REVISTA MATÉRIA V.27 N.02
ISSN 1517-7076 artigos e13206, 2022

A note on the multiscale volume changes of auxetic metamaterials assessed via digital image correlation

Julia Mentges[1], Thiago André Carniel[1]

[1]Polytechnic School, Community University of Chapecó Region, Servidão Anjo da Guarda, nº 295-D, Bairro Efapi, CEP: 89809-900, Chapecó, SC, Brazil.
e-mail: julia.mentges@unochapeco.edu.br, thiago.carniel@unochapeco.edu.br

ABSTRACT

The present manuscript approaches the mechanical interpretation about the volume changes of auxetic metamaterials within a multiscale perspective. The kinematic fields resulting from the mechanical tests were assessed via digital image correlation (DIC). The multiscale mechanical characterization is grounded within sound homogenization principles based on representative volume elements (RVE) at large deformations. The main achieved results concern in highlight that even though the Poisson's ratio still widely used to characterize the mechanical behavior of auxetic metamaterials, it does not represent a suitable variable to quantify the multiscale volume changes, mainly when finite deformations take place. In this regard, a proper homogenized kinematical quantity is used in this work. Moreover, statistical tools are employed aiming at a better understanding to what extent the multiscale kinematics influence the volume changes and the overall auxetic behavior of the material.

Keywords: Auxetic metamaterial, Poisson, Multiscale, Homogenization, Digital image correlation.

1. INTRODUCTION

With the advance of novel manufacturing technologies of lattice-like structures at different length scales [1], researches related to the so-called mechanical metamaterials spread out in the literature under a broad range of applications [2, 3]. Within the several classes of mechanical metamaterials available now-a-days, the ones which present negative Poisson's ratio (auxetic behavior) have been gained attention due to their unusual mechanical properties [4, 5]. Concerning to auxetic porous materials for biomedical purposes, the understanding to what extent the microstructural geometry responds to the external (macroscopic) loading and affects the kinematic fields at the micro-level, becomes a relevant information to the material analysis and design. For example, taking into account hydrated porous bio-structures - *e.g.*, tissue engineering scaffolds - the kinematics associated to volume changes may strongly interacts with the fluid flow behavior at the cellular level, affecting mechanotransduction mechanisms [6].

Aiming at further researches on the aforementioned topic, it has been verified in literature that the Poisson's ratio has been widely used to characterize the mechanical behavior of auxetic metamaterials. Nevertheless, its physical interpretation, generally retrieved from the linear elasticity theory [7], cannot be directly extended to finite deformation regimes. Moreover, at a multiscale perspective, if one intends to quantify the volume changes of the material and/or to investigate its relation with micro-mechanisms, a proper variable and procedure should be employed in this regard.

Motivated by these facts, the present manuscript provides a simple procedure for the experimental assessment and the mechanical interpretation about the volume changes of auxetic metamaterials within a multiscale perspective. As a representative example, unconfined compression tests were performed in an auxetic structure made of a thermoplastic polymer. The kinematic fields resulting from the mechanical tests were assessed via digital image correlation (DIC). The multiscale mechanical interpretations are grounded within sound homogenization principles based on representative volume elements (RVE) at large deformations [8–11]. In order to compare the information retrieved from the RVE-based approach, four classical strain measures of the Seth-Hill strain family [12, 13] were used to compute the apparent Poisson's ratio of the metamaterial. Moreover, statistical tools were employed aiming at a better understanding to what extent the multiscale kinematics influence in the overall auxetic behavior.

Corresponding Author: Thiago André Carniel   Received on: 16/03/2021   Accepted on: 08/02/2022

10.1590/S1517-707620220002.1306

This work should be understood as a first approach to investigate, experimentally, the volume changes of auxetic metamaterials and their relations with the multiscale fields. Moreover, the results retrieved from such approach intend to provide important data to further investigations based on computations homogenization techniques [14–16] and metamaterial design via topology optimization [17].

