Analyses of reinforced concrete beams strengthened with CFRP under bending: theorical and computational approaches

Análise de vigas de concreto armado reforçadas à flexão com fibras de carbono: abordagem teórica e computacional

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Abstract

The basic aim of this work is to compile the theoretical basis of ACI 440.2R: 2008 [1] with the NBR 6118: 2014 [2] in order to take into account the concepts of the Brazilian standard in flexural sizing of reinforced beams with CFRP (Carbon Fiber Reinforced Polymer). The contribution of the Brazilian standard is given particularly with regard to the application of its safety coefficients and material properties (steel and concrete), including its deformation limits. For this purpose, a beam is adopted as a reference for the study, where two reinforcement designs are performed with CFRP, one from the compiled formulations and another considering only the requirements of ACI 440.2R: 2008 [1]. The results obtained are compared below. Finally, through the ANSYS software, numerical modeling of the reference beam is carried out, where tensions and deformations presented by concrete, steel and carbon fiber are observed. The results of the numerical analysis are compared with those obtained from the formulations compiled in order to validate the numerical model adopted in this study. The research concludes that in flexural sizing the areas of PRFC dimensioned from the formulations of ACI 440.2R: 2008 [1] resulted in values very close to those obtained by the formulations compiled. In addition, it was concluded that the numerical modeling performed in this work represented well the behavior of the structure, because the rupture loads were approximately equal to those expected by the analytical formulations.

Keywords: reinforcement, carbon fiber, CFRP, flexure, shear.

Resumo

Este trabalho tem como objetivo compilar o embasamento teórico do ACI 440.2R:2008 [1] com o da NBR 6118:2014 [2] a fim de levar em consideração os conceitos da norma brasileira no dimensionamento à flexão de vigas reforçadas com PRFC (Polímeros Reforçados com Fibra de Carbono). A contribuição da norma brasileira é dada particularmente no que diz respeito à aplicação dos seus coeficientes de segurança e das propriedades dos materiais (aço e concreto), incluindo os seus limites de deformação. Para tanto, é adotada uma viga como referência para o estudo, onde são realizados dois dimensionamentos do reforço com PRFC, um a partir das formulações compiladas e outro considerando apenas as prescrições do ACI 440.2R:2008 [1]. Os resultados obtidos são comparados em seguida. Por fim, através do software ANSYS, é feita a modelagem numérica da viga de referência, onde são observadas as tensões e deformações apresentadas pelo concreto, pelo aço e pela fibra de carbono. Os resultados da análise numérica são comparados com aqueles obtidos a partir das formulações compiladas a fim de validar o modelo numérico adotado nesse estudo. A pesquisa teve como conclusão que, no dimensionamento à flexão, as áreas de PRFC dimensionadas a partir das formulações da ACI 440.2R:2008 [1] resultaram em valores muito próximos àqueles obtidos pelas formulações compiladas. Além disso, concluiu-se que a modelagem numérica realizada nesse trabalho representou bem o comportamento da estrutura, pois as cargas de ruptura foram aproximadamente iguais àquelas esperadas pelas formulações analíticas.

Palavras-chave: Reforço, fibra de carbono, PRFC, flexão, cisalhamento.

1. Introduction

Of course, the most widely used methods for reinforcing structural elements are those most economically viable. In contrast, most methods do not meet the limitations such as maintaining element dimensions or execution time during the reinforcement process.

Several studies are being developed to improve reinforcement techniques through the use of new materials, which allow the execution of a fast reinforcement, clean and that does not interfere significantly in the dimensions of the element, thus reducing the interference in the architecture of the building.

A decade ago, the high initial cost of producing the polymeric materials and the lack of sufficient research or technical information, limited the frequency of the use of these composites in civil construction. With the steady decline in the price of raw materials and the manufacture of these materials, they have become increasingly competitive. It is important, however, that a Brazilian standard existto define the necessary requirements for the reinforcement dimensioning of reinforced concrete elements using these materials. This work aims to contribute to the advancement of this information, by carrying out a study on the dimensioning of reinforcement of reinforced concrete beams, considering the theoretical basis of the American standard ACI 440.2R: 2008 [1], but based on the concepts and criteria of NBR 6118: 2014 [2].

2. Bending dimensioning of reinforced beams with CFRP

The calculation procedure for reinforcement with CFRP that will be presented below is based on the normative prescriptions of ACI 440.2R: 2008 [1], however, it is adapted to the recommendations of NBR 6118: 2014 [2] with regard to the properties of materials, concrete and steel. All symbology used here is in accordance with the one usually found in the Brazilian standard, except in cases where the parameters are used only by the American standards.

The complete study of the reinforcement of reinforced concrete beams with CFRP is developed in Silva [3], where are presented, in details, the prescriptions of the Brazilian norm and of the American norm for the reinforcement of beams with CFRP both the flexion and the shear.

It should be noted that in the study by Silva [3], it was observed that, for the beam studied, shear reinforcement was not necessary to increase the resistant capacity of the beam, since the existing stirrups were enough to withstand the new shear increase in overload. For this reason, shear design will not be presented in this article. For more details, see Silva [3]. Further studies on flexural sizing using carbon fibers can be found in Ferrari et al. [4] and Machado [5].

Figure 1 shows schematically the distribution of deformations and forces in a flexural reinforced section with CFRP.

In domain 2, where the deformation of the traction armature is given as 10‰, the deformations of the section can be related by:

$$\varepsilon_c = \frac{0,01x}{d-x} \tag{1}$$

$$\varepsilon_{s'} = \frac{0,01\left(x-d'\right)}{d-x} \tag{2}$$

$$\varepsilon_s = 0,01 \tag{3}$$

$$\varepsilon_b = \frac{\varepsilon_c(h-x)}{x} \tag{4}$$

Now, in domains 3 and 4, where the concrete deformation is equal to the specific concrete shortening at rupture (ε_{cu}), we have:

$$\varepsilon_c = \varepsilon_{cu} \tag{5}$$

$$\varepsilon_{s'} = \frac{\varepsilon_{cu}\left(x-d''\right)}{x} \tag{6}$$

$$\varepsilon_s = \frac{\varepsilon_b(d-x)}{h-x} \tag{7}$$

$$\varepsilon_b = \frac{\varepsilon_{cu}(h-x)}{x} \tag{8}$$

where: $\varepsilon_b = \varepsilon_{fe} + \varepsilon_{bi}$, being ε_{fe} the effective deformation of the reinforcement with CFRP and ε_{bi} the pre-existing deformation at this point of the beam, prior to the application of the reinforcement.

