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# Six pile caps reinforced concrete: numerical simulation and design by the strut and tie method

# Blocos de concreto armado sobre seis estacas: simulação numérica e dimensionamento pelo método de bielas e tirantes



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# Abstract

This paper presents the structural behavior of six reinforced concrete pile caps in rectangular arrangement, considering the ground deformability of pile support, different concrete strengths with square or rectangular cross section of column under central load. For this purpose, the analysis emphasizes a strut and tie method design and a three-dimensional numerical using the finite element method. So, the stress flow configuration and the formation of struts were seen in perspective. How much deformable is the ground, more uniform are the reactions distribution observed between the piles. The column cross section influenced the configuration of the connecting struts. The concrete strength variation had more influence in the pile caps strength than the stiffness. The analytical method has shown compatibility which obtained from the numerical simulation results.

Keywords: reinforced concrete; six pile caps; strut and tie method.

# Resumo

Este artigo analisa o comportamento estrutural de blocos de concreto armado sobre seis estacas dispostas em arranjo retangular. Considerou--se a deformabilidade do solo de apoio das estacas, diferentes resistências para o concreto e pilares com seções transversais quadradas e retangulares, solicitados por força centrada. O dimensionamento foi feito por um modelo de bielas e tirantes. Realizou-se análise numérica tridimensional por meio do método dos elementos finitos. A configuração do fluxo de tensões e a formação das bielas foram analisadas em perspectiva. Observou-se que quanto mais deformável for o solo, mais uniformes são as distribuições das reações entre as estacas. A seção transversal do pilar influenciou na configuração das bielas. A variação da resistência do concreto teve maior influência na resistência dos blocos do que na rigidez. O método analítico utilizado apresentou compatibilidade com os resultados obtidos na simulação numérica.

Palavras-chave: concreto armado; blocos sobre seis estacas; método de bielas e tirantes.

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#### 1. Introdution

Pile caps are structural foundation elements that transfer the superstructure load to the piles. Their type depends on the load on the column base, the geotechnical capacity of the ground and the available building conditions and pile strength. For pile caps containing three or four piles, it is common to adopt triangle arrangement or square arrangement for piles, respectively. However, in pile caps with many piles, the piles are equally spaced to create a rectangular base, generating situations in which the distance between the piles to the axis column is not the same.

Pile caps are special elements because their structural behavior does not follow hypothesis that sections remains plane after the structure deformation. Although they are difficult to analyze while on operation, their fundamental for the superstructure security, so it is important to know their real behavior. In situations in which the piles are not equally spaced in relation to the column axis, as the pile caps type analyzed here, the structural behavior is more complex and little known. The pile reactions may not have uniform values, because they depend on the pile cap stiffness, the ground and piles deformability.

The first researches in the area focused on experimental analysis and were crucial to the development of the first analytical methods, such as Blévot & Frémy [1] study. However, even with the increase of the years, most researches focus analysis of pile caps with a few number of piles, as the Delalibera & Giongo [2] and Miguel & Giongo [3], which analyzed pile caps containing on two and three piles, respectively. Only few studies have reported on experimental analyses of pile caps with more than four piles, focusing on piles not equally spaced from the column. Among such studies we can highlight that of Adebar et al. [4], who observed, in four and six pile caps, that reactions distribution between the piles show no uniform values, because the nearest piles from the columns receiving higher load than the other ones. However, the authors did not consider the ground deformability for the piles support.

With powerful computers and the Finite Element Method is possible to analyze models by numerical simulation, introducing more



complex situations, such as ground deformability for the piles support that is a difficult situation to do experimentally.