2. MATERIALS AND METHODS

2.1 Theoretical background

2.1.1 The Poisson's ratio and the volume changes

Within the *Continuum Mechanics* theory, the kinematics of deformable bodies is ruled by the deformation gradient $\mathbf{F} = \partial \boldsymbol{x}/\partial \mathbf{X}$, which performs a mapping between the referential $\Omega_{\mathbf{X}}$ and the spatial $\Omega_{\boldsymbol{x}}$ frames. By applying the polar decomposition $\mathbf{F} = \mathbf{RU} = \mathbf{VR}$, one obtains a pure rotation $\mathbf{R}$ and pure stretches $\mathbf{U}$ and $\mathbf{V}$. The latter are called the right and the left stretch tensors, representing referential and spatial quantities, respectively. Based on these stretches, a generalized strain measure (Seth-Hill strain family) in the referential description is defined as

$$\mathbf{E}_{(n)} = \begin{cases} \dfrac{1}{n}(\mathbf{U}^n - \mathbf{I}) & n \neq 0 \\ \ln(\mathbf{U}) & n = 0 \end{cases}, \tag{1}$$

where $\mathbf{I}$ is the second order identity tensor and n is a real number. Analogously, a spatial counterpart of this generalized strain measure can also be defined [12, 13]. Classical strain measures are derived from (1) *e.g.*: the Euler-Almansi strain $\mathbf{E}_{(-2)}$, the Biot (engineering) strain $\mathbf{E}_{(1)}$, the Green-Lagrange strain $\mathbf{E}_{(2)}$ and the Hencky (logarithmic) strain $\mathbf{E}_{(0)}$.

Representatives of the previously mentioned strain family have been widely used to investigate the Poisson's ratio of metamaterials. In this regard, it is important to state that, independently of the material response, the *Poisson's ratio* is a strictly kinematic relation defined by the quotient $-\varepsilon_t/\varepsilon_a$, where ε_t is a transverse normal strain component measured perpendicularly to an axial normal one ε_a. Based on this, and in view of Equation (1), one can note that the functional form of the Poisson's ratio strongly depends on the strain measure chosen to represent the strain.

The kinematic quantity regarding changes in volume is represented by the so-called *volumetric Jacobian*, which is formally defined as:

$$J = \frac{\mathrm{d}v}{\mathrm{d}V} = \det(\mathbf{F}) > 0. \tag{2}$$

This scalar variable represents the local ratio between the spatial volume element v and the referential one V.

2.1.2 Homogenization principles

In the present manuscript, the multiscale mechanical interpretations are grounded within sound homogenization principles based on representative volume elements (RVE) of the material and formulated under finite deformations. Further details on this theory can be found elsewhere [8-11].

The main kinematic assumption that rules this RVE-based theory is called kinematic admissibility. Briefly, the displacement $\mathbf{u}(\mathbf{X})$ and the deformation gradient $\mathbf{F}(\mathbf{X})$, at a macroscopic point $\mathbf{X}$, result from the volume average (homogenization) of their respective microscopic (RVE) fields $\mathbf{u}_\mu(\mathbf{X})$ and $\mathbf{F}_\mu(\mathbf{X})$, *i.e.*:

$$\mathbf{u} = \frac{1}{V_\mu}\int_{\Omega_{\mathbf{Y}}} \mathbf{u}_\mu \, \mathrm{d}V_\mu\,, \qquad \mathbf{F} = \frac{1}{V_\mu}\int_{\Omega_{\mathbf{Y}}} \mathbf{F}_\mu \, \mathrm{d}V_\mu, \tag{3}$$

where V_μ is the volume of the RVE in the referential domain $\Omega_{\mathbf{Y}}$ (Figure 1b). The mechanical equilibrium and the micro-to-macro transition of the stress tensor is established by the Hill-Mandel principle of macrohomogeneity [9, 10]. However, the focus herein concerns the experimental assessment and investigation of multiscale kinematic quantities only.

In view of equations (2) and (3), the macroscopic changes in volume can be assessed by:

$$J = \det(\mathbf{F}) = \det\left(\frac{1}{V_\mu}\int_{\Omega_\mathbf{Y}} \mathbf{F}_\mu \, \mathrm{d}V_\mu\right). \tag{4}$$

In particular cases of RVEs with periodic geometry [18], one can show that Equation (4) reduces to

$$J = \frac{1}{V_\mu}\int_{\Omega_\mathbf{Y}} \det(\mathbf{F}_\mu) \, \mathrm{d}V_\mu = \frac{v_\mu}{V_\mu}. \tag{5}$$

where v_μ is the volume of the RVE in the spatial configuration $\Omega_\boldsymbol{y}$ (Figure 1b).