According to Machado [5], this deformation can be calculated from an elastic analysis considering the loading existing at the time of application of the reinforcement (generally only the acting permanent loads, such as self weight and coating, are considered).
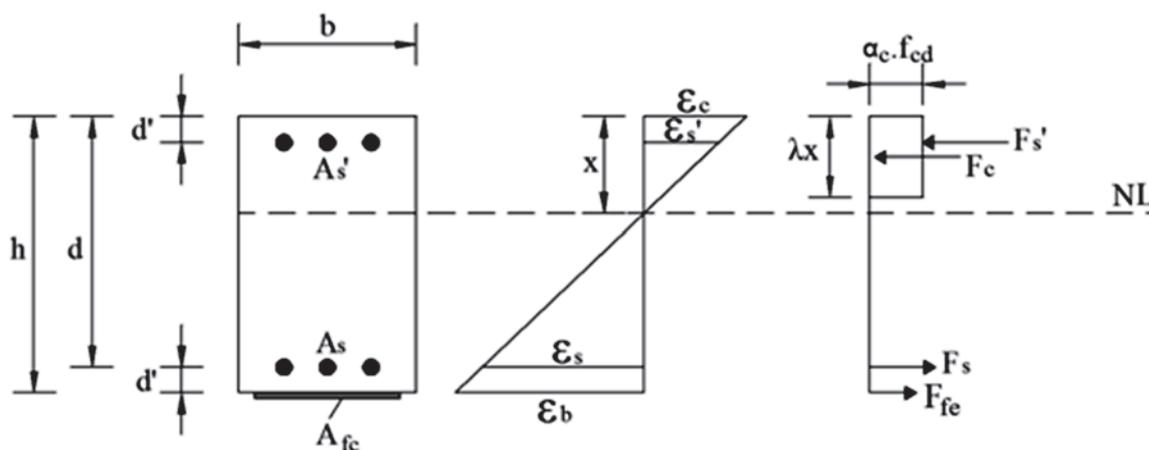

Figure 1
Distribution of deformation and forces in a section reinforced to flexion with CFRP

Now, considering the stress distribution in the section also presented in Figure 1, the following equilibrium equations can be obtained:

$$MR_{d1} = F_c \cdot \left(h - \frac{\lambda x}{2} \right) + F_{s'} \cdot \left(h - d' \right) - F_s \cdot d \qquad (9)$$

$$MR_{d2} = F_s \cdot \left(d - \frac{\lambda x}{2} \right) + F_{s'} \cdot \left(\frac{\lambda x}{2} - d' \right) + F_{fe} \cdot \left(h - \frac{\lambda x}{2} \right) \qquad (10)$$

Being that value MR_{d1} is obtained from the sum of moments at the point of application of the carbon fiber and the value MR_{d2} is obtained at the point of the force application F_c.

The forces F_c, F_s, F_s' and F_{fe} are defined by the product between the areas and the respective tensile strengths of each element. In this way, it is:

$$F_c = A_c \cdot \sigma_{cd} \qquad (11)$$

$$F_s = A_s \cdot \sigma_{sd} \qquad (12)$$

$$F_{s'} = A_{s'} \cdot \sigma'_{sd} \qquad (13)$$

$$F_{fe} = A_f \cdot \sigma_{fe} \qquad (14)$$

Machado [5] recommends that the tensile strength of the concrete should be multiplied by a reduction factor ψ to consider the effect of the influence of carbon fiber on the element. In this research, these recommendations are adapted to follow the requirements of NBR 6118: 2014 [2], which establishes that the stresses in the concrete must be calculated according to the parabola-rectangle diagram presented in item 8.1.10 of the cited standard. In this way, the factor ψ is given by:

$$\psi = 1, \text{ for } \varepsilon_c = \varepsilon_{cu} \qquad (15)$$

$$\psi = 1{,}25 \cdot \left(1 - \frac{\varepsilon_{c2}}{3\varepsilon_c} \right), \text{ for } \varepsilon_{c2} < \varepsilon_c < \varepsilon_{cu} \qquad (16)$$

$$\psi = \frac{2{,}5}{3} \cdot \sqrt{\frac{\varepsilon_c}{\varepsilon_{c2}}}, \text{ for } \varepsilon_c < \varepsilon_{c2} \text{ (2,5/3)} \qquad (17)$$

Being ε_{c2} the specific shortening deformation of the concrete at the beginning of the plastic landing.

Thus, since the compressed concrete area is equal to $\lambda x.b$, the equations (11) and (14) can be rewritten as:

$$F_c = \psi \cdot \alpha_c \cdot f_{cd} \cdot \lambda x \cdot b \qquad (18)$$

$$F_s = A_s \cdot f_s \qquad (19)$$

$$F_{s'} = A_{s'} \cdot f_{s'} \qquad (20)$$

$$F_{fe} = \psi_f \cdot A_{fc} \cdot f_{fe} \qquad (21)$$

being $\psi_f = 0{,}85$ a reduction factor applied to the fibers, as defined in item 10.2.10 of ACI 440.2R: 2018 [1], and f_{cd} the design strength of the concrete.

The stresses f_s', f_s, f_{fe} and $\alpha_c \cdot f_{cd}$ are, respectively, the stresses acting on the lower reinforcement, the upper reinforcement, the CFRP and the compressed concrete area.

Considering that:

$$f_{fe} = E_f \cdot \varepsilon_{fe} \qquad (22)$$

And, remembering that $\varepsilon_{fe} = \varepsilon_b - \varepsilon_{bi}$, one can define f_{fe} as being equal to:

$$f_{fe} = E_f \cdot \left(\varepsilon_c \cdot \frac{(h-x)}{x} - \varepsilon_{bi} \right) \qquad (23)$$

then, by developing equations (9) and (10), one has to:

$$MR_{d1} = \psi \cdot \alpha_c \cdot f_{cd} \cdot \lambda x \cdot b \left(h - \frac{\lambda x}{2} \right) + A_{s'} \cdot f_{s'} \cdot \left(h - d' \right) - A_s \cdot f_s \cdot d \qquad (24)$$

$$MR_{d2} = A_s \cdot f_s \cdot \left(d - \frac{\lambda x}{2} \right) + A_{s'} \cdot f_{s'} \cdot \left(\frac{\lambda x}{2} - d' \right) + \psi_f \cdot A_f \cdot (E_f \cdot \varepsilon_{fe}) \cdot \left(h - \frac{\lambda x}{2} \right) \qquad (25)$$

The depth of the neutral line (x) could be obtained by equating the equation of MR_{d1} with the value of the design requesting moment MS_d. However, the deformations used to calculate the stresses in the reinforcement also vary as a function of the depth of the neutral line, as well as the value of ψ, which depends on the deformation of the concrete and, moreover, can be defined by three different equations. Therefore, it is necessary to estimate a value of x and, iteratively, verify that the difference between MR_{d1} and MS_d is below a tolerance adopted.