Ramos & Giongo [5] analyzed pile caps over ten piles by numerical simulation and found that the piles closer to the column were the most loaded ones, even considering the ground deformability for piles support. They also observed that methods that consider flexural behavior, with shear force and bending moment design, in reference sections, were not compatible with the pile caps behavior, nevertheless they did not present an appropriate model for design. The analytical methods for pile caps design available in the literature follow basically two different ways. The first and more accepted in the technical environment is the strut and tie method, which represents the stresses flow idealized by the truss model. The internal structure consists of compressed and tensile bars, which are the strut and ties, interconnected by nodes. This method has become more employed after Blévot & Frémy [1] research, but regarding pile caps with many piles, the literature lacks studies that show the struts configuration and criteria to define strength of struts. Therefore, it is still common to use a second way to design those types of pile caps. It consists in associating the behavior of the pile caps with the bending theory of beams. Although studies have shown that this option is not so compatible with pile caps structural behavior. Analytical methods based on this principle are still used because they are practical and easy to understand. As the method presented in Bulletin number 73 of the CEB- FIP [6] that consists of computation of bending moment and shear force in reference sections.

ABNT NBR 6118:2007 [7] classifies blocks as rigid or flexible and considers the pile caps special elements, which are characterized by a behavior that does not follow the hypothesis that the cross sections remain planes after structure become deformed because they are not long enough to dissipate located disturbances. For a pile caps design, the Brazilian standard recommends the strut and tie method as the best option to represent the stress distribution, more appropriately. Despite the recommendation, the Brazilian regulation does not provide a clear guide for pile caps design. Brazilian technical lacks researches about pile caps, especially about pile caps with many piles and when piles are not equally spaced to the column axis.

This paper reports a structural behavior analysis of pile caps congaing six piles, arranged in two rows of three piles. Presents an analytical method based on the strut and tie method and evaluates the parameters that influences structural behavior of this type of pile caps though the finite element method.

### 2. Pile caps design by analytical models

For design of the six pile caps Andrade [8] recommendations were considered, as it follows similar criteria to those proposed by Blévot & Frémy [1] and Machado [9]. However, Andrade [8] indicate parameters for the pile caps with any number of piles, different arrangements for the piles and square or rectangular cross section for the column. According to the author, in general the cross section columns are not square, but rectangular and very elongated. For very elongated sections it is more correct to consider the struts position near the column base, which must be determined in accordance with the engineer's analysis in each specific situation. The struts positions scheme near the column is shown in Figure [1].



The design was based on the strut and tie model shown in Figure [2] for the six pile caps. Was computed the required reinforcement and checked the compressive stresses in the ends of the struts near the column and piles. Andrade [8] recommends that the angles between struts and ties should be higher than 40° and less than 55°.

The concrete stresses for the struts are computed by equations [1] and [2], always considering the angle for the less inclined strut. Limit compressive stress in struts near the column:

$$\sigma_{cb,p} = \frac{F_{sd}}{A_p \cdot sen^2 \theta} \le \alpha_p \cdot 0.85 \cdot f_{cd}$$
(1)

Limit compressive stress in struts near the piles:

$$\sigma_{cb,e} = \frac{F_{sd}}{6 \cdot A_e \cdot sen^2 \theta} \le \alpha_e \cdot 0.85 \cdot f_{cd}$$
<sup>(2)</sup>

where:

- $A_e$  Pile cross-sectional area;
- $A_p$  Column cross-sectional area;
- $F_{sd}^{-}$  Normal force from the column;  $f_{sd}^{sd}$  Concrete compression
- $f_{cd}^{sa}$  Concrete compressive strength;  $oldsymbol{ heta}$  Inclination strut angle;
- $\alpha_p$  Adjust coefficient equal 2.6 (indicated by Andrade [8]);
- $\alpha_{e}^{\prime}$  Adjust coefficient equal 1.0 (indicated by Andrade [8]);
- The areas of reinforcements are calculated by the equation [3].



where:  $f_{vd}$  - Steel yielding strength;  $R_{\rm ad}$  - Tensile force in the tie.

#### Properties of the pile caps analyzed 3.

The structural behavior of the six pile caps was analyzed considering the following parameters variation: column cross section, ground deformability for the piles support and compression strength of the concrete.