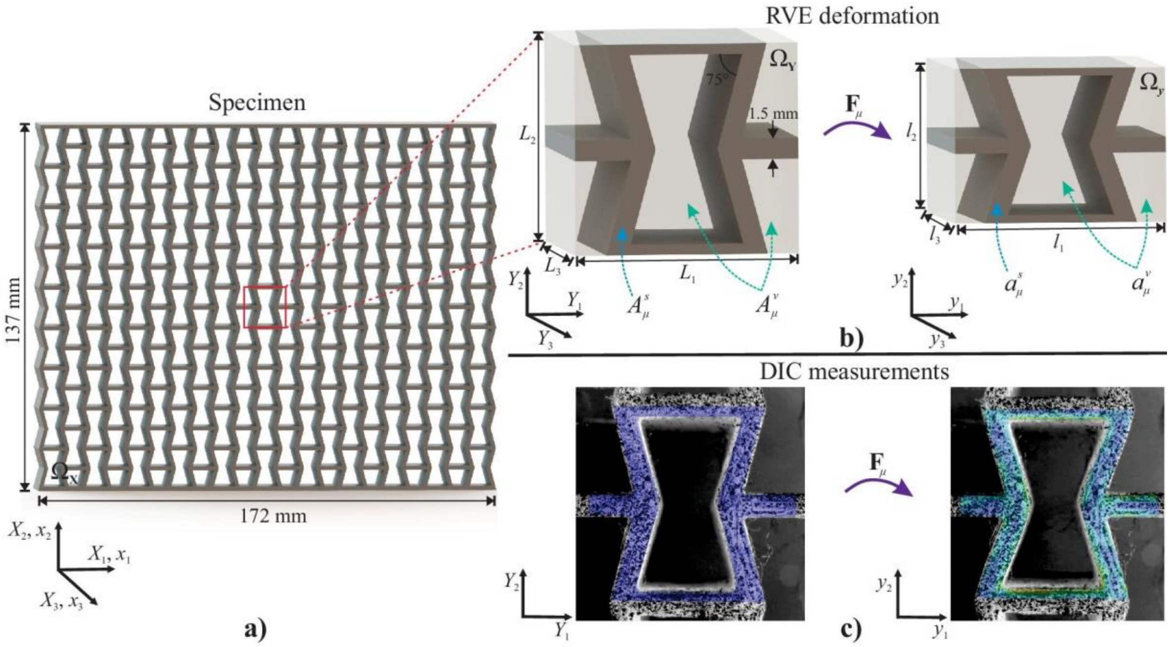

Figure 1: a) The auxetic specimen. b) Illustrative representation of the RVE deformation and the surface areas of solid phase $(\cdot)^s$ and voids $(\cdot)^v$ in the reference and spatial configurations. c) The stochastic speckle pattern on the surface of the RVE and an example of a strain field computed by the DIC software.

2.2 Experimental procedure

2.2.1 The specimen and the RVE

The auxetic metamaterial investigated in the present study is shown in Figure 1a, which is composed of a periodic arrangement of a well-known unit cell. This specimen was made of thermoplastic polyurethane via additive manufacturing with 100% infill and 0.2 mm layer resolution.

The RVE concerns a rectangular volume with the following dimensions in the reference domain: $L_1 = 10$ mm, $L_2 = 15$ mm and $L_3 = 20$ mm (Figure 1b). Note that the overall volume of the RVE is composed of a solid phase and voids.

2.2.2 The testing setup

Monotonic compression tests at 5.0 mm/s were performed in a universal testing machine (AG-X Plus, Shimadzu). In order to mitigate friction issues, a polytetrafluoroethylene film was used in the interface between the metallic compression plate and the specimen. The analyses shown in this manuscript were constrained to 2.6 mm displacement, once, after this value, the specimen underwent buckling.

2.2.3 The DIC features

The deformation of the RVE was recorded by a CMOS sensor camera (monochromatic) with 2 megapixel resolution at 30 frames per second. The kinematic measurements were performed in the DIC software GOM Correlate (ZEISS Group). The stochastic speckle pattern used for the DIC was obtained by spraying a black paint on the surface of the specimen (Figure 1c).

2.2.4 The kinematic measurements

Concerning the multiscale volume changes, the proposed strategy consists of estimating the changes of the RVE volume during the test and obtaining the macroscopic volumetric Jacobian by Equation (5). In view of Figure 1b, one can note that the domain of the RVE is composed of a solid phase and voids. Based on this, the volume of the RVE in the reference and spatial configurations are expressed, respectively, by:

$$V_\mu = \left(A_\mu^s + A_\mu^v\right)L_3, \qquad v_\mu = \left(a_\mu^s + a_\mu^v\right)l_3 \tag{6}$$