In this work, the neutral line was obtained using Newton's method, making $MS_d - MR_{d1} = 0$. Newton's Method and its variations (Quasi-Newton Methods) are used to solve nonlinear problems without restrictions, as is the case of the problem in question.

In a numerical analysis, the objective of this method is to obtain the value of the variable x such that the function f(x) is equal to 0, from an iterative process. For this, it is necessary that the equation be differentiable in x_n, where n indicates the nth iteration of the algorithm. The iterative process of Newton's Method is best illustrated in the work of Bandeira [6].

As a premise for dimensioning, the value of MS_d must be less than or equal to the design resistance MR_d, and, according to the limitation presented in the introduction of chapter 10 of ACI 440.2R: 2008 [1], the strength of the reinforced part (MR_d) shall not have a resistance greater than 40% of the initial strength of the part (MR_{di}), prior to reinforcement, calculated on the basis of the existing cross section reinforcement. That is, $MR_d \leq 1{,}4 \, MR_{di}$.

Knowing that $MR_{d1} = MR_{d2}$, we can define the force (F_{fe}) of the carbon fiber considering the value of x previously calculated. As $F_{fe} = \psi_f \cdot A_{fc} \cdot f_{fe}$, the required carbon fiber area (A_f) for the reinforcement is calculated by:

$$A_f = b_f \cdot t_f \cdot n \qquad (26)$$

Being b_f the width of the CFRP, t_f the thickness of the layer and n the number of layers.

It is recommended in item 10.2.10 of ACI 440.2R: 2008 [1] that the deformation of the carbon fiber (ε_{fe}) must be limited by the ultimate deformation of the fiber (ε_{fud}), which is given by:

$$\varepsilon_{fud} = 0{,}41 \sqrt{\frac{f_{cd}}{nE_f t_f}} \leq 0{,}9\varepsilon_{fu} \qquad (27)$$

If an element is being reinforced by bending, it is assumed that there was an increase in beam loading relative to that estimated at the initial design. Therefore, it is essential that a verification of its shear strength be made for the new stresses presented as a result of the increased loads.
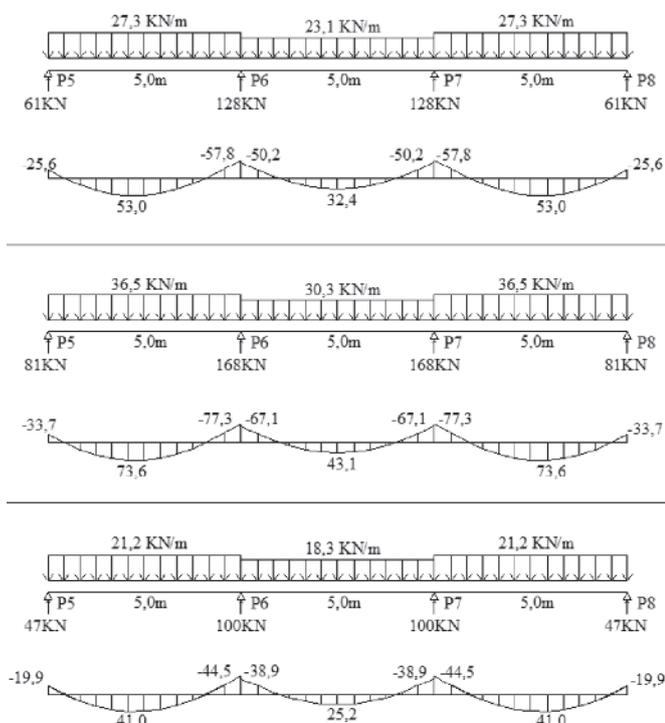
To consider the ductility reduction of the original element caused
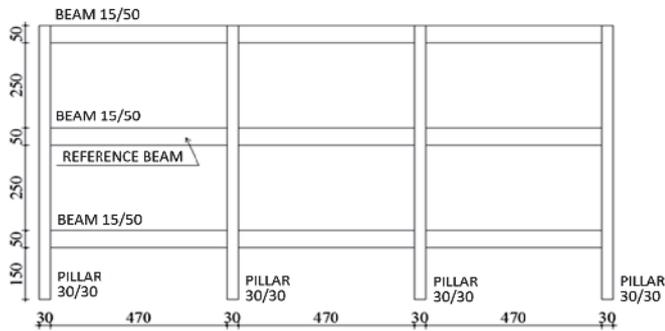
Figure 2
Portico indicating the reference beam for the case study. Dimensions in centimeters
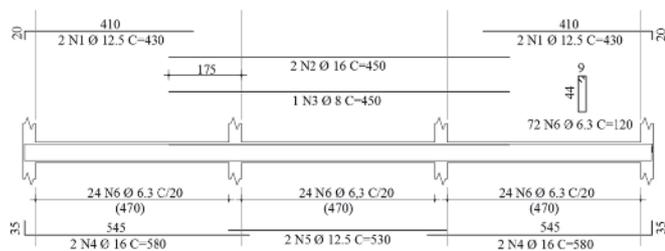
Figure 3
Detail of the reference beam

Figure 4
Acting loads on the reference beam

by the use of reinforcement with CFRP, according to item 10.2.7 of ACI 440.2R: 2018 [1], a factor φ is defined to reducethe tensile strength of the structural elements. This factor varies as a function of steel deformation (ε_s), as defined below.

$$\phi = 0{,}65, \text{ for } \varepsilon_s \leq \varepsilon_{sy} \tag{28}$$

$$\phi = 0{,}65 + 0{,}25 \cdot \left(\frac{\varepsilon_s - \varepsilon_{sy}}{0{,}005 - \varepsilon_{sy}}\right), \text{ for } \varepsilon_{sy} < \varepsilon_s < 0{,}005 \tag{29}$$

Where ε_{sy} is the steel flow deformation defined by $\varepsilon_{sy} = f_{yd}/E_s$, where f_{yd} represents the plasticity stress of the steel and E_s represents the elasticity modulus of the steel.

$$\phi = 0{,}9, \text{ for } \varepsilon_s \geq 0{,}005 \tag{30}$$

3. Presentation of the reference beam and sizing of the reinforcement with CFRP

The beam used as a reference for study in this work was obtained from the analysis of a model building, where the loads adopted were in accordance with NBR 6120: 1980 [7] for a commercial building. To better represent the behavior of the beam, it was analyzed in conjunction with the portico in which it was inserted. Figure 2 below shows the characteristics of the portico and the dimensions of each element that compose it.