The geometric parameters were defined based on an example of a building project, just to set the magnitude of the pile cap dimensions and the adjacent column cross-sectional area. Piles with 60 cm of diameter (continuous helix piles) were considered. They have spaced each other by a distance equal a three times their diameter. The distance between the tangent plan of the outer pile to the pile cap ends was equal to 30 cm. The pile caps plan dimensions are shown in Figure [3].

Three different column cross sections shape were considered in the analysis: square  $(b_p = a_p)$ ; rectangular slightly elongated  $(b_p = 4 \cdot a_p)$ , rectangular very elongated  $(b_p = 8 \cdot a_p)$ . Furthermore, the cross-sectional area was kept constant and the magnitude of the value was defined based on the pile caps taken as examples, as previously mentioned. Table [1] shows the cross sections of the dimensions for the columns.

The height was defined considering that the angle between the strut and tie relative to the farthest pile to column axes is 40°. Each strut is located at the midpoint of each portion columns area, as shown in Figure [1]. Nonetheless, in cases in which there was a variation in the column cross section, the struts became different of 40°, as shown in Table [1]. For all pile caps, the heights obtained satisfied the rigid criteria indicated in Bulletin number 73 of the CEB- FIP [6] and ABNT NBR 6118:2007 [7]. The piles were considered 10 cm embedded in pile cap. Reinforcement with 25 mm diameter was considered and the lower face of the reinforcement were placed 2 cm above the top face of the pile.

Pile caps with tree concrete strengths (25, 30 and 35 MPa) were analyzed. The yield strength for the reinforcement was considered equal to 500 MPa.



Table 1 - Parameters evaluated on six pile caps								
Pile cap	b <sub>p</sub> (cm)	a <sub>,</sub> (cm)	h² (cm)	f <sub>ck</sub> (MPa)	k (kN/mm)	θ (degree)		
B-4-30-600 <sup>1</sup>	143	35	145	30	600	40		
B-4-25-600	143	35	145	25	600	40		
B-4-35-600	144	36	145	35	600	40		
B-4-30-rig	143	35	145	30	Rigid support	40		
B-4-30-300	143	35	145	30	300	40		
B-4-30-900	143	35	145	30	900	40		
B-8-30-600	200	25	145	30	600	43		
B-1-30-600	71	71	145	30	600	37		

<sup>1</sup> B pile cap or Bloco in Portuguese, 4 relation between the lowest side ( $a_p$ ) and biggest side ( $b_p$ ) of columns cross section, 30 concrete strength, 600 spring coefficient; <sup>2</sup> Pile cap height.

The ground deformability was considered by elastic springs, located in piles base. Four cases were examined: a rigid pile support, considered the most unfavorable for the pile cap, and three other situations that considered piles supported by elastic springs with the following coefficients: 300 kN/mm, 600 kN/mm and 900 kN/ mm. For more information about the spring coefficients obtained considering the piles settlement, consult Oliveira [14].

Eight different pile caps were obtained with the parameters variation mentioned above. The first pile cap (Table [1]) was considered the reference pile cap. The remaining ones were defined by parameters variation from the reference pile cap and are bolded in Table [1].

The results of the pile caps design are provided in Table [2]. The pile caps were designed always considering the uniform distribution of the piles reactions, independently of the ground deformability. Figure [4] shows positions for the main reinforcements computed in Table [2].

The reinforcements were detailed as recommended by ABNT NBR 6118:2007 [7], which indicates that tensile stresses are concentrated mainly on connecting lines between piles, in a band of width equal to 1.2 times the pile diameter.

There was neither anchoring check for reinforcements nor hooks at their ends, because the numerical finite element model considers perfect adhesion between the reinforcement and concrete. For the column reinforcement, was adopted area equal to 3% of the columns cross section area and reinforcements with diameter equal to 25 mm distributed on the column perimeter. Stirrups of 8 mm diameter and spaced 20 cm each other were also placed.