where A_μ^s, A_μ^v, a_μ^s and a_μ^v are the surface areas (in the measuring plane) of the solid phase $(\cdot)^s$ and voids $(\cdot)^v$ in the reference and spatial domains, respectively (Figure 1b). The area of the solid phase is directly obtained by the DIC (Figure 1c). On the other hand, the area of voids results from the intersection between the total surface areas, A_μ and a_μ, and the areas of the solid phase, such that: $A_\mu^v = A_\mu \cap A_\mu^s$ and $a_\mu^v = a_\mu \cap a_\mu^s$. Since the total surface area in the reference domain is trivial to compute, *i.e.*, $A_\mu = L_1 L_2$, that of the spatial domain is estimated by $a_\mu \approx l_1 l_2$, where l_1 and l_2 are retrieved by the DIC measurements (Figure 1b).

Since no measurements were performed in the out-of-plane direction y_3, an incompressible (volume preserving) behavior for the solid phase is assumed to estimate l_3, resulting in,

$$V_\mu^s = v_\mu^s \quad \Rightarrow \quad l_3 \approx \frac{A_\mu^s}{a_\mu^s} L_3 \tag{7}$$

The previous mentioned incompressible assumption is a suitable approximation for elastomeric materials. Otherwise, the out-of-plane deformation should be also experimentally measured.

Four apparent Poissons' ratios were estimated based on the in-plane strain measures obtained from Equation (1): the Euler-Almansi strain, the Engineering strain, the Green-Lagrange strain and the Hencky strain.

3. RESULTS AND DISCUSSION

One can see in Figure 2a that the evolutions of the Poisson's ratios show a nonlinear shape, and they considerably differ from one strain measure to other. However, independently of the strain measure chosen to compute the Poisson's ratio, it represents a reasonable variable to investigate, *qualitatively*, the auxetic behavior of metamaterials, *i.e.*, pointing out negative values. On the other hand, the use of the Poisson's ratio to *quantify* the changes in volume is highly questionable in this case. For linear elastic materials under small strains, this ratio coincides with the Poisson's parameter ν of the classical Hooke's model, and it can be used to assess the volume changes of the material under compression, *i.e.*: volume augmentation $0 < \nu < 0.5$, volume reduction $-1 < \nu < 0$ and volume preserving $\nu \approx 0.5$. Nevertheless, when finite strains take place, it is likely that the Poisson's ratio does not provide a constant value ν, loosing its physical meaning as a constitutive parameter and strongly depending on the strain measure. Moreover, if the material is anisotropic (as in the present case), different values of this ratio could be achieved.

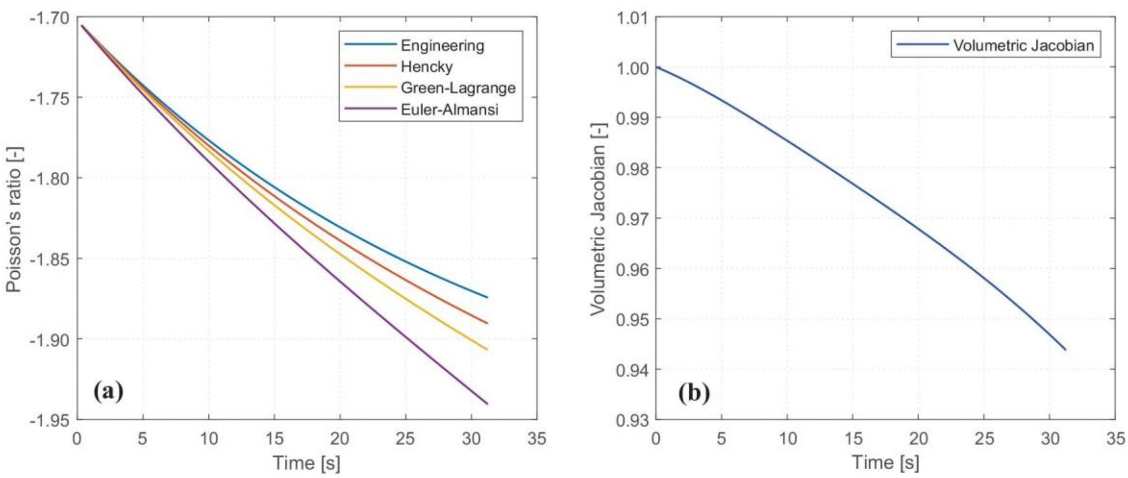

Figure 2: a) Poisson's ratio computed from different strain measures. b) Evolution of the volumetric Jacobian.