The beam was detailed according to the initial design made considering the efforts anticipated before the increase of the overload, that is, before the need to apply the reinforcement. Figure 3 shows the detail of the beam.

In order to make the analysis and the dimensioning of the reinforcement possible, it was considered that the loading of the beam suffered an increase of the overload due to a change of use of the model building. Figure 4 below shows the loadings on the beam in three situations, the first one considering the predicted loads before the increase of the load, the second after the increase of the load, and the third considering only the permanent loads acting. In summary, the efforts of the first situation were used for the dimensioning of the reinforcement, the efforts of the second situation for the dimensioning of the CFRP and those of the third situation for the definition of the preexisting deformation ε_{bi}, prior to the application of the reinforcement.

The dimensioning of the reinforcement of the reference beam was performed in two ways. The first, considering only the requirements of ACI 440.2R: 2014 [1] and the second one considering the formulations compiled presented in this paper. Table 1 shows the characteristics of the materials adopted during the study.

Initially, it is necessary to determine the strength of the cross-section prior to the application of the reinforcement with the CFRP, since, according to the guidance of ACI 440.2R: 2008 [1], it is not recommended to use this type of reinforcement in cases where the new working force is greater than 40% of the strength of the reinforced concrete part (part without the use of the CFRP). For this, the section of the beam next to the support is verified, where the moment tends to be greater. In this situation, after the reinforcement, the beam presents the configuration of the cross section shown in Figure 5. It can be observed that, in this case, the tractioned reinforcement is the upper reinforcement and, therefore,

Table 1
Properties of materials adopted in the case study

Concrete	f_{ck}	$= 30 MPa = 3\ KN/cm^2$
	E_{ci}	$= 5{,}600\ \sqrt{f_{ck}} = 30{,}672\ MPa = 3{,}067\ KN/cm^2$
	α_i	$= 0.8 + 0.2 \times f_{ck} = 0.875$
	E_{cs}	$= \alpha_i.E_{ci} = 0.875 \times 26{,}838\ MPa = 2{,}684\ KN/cm^2$
Steel	f_{yk}	$= 500\ MPa = 50\ KN/cm^2$
	E_s	$= 210{,}000\ MPa = 21{,}000\ KN/cm^2$
	$\varepsilon_{s,u}$	$= 10‰$
	$\varepsilon'_{s,u}$	$= 10‰$
Fiber **(CF-30 from MbraceTM sistem)**	$\varepsilon_{fu,k,MANUFACTURER}$	$= 17‰$
	$f_{fu,k,MANUFACTURER}$	$= 3{,}790\ MPa = 379\ KN/cm^2$
	E_f	$= 228{,}000\ MPa = 22{,}800\ KN/cm^2$
	t_f	$= 0.165mm$
	CE	$= 0{,}95$ (coefficient of reduction according to the enviroment)
	$\varepsilon_{fu,k}$	$= \varepsilon_{fu,MANUFACTURER} \times CE = 16.15‰$
	$f_{fu,d}$	$= f_{fu,MANUFACTURER} \times CE = 3{,}600\ MPa = 360\ KN/cm$

it is in this region that the carbon fiber reinforcement is needed when the active efforts are increased.

At the moment of application of the carbon fiber reinforcement, the beam is already subjected to a certain load, which in this case was given by the load coming from the permanent loads. Thus, the beam already presents a table of initial deformations in the cross section under analysis before the application of the carbon fiber reinforcement. In the most tractioned face, or rather, at the point where the CFRP is applied, the deformation ε_b can be called, in this case, ε_{bi}. This deformation must be calculated considering that the part is working in Stage 2 of sizing, which occurs before the part reaches its state of plastification.
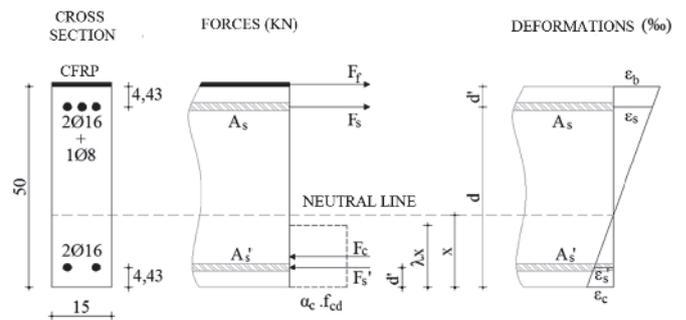
It is important to verify if the deformation of the carbon fiber is lower than the allowable, that is, if $\varepsilon_{fe} < \varepsilon_{fu,d}$. Then, it is necessary to verify if the tensioning stress in the carbon fiber is inferior to the admissible, that is, if $f_{fe} < f_{fu,d}$.

Table 2 presents a comparison between the parameters involved in the calculation of the initial strength of the beam, both by ACI 440.2R: 2014 [1] and by the formulations compiled in this paper.

