

# STATE-SPACE APPROACH FOR THE ANALYSIS OF SOIL WATER CONTENT AND TEMPERATURE IN A SUGARCANE CROP

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**ABSTRACT:** The state-space approach is used to describe surface soil water content and temperature behaviour, in a field experiment in which sugarcane is submitted to different management practices. The treatments consisted of harvest trash mulching, bare soil, and burned trash, all three in a ratoon crop, after first cane harvest. One transect of 84 points was sampled, meter by meter, covering all treatments and borders. The state-space approach is described in detail and the results show that soil water contents measured along the transect could successfully be estimated from water content and temperature observations made at the first neighbour.

**Key words:** soil water, soil temperature, state-space, transect

## A ABORDAGEM DE ESTADO-ESPAÇO NA ANÁLISE DO CONTEÚDO DE ÁGUA E TEMPERATURA DO SOLO EM UMA CULTURA DE CANA

**RESUMO:** Para estudar o comportamento do conteúdo de água e da temperatura na camada superficial do solo, a abordagem "state-space" foi empregada em dados obtidos em uma cultura de cana submetida à práticas distintas de manejo. Os tratamentos constaram de cobertura morta com palha (e ponteiros), solo nu e palha queimada, todos no início da primeira soca. As amostragens foram realizadas em uma transeção de 84 pontos, metro a metro, cobrindo todos os tratamentos e bordaduras. A metodologia "state-space" é descrita em detalhe e os resultados mostram que o conteúdo de água no solo pode ser estimado com sucesso a partir de dados de conteúdo de água e de temperatura observados no primeiro vizinho.

**Palavras-chave:** água no solo, temperatura do solo, estado-espaço, transeção

### INTRODUCTION

Several physical, chemical and biological phenomena are observed in a form that leads to a numerical quantification in a sequence, distributed in time, called time series. The simplest form used to define a time series  $Z_t$ ;  $t=1,2,\dots$ , is by understanding  $Z_t$  as a set of discrete observations, obtained in equal time intervals and that present a serial dependence among themselves. This concept, although simple, gives a special emphasis to the "Time Series Analysis" as a well defined area inside Statistics, since independent and identically distributed data are clearly discarded, in contrast

to most statistical models (Souza, 1989). In soil science this concept is also applied for space series, corresponding to a set of discrete observations, obtained in equal (or not) space intervals, along transects or on grids, and that present spatial dependence. The space representation of different states is a form of representing a linear system through a system of two equations: one for an observations vector and another for the evolution of the states. Once a model is represented by states distributed on space, the Kalman (1960) filter can be applied, to obtain predictions and estimatives (Motta & Hotta, 1998). In contrast to the "classical statistics" that presupposes the independence

among observations and disregard their location intra-field, these newly applied methods in soil science, take advantage of the spatial dependence making use of the location of each observation. Several statistical tools, like autocorrelation function, semivariograms and state-space, have been used more recently to evaluate the structure of spatial distributions of soil properties (Wendroth et al., 1997; Hui et al., 1998). According to Bresler et al. (1981), research of the last two decades has focused the study of soil spatial variability with the aim of better understanding the processes that influence the variability of crop production. Nielsen & Alemi (1989) comment that in several reports using classical statistics, observations within and between treatments are not always independent, making the used statistical design inadequate.

Vauclin et al. (1982) studied the variability of soil water and temperature using an autoregressive model of first order. Shumway (1988) states that the state-space analysis is a special kind of autoregressive model. Some of the early applications of the state-space approach are given in Morkoc et al. (1985) and Wendroth et al. (1992). In this report, we present a state-space analysis of sets of soil water content and of soil temperature data, collected in a sugarcane field, submitted to different management treatments. It is intended to contribute for a better understanding of the relation between water and temperature behavior in the soil.

### THEORETICAL ASPECTS

In the state-space analysis, the state of a system, of a variable, or of a set of variables measured at a location  $i$ , is related to the state of the same and other variables, at location  $i-h$ , where  $h=1,2,3,\dots,n$ , called the lag between neighbour observations. This autoregressive model is used for several types of prediction (and forecast) based on a space or time series, to identify coefficients that join these state systems (Wendroth et al., 1997).

The basic equation is called state equation, which for  $h=1$  can be written as:

$$X_i = \phi X_{i-1} + W_i \quad (1)$$

where  $X_i$  is the state vector (of a set of  $p$  variables) at location  $i$ ;  $\phi$  is a  $p \times p$  matrix of state coefficients, which indicate the measure of the regression; and  $W_i$  are noises of the system for  $i=1,2,3,\dots,n$ , assumed to have zero mean, not

correlated and normally distributed. This is the usual structure of a common autoregressive model, in which the coefficients of the matrix  $\phi$  could be calculated by multiple regression, taking  $X_i$  as the dependent variable and  $X_{i-1}$  as independent one. In the state-space model, however, the true state of the variable, or of the state vector, is considered "embedded" in the following observation equation:

$$Y_i = A_i X_i + V_i \quad (2)$$

where the observation vector  $Y_i$  is related to the state vector  $X_i$  by an observation matrix  $A_i$  (unit matrix,  $p \times p$ ) and observation noises  $V_i$ , of zero mean, not correlated and normally distributed. The noises  $W_i$  and  $V_i$  are considered independent of each other, in other words, what is measured does not have to be taken as true, but can be considered as an indirect measurement, reflecting the true state of the variable added to the noise (Wendroth et al, 1997).