In this regard, the volumetric Jacobian (Figure 2b) is a proper kinematic variable to quantify changes in volume, *i.e.*, volume augmentation $J > 1$, volume reduction $J < 1$ and volume preserving $J \approx 1$. In view of the results shown in Figure 2b, one can see that the metamaterial experiences a nonlinear volume reduction in the measured region (RVE site) with a maximum of ~5.5%. Knowing that ~1.9% compression was imposed to the specimen, this volume loss of 5.5% highlights a significant compressible behavior, corroborating with the high negative Poisson's values shown in Figure 2a.

In order to provide further insights about the micro-mechanisms that result in the macroscopic auxetic behavior, kinematic fields provided by the DIC measurements may be investigated in details at the RVE level, which are shown in figures 3 and 4.

Figure 4 displays the equivalent (von Mises) measure of the Hencky strain and the related distributions at five instants of time. Discussion on these results are twofold. Firstly, based on the strain fields depicted on the solid part of the RVE, one can see that the fields present a nearly symmetry pattern on the RVE, with maximum of 10% strain at 31.25s. On the other hand, the distributions highlight that, when time spans, the overall strains are considerably lower than the maximum values observed. Particularly at 31.25s, 75% of the strains on the surface of the RVE are found below 0.03 (see the quartiles in Figure 3). Secondly, a multiscale relation may be established between the percentage of the specimen compression (red vertical lines on the distributions of Figure 3) with the resulting strain distributions on the RVE. One can note that when the specimen is compressed, the medians tend to locate below of the imposed macro compression. Considering, again, the time of 31.25s, more than 50% of the strains on the RVE are found below of the specimen compression. These results emphasize that the macro-deformations induce low strain localization effects on this metamaterial.

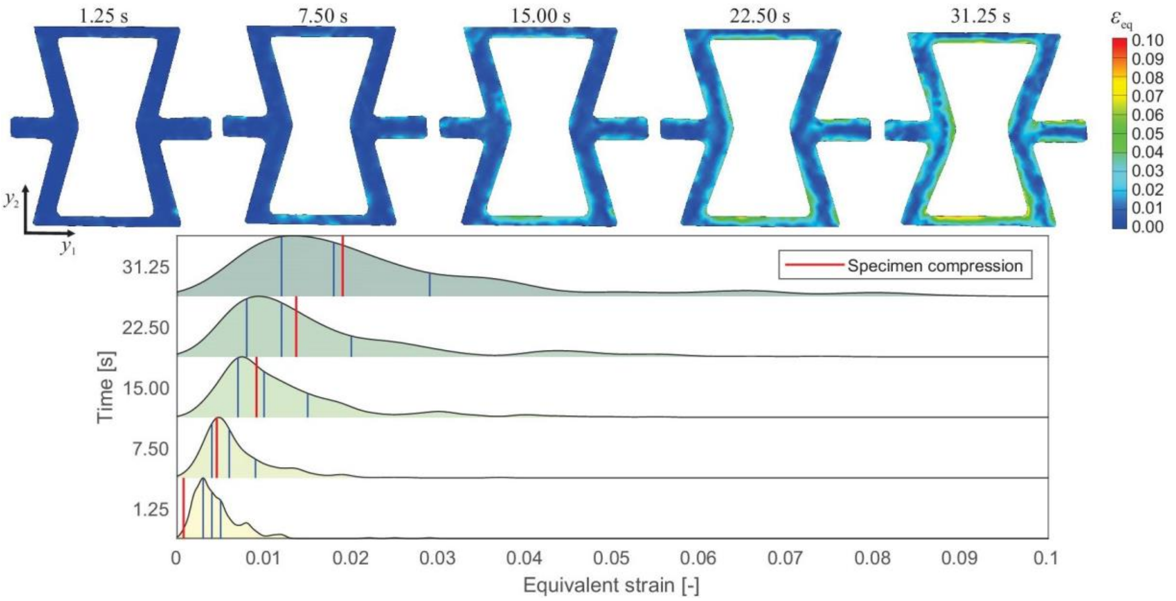

Figure 3: Equivalent (von Mises) measure of the Hencky strain and the related distributions at five instants of time.

In Figure 4, the evolution of the normal (in-plane) components of the Hencky strain tensor are plotted. In addition, these fields are depicted on the solid part of RVE at 31.25s. Based on these strain fields, one can verify that the maximum values previously observed in Figure 4 are ruled from the second normal strain component. Moreover, the distribution of both normal strain components over time (boxplots) are lower in magnitude than the compression values imposed to the specimen, corroborating with the results of Figure 3.