Table 3 below presents a comparison between parameters involved in the calculation of the CFRP area required for reinforcement.
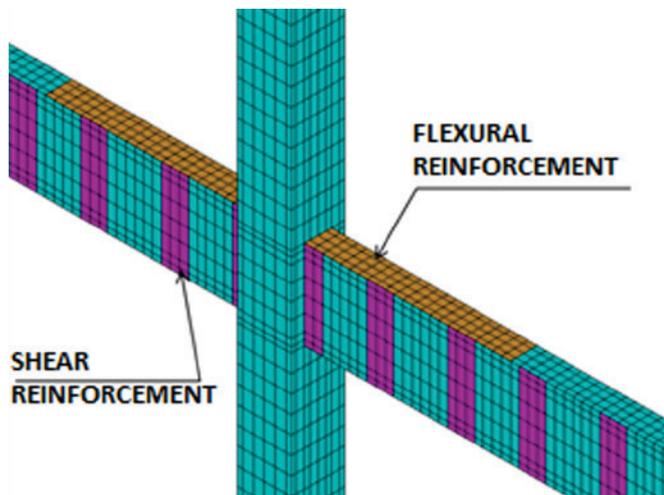

Figure 5
Cross section of maximum bending moment

4. Presentation of the reference beam and dimensioning of the reinforcement

Version 16 of the ANSYS software was used to evaluate the reference beam of the case study. In the modeling, three types of discrete elements were defined: to represent the concrete, the element SOLID 65 was adopted; to represent the armatures, the

Table 2
Parameters involved in the calculation of the initial resistent momentum of the beam

Parameter	Results by:		Percentage difference (%)	Absolute difference
	ACI 440.2R:2014	Compiled formulations		
x (cm)	6.22	6.70	-8%	-0.49
ε_c (‰)	1.58	1.72	-9%	-0.14
ε_s (‰)	10.00	10.00	0%	0.00
ε'_s (‰)	0.45	0.58	-29%	-0.13
ε_b (‰)	11.13	11.14	0%	-0.01
F_{cd} (KN)	170	147	14%	23
F_{sd} (KN)	203	196	3%	7
F'_{sd} (KN)	33	49	-50%	-16
M_{Rd} (KN.cm)	8,634	8,305	4%	329

Table 3
Parameters involved in the calculation of the CFRP area required for reinforcement

| Parameter	Results by:		Percentage difference (%)	Absolute difference
	ACI 440.2R:2014	Compiled formulations		
x (cm)	7.26	8.42	-16%	-1.17
ε_c (‰)	1.89	2.27	-20%	-0.37
ε_{cs} (‰)	10.00	10.00	0%	0.00
ε'_s (‰)	0.74	1.08	-46%	-0.34
ε_b (‰)	11.16	11.19	0%	-0.04
ε_{bi} (‰)	1.34	1.28	5%	0.07
ε_{fe} (‰)	9.81	9.92	-1%	-0.10
F_{cd} (KN)	198	162	18%	36
F_{sd} (KN)	203	196	3%	7
F'_{sd} (KN)	53	90	-70%	-37
F_{fe} (KN)	64	67	-5%	-3
A_{fc} (cm²)	0.32	0.33	-4%	-0.01

Figure 6
Positioning of reinforcement with CFRP – top view
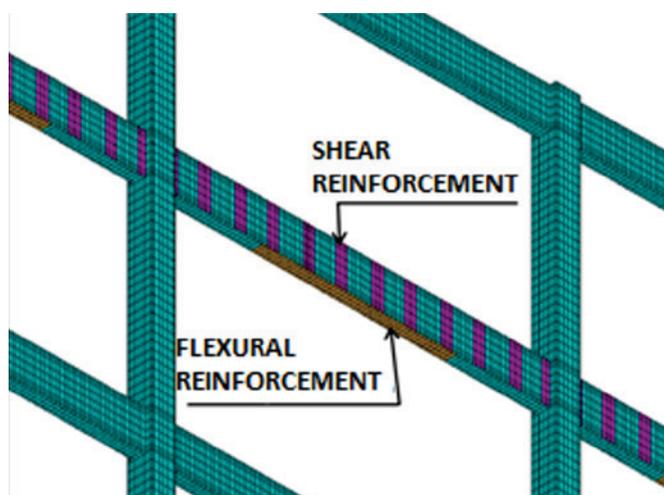
Figure 7
Positioning of reinforcement with CFRP – bottom view

element BEAM 188; and, to represent the CFRP, the element SHELL 181, all as presented in [8]. In summary, BEAM 188 is a one-dimensional element that is based on the Thimoshenko beam theory, which takes into account the cross-sectional deformations caused by the shear stresses of the element. This element is linear, having 2 nodes with 6 degrees of freedom each (three translations in x, y and z, and three rotations in x, y and z). SOLID 65 is an element used for three-dimensional modeling of solids with or without reinforcing bars. The element consists of 8 nodes with three degrees of freedom each (translations in x, y and z). The SHELL 181 is capable of representing the flexion and membrane behavior of a given material. This element supports in-plane and normal loads. This element has 4 nodes where in each node there are 6 degrees of freedom each (three translations and three rotations). Similar studies were carried out by Bandeira [9]. However, in the work of this author only the longitudinal reinforcement of the beam
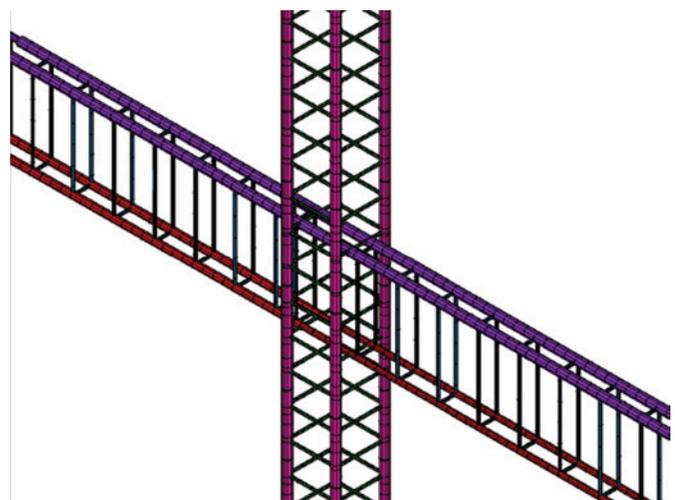
Figure 8
View of the reinforcement of the elements modeled on the beam-pillar connection
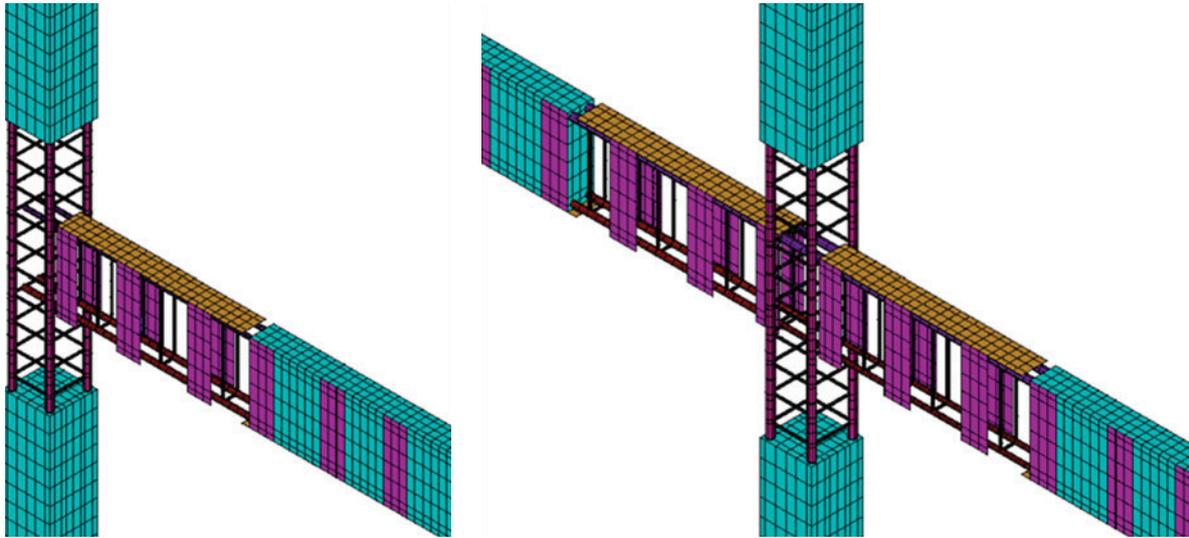
Figure 9
View of the elements modeled on the beam-pillar connection
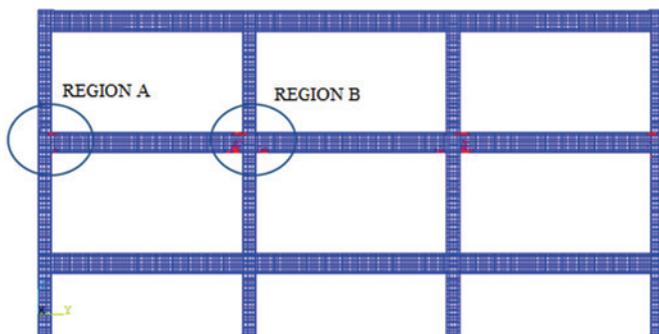
Figure 10
Points of cracking and crushing of the concrete at the moment of rupture (1st Model) and at the instant of the final increment of load (2nd Model)