#### 4. Aspects of finite element numerical simulation

The finite element models were simulated using DIANA software [10]. To simulate the physical nonlinearity of the concrete structural behavior, the total strain model of the smeared cracking was considered. It treats the concrete as a continuous material and retains the original mesh discretization even with the appearing of crack. The parabolic model was considered for the concrete compression behavior and for tensile behavior was used the exponential model, available in DIANA software [10]. To account the beneficial effect of lateral confinement and a strength reduction, because of lateral cracking, was used the model proposed by Vecchio and Collins [11] and available in DIANA software [10]. The inelastic process zone was defined by the length of the cracks band, which was calculated from the cubic root of the

Table 2 – Results of pile caps design									
Pile cap	F <sub>teo</sub> <sup>1</sup> (kN)	$A_{s,teo}^2$ (cm <sup>2</sup> )	Φ³ (mm)	n⁴	A <sub>s,effe</sub> <sup>5</sup> (cm <sup>2</sup> )				
B-4-30-600	16159	x° 54,66 y° 33,56	25 25	12 7	58,90 34,36				
B-4-25-600	13466	x 45,55 v 27,96	25 25	10 6	49,09 29,45				
B-4-35-600	18852	x 63,76 y 39,15	25 25	13 8	63,81 39,27				
B-4-30-rig B-4-30-300 B-4-30-900	16159	x 54,66 y 33,56	25 25	12 7	58,90 34,36				
B-8-30-600	18005	x 52,14 v 38,53	25 25	11 8	54,00 39,27				
B-1-30-600	14342	x 57,33 y 26,49	25 25	12 6	58,90 29,45				

<sup>1</sup> Analytical pile cap strength; <sup>2</sup> Analytical reinforcement; <sup>3</sup> Diameters of reinforcements; <sup>4</sup> Number of reinforcements; <sup>5</sup> Steel reinforcements area effectively considered; <sup>6</sup> Positions of reinforcements shown in Figure 4.



finite element volume. The elasticity modulus, the tensile strength, the tensile fracture energy and poisson coefficient, were adopted considering CEB-FIP Model Code 1990 [12] and they are show in Table [3]. The compressive fracture energy was considered equal to 50 times the tensile fracture energy, as shown in Feenstra and Borst [13].

The linear elastic model was considered for the simulation for the column and the piles concrete. With this consideration, just the pile cap will reach the ruin.

Von Mises elastoplastic model was considered for steel, with elasticity adopted equal 210 GPa as shown in ABNT NBR 6118:2007 [7], and the Poisson's ratio was 0.3.

The solid finite element CHX60 available in DIANA software [10] was used to represents the concrete. This element has quadratic interpolation to compute the displacements. The finite element SP-2TR was used to represent springs for piles supporting available in DIANA software [10], acting only with vertical translation and linear elastic behavior between strength and deformation. Only 1/4 of the pile caps were simulated considering their symmetry and the mapped mesh was used with maximum dimensions of 15 cm for solid elements, as shown in Figure [5]. The piles were simulated with square cross section, keeping an area of 2827.4 cm<sup>2</sup>.

The reinforcement was simulated functioning only as stiffeners for the solid finite elements. Consequently, the perfect adhesion between the



#### Figure 5 - Finite elements mesh for concrete



reinforcement and the surrounding concrete was considered. Figure [6] shows the modeling of the pile cap and the column reinforcements. Finally, the loading was applied through displacement steps, considering just centered normal force on column top. The strategy used for solving nonlinear systems equations was the "Newton-Raphson Regular" method with convergence criterion for energy and tolerance equal to 0.01.









#### 5. Results

Figure [7] shows a graph that compares the total load percentage acting on each pile considering different values for coefficient of the springs. For initial loading stages, the reactions remained almost uniform in the pile cap supported on the springs, independently of spring coefficient. However, for rigid support, the reactions were significantly different since the beginning of the load. With load increase, the piles, whose struts are more inclined, become more loaded than the other ones, whose struts are more slaughtered. Therefore, the higher the value of the spring coefficient, the worse the reactions distribution between piles.