Correlating the aforementioned results with the high compressible behavior quantified by the volumetric Jacobian shown in Figure 2b, becomes clear that, for this particular metamaterial, the observed macroscopic auxetic behavior results from mechanisms at the RVE level ruled by displacements and rotations that induce strain distribution statistically lower than those imposed at the specimen level.

The rationale behind the analyses performed herein can be extended to other metamaterials with different geometries and length scales. Moreover, such data provides important information to validate computational homogenization models to further design metamaterials with particular volumetric behaviors, *e.g.*, via topology optimization techniques [17].

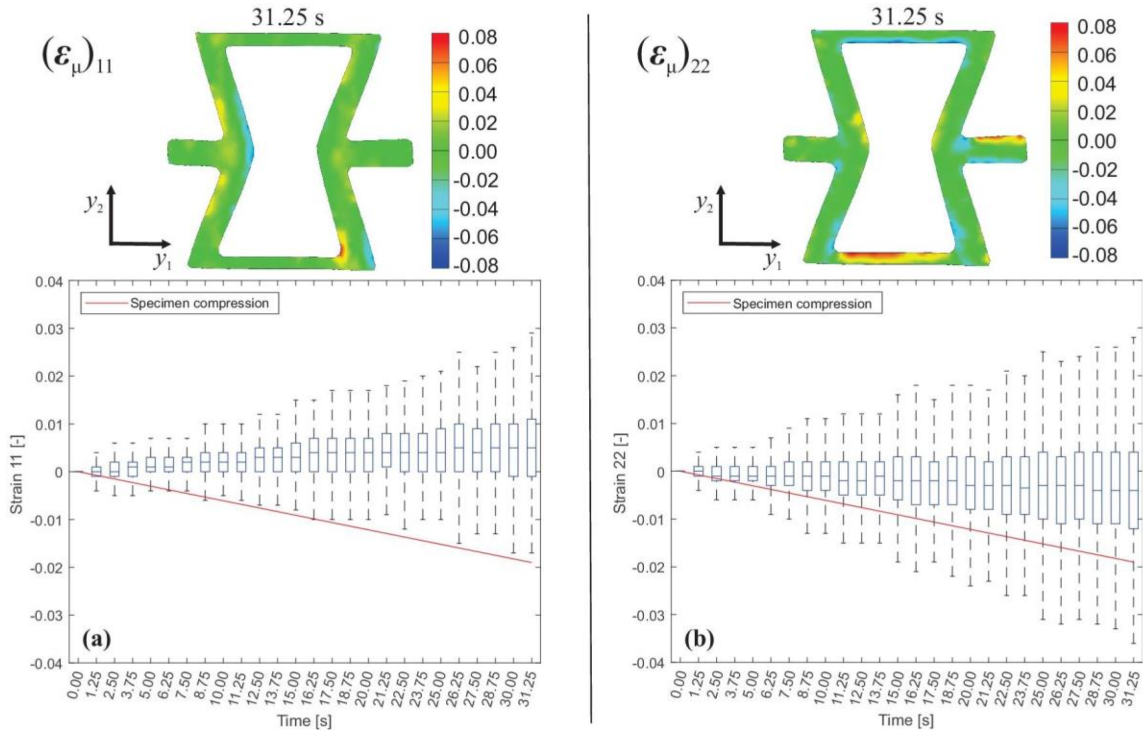

Figure 4: The evolution of the normal (in-plane) components of the Hencky strain field.

4. CONCLUSIONS

A multiscale approach, based on DIC measurements and homogenization principles, was proposed to experimentally assess the volume changes of auxetic metamaterials. The achieved results concern in highlight that, even though the Poisson's ratio still widely used to characterize the mechanical behavior of auxetic metamaterials, it does not represent a suitable variable to quantify the multiscale volume changes, mainly when finite deformations take place. In this regard, the homogenized volumetric Jacobian is a proper variable to quantify the volume changes of such materials. Moreover, in order to provide further insights to what extent the multiscale kinematics influence the volume changes, a statistical-based approach of the strain fields retrieved from the DIC measurements was presented. The rationale behind the proposed approach can be extended to other metamaterials, providing important data to validate computational homogenization models at different applications.

5. ACKNOWLEDGMENTS

The authors would like to thank the Community University of Chapecó Region, Altair Filipini Júnior and Gustavo Thomas Hoehn.

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ORCID

Julia Mentges	https://orcid.org/0000-0003-4312-6941
Thiago André Carniel	https://orcid.org/0000-0002-5040-2556