was modeled. A major contribution of this article is the closer consideration of the actual beam situation, i.e. considering the stirrups and the connection of the beam with the pillars.

It was considered a perfect integration between the materials (concrete-steel or concrete-fiber), no transition zone in the bonding region, such as the resin in the bond between the concrete and the CFRP. In this way, it was imposed in the modeling that there is no rupture by taking off the CFRP or by displacement of the concrete cover.

It is worth noting that the objective of this work is not to present the formulations used in the ANSYS software, as well as the methodology applied through the Finite Element Method.

Figure 6 illustrates the positioning of the CFRP reinforcement used for flexing on the upper face of the beam and for shearing on the lateral faces of the beam.

It is observed that the flexural reinforcement on the upper face of the beam was arranged only in the part of the stretch where the beam is subjected to the traction. In this example, the length of the upper reinforcement strip was adopted as 1/4 of the span between pillars.
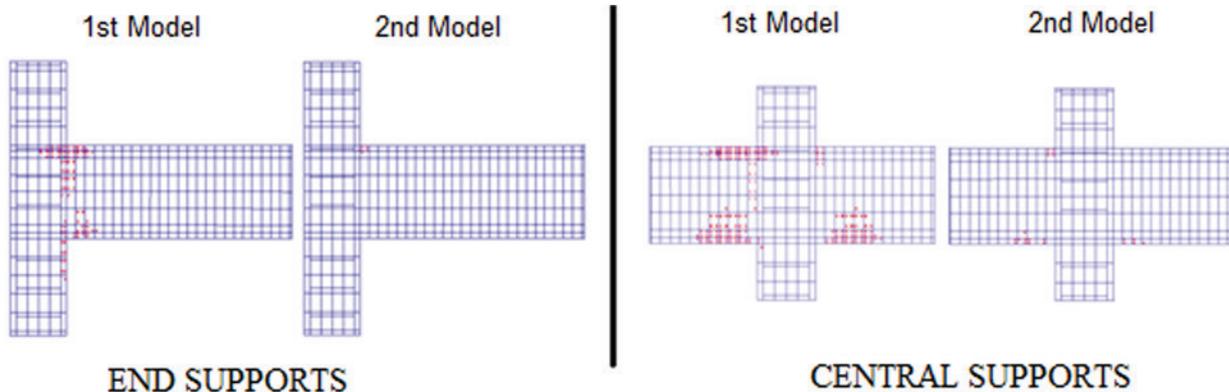
Figure 11
Detailed view of regions A and B

In order to have a real representation of the behavior of the structure, the reinforcement was also positioned in the underside of the beam, in the section where the cross sections are traced. The CFRP arrangement was made between 1/4 and 3/4 of the span, as shown in Figure 7. The armatures, represented in ANSYS by the element BEAM 188, are shown in Figure 8.

Figure 9 shows the modeling of all elements in the beam-column connection. In this figure a part of the concrete was hidden to allow the visualization of the reinforcements.

To avoid lateral instability of the portico, the displacement in the orthogonal direction to the beam axis was restricted in all elements of the pillars. It was also defined that the pillars are fully constrained at their base, so that any translations or rotations at these points were prevented.

Two models were simulated, as described below:

- 1st Model: No reinforced beam with CFRP;
- 2nd Model: Reinforced Beam with CFRP.

In both models, the geometry of the elements, the detail of the reinforcements and the properties of the materials were maintained. The load used in both models already contemplates the increase of the overload.

In the ANSYS software, the load is applied gradually to the structure until the limit of rupture of any of the finite elements is reached. In this way, we tried to verify, for each model analyzed, the tensions and deformations of each material in the section of maximum mo-

ment. The aim is to compare these values obtained numerically with those obtained from the analytical formulations.

It was observed that the 1st Model (non-reinforced beam with CFRP) did not withstand the total load imposed, breaking with approximately 67% of the total load, which roughly corresponds to the initial load used for designing the reinforcement of the initially designed structure. The 2nd Model supported the total load imposed, not breaking due to flexural reinforcement.

The rupture of the Model 1 occurred due to the crushing of the concrete in the regions close to the support, as shown in Figure 10. The regions in red represent the points of cracks in the traction region and crushing of the concrete in the compressed region. Regions A and B identified in Figure 10 are detailed, for each model, in Figure 11. For Model 1, where there was no flexural reinforcement, it can be observed that in the traction region there are crack openings that prove that the tensile strength in the concrete bending was reached. Also, for this model, it is observed that the crushing of the concrete in the compressed zone is excessively high.