Analyzing the graph shown in Figure [8], the obtained pile caps strength was similar, but the pile cap with spring coefficient k = 300 kN/mm showing a small strength increase. This increase can be attributed to a better reactions distribution among piles, provided by springs with k = 300 kN/mm, which enable a significantly contribution of all piles to the pile cap strength. However, for other pile caps, this contribution did not occur in same proportion.

Figure [8] shows that for more deformable ground, the pile cap

have a more continuous increase in load, with a large settlement until the ruin. The higher the spring coefficient, the lower the settlement in ruin. The lower the ground deformability, the more discontinuous the pile cap behavior until ruin, which may have been caused by cracking and localized fractures. This may be occurred because the pile caps have large capacity to redistribute internal stresses and they are very statically indeterminate.

The coefficient k = 300 kN/mm enable the best reactions distribution for the pile cap, with all piles contributing with significant strength until the pile cap ruin. Thus, is expected that for coefficient lower than k = 300 kN/mm, the reactions distribution tend to be more uniform.

Figure [9] shows that for higher ground deformability better is the tensile stresses distribution in reinforcement, with similar behavior to that considerate in strut and tie model. With the higher spring coefficient higher is the tensile stress concentration in central reinforcement, and this concentration was even more critical where the piles support is rigid, with the central reinforcement reaching the yield stress while tensile stresses in the other reinforcements keeping relatively small values.

Figure [10] shows pile caps near a ruin stage. For pile caps sup-





ported on springs, better are the distribution of the compressive stress forming struts for all piles. This behavior is similar to that supposed by the strut and tie model. Therefore, the lower the spring coefficient, the better the stress distribution. However, for rigid support occurred concentration of the compressive stress predominantly for the two piles closest to column.

Regarding column cross section variation, the column cross section elongation improved the piles reactions distribution, contributing to a more uniform distribution in comparison with the square cross section, as shown in the graph of Figure [11]. The elongated cross section provided a continuous behavior, while square column cross section there was a series of brusquely redistributions, as the load increased.

The variation in the shape of the column cross section does not



change the pile caps strength significantly, displaying the same order of magnitude for resistance, as shown in Figure [12]. But for highly elongated columns cross section there was a small increase in strength. This can be associated to a better distribution of compressed stresses for the farthest piles to the column, forming more inclined struts.

The elongation of the column cross section alters the tensile stress distribution on ties reinforcements, as can be seen by comparing Figures [13a], [13b] and [9b]. For the pile cap with square section, the tensile stresses concentrated on the reinforcement between the two middle piles. On the other hand, with the column cross section elongation tensile stresses was better distributed in all reinforcements.

The diagrams of surfaces with same stress (see Figures [14] through [16]) show that for all situations, the flow stress has





Figure 15 – Principal compressive stress. Surface where acts a stress of -6MPa on pile cap B-4-30-600: (a) entire pile cap and (b) vertical section. Loading stage with 13.3 mm of displacement applied



Figure 16 – Principal compressive stress. Surface where acts a stress of -6MPa on pile cap B-8-30-600: (a) entire pile cap (b) vertical section. Loading stage with 10.4 mm of displacement applied







adapted to the column cross section shape. The compressive stresses trajectory accompanied the column cross section elongation, enabling the formation of more inclined struts relative to the horizontal plan, starting near the ends of the column cross section. Conversely, for square columns cross section struts were less inclined to the horizontal plan. Furthermore, the columns cross section elongation enable a reduction in the concentration of the compressive stresses related to piles near the column and a redistribution of these stresses to other piles more distant to the column. Figure [17] shows that increasing in strength of concrete did not improve the reactions distribution, significantly, for the pile caps. The influences in reactions distributions caused by ground deformability were more significant than the concrete strength variation. The graph in Figure [18] shows the increase of concrete strength resulted in a pile cap strength increase. The initial portion of force versus displacement curves practically coincided in the early stages, diverging only for the lasts stages of load. Therewith it is clear that the variation of concrete strength had negligible influence in the stiffness of this pile caps here analyzed.

