example I, when both limit states are considered as optimization constraints, the result is the same as when only the constraints of the ULS are taken into account; different from what occurred when considering only the SLS constraints, which leads to an optimal structure with a lower cost (27.10% less expensive). Therefore, the optimal structure for the ULS also checks the SLS constraints, but the optimal structure for the SLS does not meet all the constraints of the ULS. This situation is directly related to the design practice, in which the structure is usually designed for the ultimate conditions and then the service checks are performed. Unlike example I, cases I, II and III of example II led to different optimal sections, that is, the optimal structure for the ULS does not satisfy all the constraints of the SLS, in the same way that the optimal structure for the SLS does not satisfy all the constraints of the ULS. However, the optimal sections for case I are like the optimal sections for case II. In this example, when disregarding the SLS constraints (case II), the reduction in the total cost of the structure in relation to case I was of 2%, and when disregarding the ULS constraints (case III), this reduction was of 4%.

Among the specific constraints of both limit states, in example I, the constraint related to the design bending strength of C_2 was identified as a limiting constraint for cases I and II. For case III, the limiting constraint was related to the horizontal displacement at the top of the frame. By limiting constraint, it is understood that the constraint that was closer to being violated, since when using discrete variables in problems like the one proposed, there will be hardly any active constraints. For example II, the limiting constraint for cases I and III was the horizontal displacement at the top of the frame. For case II, the limiting constraint was related to the design bending strength of the top face of beam B_1 of the first story.

In both examples, the optimal columns related to ULS presented widths and amounts of longitudinal reinforcement equal to or greater than those related to SLS. On the other hand, the heights of the cross sections of the optimal columns according to the SLS tended to be equal to or larger than those of the ULS. It should be noted that, in case I of example I, the widths of the columns were greater than the heights due to the consideration that the columns have only one longitudinal reinforcement layer. Since the constraint of the transverse reinforcement for the columns concerns the minimum amount defined in the code, it was expected that all cases would have the same amount of such reinforcement, which happened for most situations. The optimal beams for the ULS have heights equal to or greater than the optimal beams for the SLS, just as they are equally or more reinforced in both directions. Also, there was a preponderance of minimum widths in all the elements optimum for SLS, except for one beam in each example.

In all cases of both examples, the formwork accounted for the most significant part of the optimal total cost, followed by the reinforcement.

5 CONCLUSIONS

This paper presented the optimization of reinforced concrete frames applying three different cases of constraints: related to the ULS and SLS constraints simultaneously; only those related to the ULS; and only those related to the SLS. All constraints were based on the requirements presented by the Brazilian design code, NBR 6118 [26].

The results obtained for two different examples indicated a tendency of the optimal characteristics of the structures, depending on the constraints applied. Thus, by evaluating the set of structures, it can be said that the cross sections tend to be larger and more reinforced in the case of the ULS. This is because the combinations of loads for the ULS lead to larger loads than those of the SLS.

The optimization processes of case III presented a greater reduction of cost in relation to the optimum of case I than the optimization processes of case II. It is also observed that the optimal structure considering all constraints is more like the optimal structure for the ULS than for the SLS, which indicates that SLS is generally less restrictive than ULS. Even so, optimal structure related to the ULS often does not satisfy SLS and vice versa. In addition, it is observed that, in the first example, the limiting constraint of the optimization of case I is related to section resistance (ULS). For example II, which is a higher structure, the limiting constraint in case I refers to the horizontal displacement at the top of the frame (SLS).

It is noteworthy that large differences between the costs of the optimal structures for cases I, II and III, may indicate that the structural design as a whole needs adjustments and/or modifications. Thus, making other changes to the design can be a more efficient alternative. These changes may include, for example, the adoption of different cross sections (T or hollow) and modifications in the topology (positions of the columns and different lengths of the structural elements).

Regarding to costs, there was a predominance of the formwork in the costs of the structures. It is noteworthy that the use of concrete with larger f_{ck} could result in smaller cross-sections, and consequently decrease the contribution of the formwork to the cost of the structure.

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