However, for Model 2, where there was flexural reinforcement, existing crack openings are significantly smaller and fully acceptable in a reinforced concrete structure. In this case, it is observed that
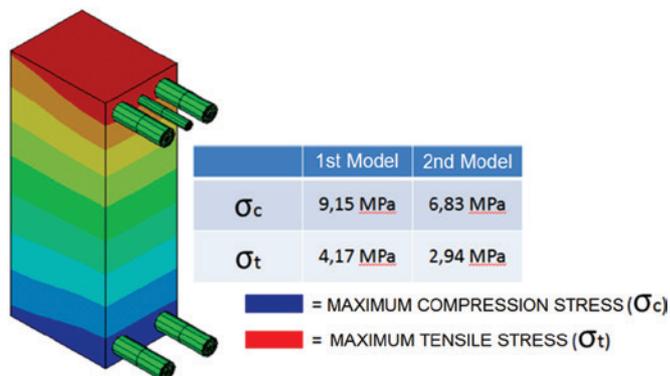

Figure 12
Deformation in the concrete in the maximum momentum section
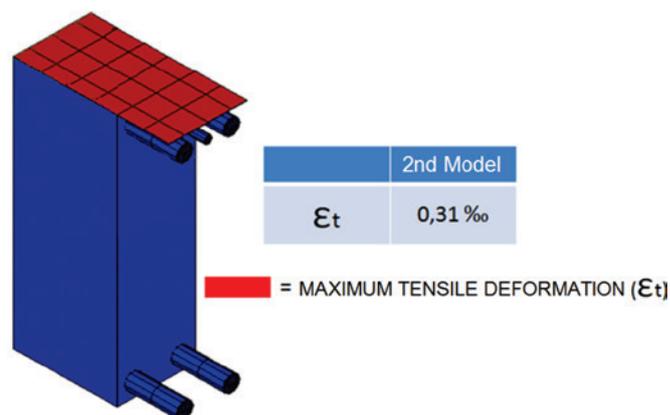
Figure 14
Deformation in the CFRP in the maximum momentum section

Figure 13
Deformation in the steel in the maximum momentum section

Figure 15
Stress in the concrete in the maximum momentum section

the crushed concrete is limited approximately to the covering region of the reinforcements.

For the maximum moment section, the Figure 12 shows the deformations in the concrete beam, the Figure 13 the deformations in the steel, the Figure 14 the deformations in the CFRP, the Figure 15 the stresses in the concrete, the Figure 16 the stresses in the steel and Figure 17 the stresses in the CFRP.

The 1st Model was expected to break before reaching the maximum load. Therefore, in order to verify that the numerical modeling is in conformity with the analytical formulations, this same modelwasre-analysed, however, with the initially estimated loading for the dimensioning of the reinforced concrete structure. The structure was found to have withstood the maximum loading, implying that the rupture loads of the numerical formulations are in accordance with the requirements of the analytical prescriptions.
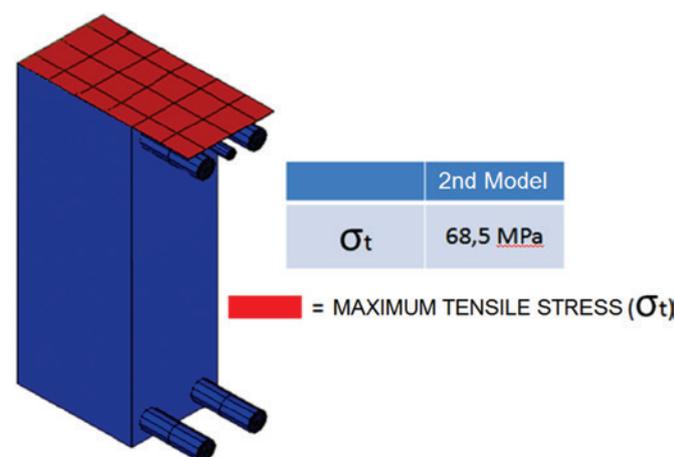
5. Results discussion

In this section we will present the discussions about computational modeling and modeling mentioned previously.

The values of the forces obtained through the formulations based

Figure 16
Steel tensions in the maximum momentum section

Figure 17
Stresses in the CFRP in the maximum momentum section

on NBR 6118: 2014 [2] are already design values. However, the corresponding values calculated by ACI 440.2R: 2008 [1] are characteristic values. Thus, to allow for comparison, the characteristic forces from the American formulation were multiplied by the reduction coefficient $\varnothing$.

It is observed that, although there is a great variation between the calculated forces and deformations, the initial strength of the beam is very close to both standards (difference of approximately 4%), as shown in Table 2.

Obviously, because the two standards employ different stress factors for material strength, different results can be expected to be found for the calculated forces and deformations. This difference was more evident for the steel located in the compressed zone, where the difference was 29% for the deformation and 50% for the strength resistant. However, it is observed that the absolute difference is not so great (0.13 ‰ for deformations and 16KN for force). What we see is that the depth of the neutral line is greater when considering the requirements of NBR 6118: 2014 [2]. This is due to the fact that the safety coefficients used by the Brazilian standard to reduce the resistance (1.4 for concrete and 1.15 for steel) are lower than those presented by ACI 318: 2014 [10]. Numerically, if the coefficients of NBR 6118: 2014 [2] obey the American standards (where the safety coefficient must be multiplied by the resistance and not divided), they would be 1 / 1.4 = 0.71 for the concrete and 1 / 1.15 = 0.87 for steel. It is observed that both values are lower than the coefficient $\varnothing$ = 0.9 defined by the American standard for both steel and concrete. As NBR 6118: 2014 [2] reduces the strength of the concrete more than the strength of the steel, certainly, for there to be equilibrium in the section, the compressed concrete area should be larger in the sizing done by this standard than when done by American standard. Consequently, there is a tendency to increase the height of the neutral line to the Brazilian standard in relation to the American standard. The position of the neutral line is what defines the deformations of the materials and, consequently, their resistive force.

It is also observed in Table 3 that there is a large variation between the values of forces and deformations calculated for the parameters involved in the calculation of the CFRP area required for reinforcement. However, the areas of CFRP determined by both standards are very close (difference of only 0.01cm²).