In relation to the analytical method used, Table [4] shows that the criteria recommended by Andrade [8] for the strut and tie model enable a good approximation for the strength than those obtained by numerical models, for all pile caps.

The compatibility between the results obtained by analytical method and by the numerical models was not only relative to strength of the pile caps. Figure [15] shows that the configuration of the compressive stresses are compatible to the struts configuration indicated by Andrade [8]. The struts origins are in the contact region between column and pile cap and connect to the top of piles. Several pile caps revealed that struts have not converged to a single point at columns axis, they change with the variation of the shape of the column cross section. This fact confirms the analytical method hypothesis that considers the origins points of each strut in centers of portions of areas of the column cross section. This can be



observed when we compare Figure [1] with Figure [19]. Therefore, this shows the importance of considering the real columns cross section for pile caps design.

The analytical method had good approximation for the strength of the pile caps relative to the increase in concrete strength. Moreover, this method proved to be easy to use, enabling the determination of the truss model according to any pile caps arrangement and considers the real columns cross section and pile caps. However, an important aspect should be considered that is relative to struts angle. This type of pile cap do not have symmetrical arrangements for the piles and because of it, struts will have different angles. So, is better to consider the less inclined strut having 40°. Thereby, maybe the angles of the more inclined struts can be below the upper limit of 55°.

Table 4 – Comparison between numerical and analytical models								
Pile cap	F <sub>teo</sub> (kN)	F <sub>num</sub> <sup>1</sup> (kN)	F <sub>num</sub> /F <sub>teo</sub> (kN)					
B-4-30-600	16159	16744	1,036					
B-4-25-600	13466	14929	1,109					
B-4-35-600	18852	20865	1,107					
B-4-30-rig	16159	17247	1,067					
B-4-30-300	16159	18775	1,162					
B-4-30-900	16159	17019	1,053					
B-8-30-600	18005	19195	1,066					
B-1-30-600	14342	16183	1,128					
<sup>1</sup> Pile caps strength obtained by numerical simulation.								

Oliveira [14] analyzed the pile caps structural behavior with six and also five pile arrangement. This author found compatibility of results obtained by Andrade [8] and by numerical models and analyzed pile caps with different column cross sections, different heights and irregular piles arrangement.

### 6. Conclusions

The results show that the ground deformability for piles support, represented by elastic springs, significantly influences the structural behavior of the pile caps here analyzed, especially regarding the distribution of pile reactions. This influence was mainly observed in compressive stresses configuration and tensile stress distribution on reinforcement. These results sustain the hypothesis commonly adopted for design that uniform distribution for pile reactions may not be appropriate for situations where the ground is very rigid. Therefore, is better to do soil structure interaction study to check the hypothesis that considers uniform reactions is a reasonable approximation. Otherwise, should be necessary design the pile cap considering non uniform distribution reactions for the piles.

In any case, ground deformability did not affect the pile caps strength significantly because of the ability of these pile caps to redistribute internal stresses. Although the analytical method showed good strength approximation for all cases, the distribution pile reactions should be analyzed for each specific case.

The concrete strength increase enabled a pile cap strength gain, which is consistent with the hypothesis of analytical methods. However, varying the concrete strength did not alter the pile caps stiffness significantly.

Concerning the column cross section variation, the struts configuration were modified by the cross section elongation. Furthermore, consider the struts origins in the center of portions columns cross sections relative to each pile area is a reasonable approximation. Andrade [8] criteria enabled a pile caps strength prediction with good



approximation of the numerical simulation results. There was also compatibility between the strut and tie model compressive stress configuration observed and the tensile stresses distribution in the reinforcements.

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