In the sizing of the reinforcement, the depth of the neutral line calculated following the recommendations of NBR 6118: 2014 [2] continues to be greater than that calculated according to the requirements of ACI 440.2R: 2008 [1]. This is justified in the same way, because if the safety coefficients of the Brazilian standard are more conservative, it is expected that in the sizing done by this standard a larger area of compressed concrete will be needed to balance the section.

During the numerical modeling in the ANSYS software, some concepts were observed that led to questions about the design hypotheses defined from the analytical formulations.

It is important to mention that the ANSYS modeling is done in a non-linear, physical and geometric, way. The concrete material is represented by the element SOLID 65, associated with the constitutive equation defined in WILLAM and WARNKE [10], p.174. This material law has the objective of representing the physical behavior of the concrete, establishing a criterion of resistance to the

same. This criterion takes into account the maximum tensile stress established in the standards, in this case by NBR6118: 2014 [2], which is defined by $0,3.\sqrt[3]{f_{ck}^2}$. In addition, it takes into account the maximum compression stress, defined by $0,85.f_{cd}$. This implies that when the tensile or compressive stress of the concrete reaches the maximum limit, there will be a rupture of the material.

It is important to mention that the ANSYS program is based on the formulation of the Finite Element Method and therefore can not achieve convergence in its analyzes when the stiffness matrix is not enough to support the efforts. In this case, the loss of strength of the stiffness matrix is associated with cracking (traction) and crushing of the concrete (compression). This is one of the limitations of the numerical model proposed in this paper.

In the design of the reinforcement in the Ultimate Limit State (ELU) predicted by NBR6118: 2014 [2], it is understood that the reinforcement is in flow, reaching its maximum tensions with all its resistant capacity, completely neglecting the resistance of the concrete in the traction zone. However, by the numerical formulation, part of the tensile stress is absorbed by the concrete, so that the tensile strength in the reinforcement is relieved.

The Brazilian standard does not evaluate the element in a global way, evaluating only the resistant capacity of the most requested sections. When the armatures reach the calculation stress of 435MPa (in the case of CA50 steel), i.e. when they are in a flow regime, the traction region of the part already has an excessive opening of cracks. This can not be represented in the physical model used in ANSYS, since the program understands that the part went into rupture and, with the excessive cracking of the material, there is no convergence to obtain a numerical solution.

Analytical formulations lead to failure results similar to those expected by numerical modeling, since the loading supported by this modeling is very similar to that used in sizing. This is proven when analyzing the 1st Model, where the structure broke with approximately 67% of the load in the situation where there was no reinforcement. When you run the same model for the initial load (load used when sizing), it has already supported the full load.

The SHELL 181 element represented the behavior of the CFRP well. It is important to mention that the material was defined as orthotropic because its physical properties in the longitudinal direction (in the fiber direction) are different from those defined for the transverse direction.

The values of stresses and deformations in the maximum moment section are different when comparing the analytical theory with numerical modeling. Considering that the analytical formulations are made for the final phase of rupture of the element, it can be said that the numerical model better represents the situation of the structure in use, respecting the maximum openings of cracks and admissible tensile and compression tensions and considering the gradual loss of the resistant capacity of each element in function of the increase of loads.

The analyzed Model 1 ruptures when the tensile stress of the concrete is reached and the crack opening is excessive. As a consequence, there is a reduction of the neutral line and the crushing of the compressed region. In this situation, ANSYS considers that the structure collapses. Differently, NBR6118: 2014 [2] presents in the analytical formulation the imposition that the tensile strength of the concrete should be neglected. It is known, however, that the con-

crete in this region works until its tensile strength limit is reached, at which point the part begins to crack. What NBR6118: 2014 [2] does is to control the opening limit of these cracks from a Service State Limit analysis.

6.   Conclusions

According to the sizing made, with the computational analysis and the discussions about the results, it is possible to reach some conclusions regarding the reinforcement dimensioning of reinforced concrete beams with CFRP, as shown below.

As expected, the sizing done from the requirements of ACI 440.2R: 2008 [1] provides results different from those obtained from the adapted formulations considering the concepts of NBR 6118: 2014 [2]. In the dimensioning of the flexural reinforcement, the difference between the CFRP areas obtained was largely due to the variation between the coefficients used by both standards. The ACI 440.2R: 2008 [1] uses different resistance factors than those applied by NBR 6118: 2014 [2], so that, according to the American standard, the materials resist more than the Brazilian standard. Thus, the sizing done by the Brazilian standard provides more conservative results, generating necessary reinforcement areas slightly higher than those obtained through the design of the ACI 440.2R: 2018 [1].

For NBR 6118: 2014 [2], the ultimate deformation of the concrete (particularly for concrete of class C20 to C50) has a value of 3.5 ‰ whereas for the ACI 440.2R: 2008 [1] this deformation has a value of 3 ‰. This makes the resistant capacity of the concrete larger by the Brazilian standard, since the tension in the concrete is a function of its deformation. Thus, the depth of the neutral line could be smaller by the Brazilian standard. However, this does not happen, because the influence caused by the safety factors used by both standards is more significant. Since the safety factors of NBR 6118: 2014 [2] are more conservative, the tensile strength of the concrete ends up being lower by this standard. For this reason, the area of compressed concrete must be larger, so that the neutral line consequently has a greater depth.

However, in spite of the previously discussed discrepancies, it is observed that the formulations based on the principles of NBR 6118: 2014 [2] presented satisfactory results due to the proximity of the results found in both standards, both for the initial resistant moment of the concrete part reinforced as to the area of CFRP required for reinforcement. Therefore, it is concluded that the formulations generated from the compilation between the requirements of ACI 440.2R: 2008 [1] and the requirements of NBR 6118: 2014 [2] have generated satisfactory results with respect to the dimensioning of the flexural reinforcement.

The ANSYS program is based on the Finite Element Method formulation and, therefore, can not achieve convergence in its analyzes when the stiffness matrix is not sufficient to support the efforts. In this way, when the tensile or compressive stress of the concrete reaches the determined maximum limit, there will be a rupture of the material. With excessive cracking of the material, there is no convergence to obtain a numerical solution. In this situation, ANSYS considers that the structure collapses.

Thus, it can be concluded that the numerical modeling performed in this work represented well the behavior of the structure, because

the rupture loads were approximately equal to those expected by the analytical formulations. However, evaluating only the maximum moment section, it was observed that the forces and deformations presented by the numerical model are significantly different from those presented by the analytical model.

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