Considering the case of this study,  $X_i$  is the soil water content  $\theta_i$  ( $\text{cm}^3.\text{cm}^{-3}$ ) and the soil temperature  $T_i$  ( $^\circ\text{C}$ ), and equation (1) can be written in the following matrix form:

$$\begin{bmatrix} \theta_i \\ T_i \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \times \begin{bmatrix} \theta_{i-1} \\ T_{i-1} \end{bmatrix} + \begin{bmatrix} W_{\theta_i} \\ W_{T_i} \end{bmatrix} \quad (1a)$$

The state coefficients  $\phi_{k,j}$  and noises of equation (1a) are estimated through a recursive procedure given by Shumway & Stoffer (1982). They are optimised using the Kalman (1960) filter, with an iterative algorithm. This filter is a recursive procedure to find optimised estimators for the state vector at position  $i$ . Motta & Hotta (1998) state that this filter is frequently used in engineering (Mine, 1984; Gomes & Mine, 1989; 1991; Alves et al., 1991) since it permits the estimation of the state vector with its constant renewal as new observations are obtained, the actual value of the state vector being of most interest.

Equation (2) can also be written as:

$$\begin{bmatrix} \theta_i^o \\ T_i^o \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} \theta_i \\ T_i \end{bmatrix} + \begin{bmatrix} V_{\theta_i} \\ V_{T_i} \end{bmatrix} \quad (2a)$$

in which  $\theta_i^o$  e  $T_i^o$  are observed values at location  $i$ . Equation (2a) indicates that the observations of soil water content and temperature consist of two parts:  $\theta_i$  and  $T_i$ , as well as  $V_{\theta_i}$  e  $V_{T_i}$  (noises). The noises can be due to measurement

uncertainties of  $\theta$  and  $T$  and to the influence of other not measured variables that can affect them (Morkoc et al., 1985).

## MATERIAL AND METHODS

Soil water content ( $\theta$ ) and soil temperature ( $T$ ) data were collected in a field study carried out at Piracicaba, SP, Brazil (22° 42' S and 47° 38' W) on an area of a dark red latosol, called "terra roxa estruturada" (Rhodic Kandindex) (Figure 1). The crop was planted in October 1997, harvested in October 1998, and after this the soil temperature and moisture study began, using the first ratoon crop. Three management treatments were compared: i. mulching with trash (cane tips and straw from the last harvest,  $T_1$  and  $T_2$ ); ii. bare soil between rows ( $T_3$ ); and iii. soil surface with the residues left by the traditional practice of straw burning before harvest ( $T_4$ ). The treatments  $T_1$  and  $T_2$  are similar in respect to the mulching with trash, and therefore replicates in terms of this soil temperature study, being different only in terms of  $^{15}\text{N}$  label which was used in an additional organic matter residue study.

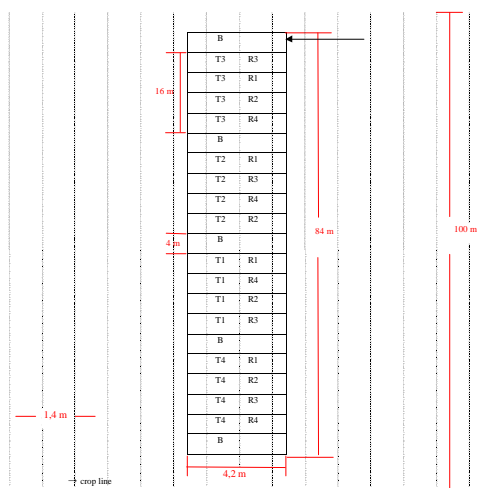


Figure 1 - Schematic experimental design showing the 15 cane lines, 100 m long, indicating the 3 central lines that received  $^{15}\text{N}$  labeled fertilizer. B= border; T= treatment; R= replicate.

Soil temperatures were measured with digital stick thermometers, introduced into the soil down to three depths: 3, 6 and 9 cm. Readings were made after equilibrium (about 3 minutes). In this study, the average of the three depths was used. Soil water contents were obtained using a single gamma-neutron surface probe, model CPN, MC-3. This gauge operates in such a way that water contents are measured by neutron moderation, sampling a semi-sphere of radius of about 15 cm, so that  $\theta$  measurements are averages of the 0 – 15 cm soil layer.

Data of  $\theta$  and  $T$  were simultaneously collected, at noon (11:00 am to 12:00), along a 84 point transect that covers all treatments and borders. Data collected on Nov. 1998 were analysed through the state-space approach, using the software ASTSA (Applied Statistical Time Series Analysis), developed by Shumway (1988).

According to Hui et al. (1998), if for the state-space analysis data are transformed by:

$$x_i = [X_i - (m - 2s)] / 4s \quad (3)$$

the state coefficients  $\phi_{k,j}$  in equation (1) will have magnitudes directly proportional to the contribution of each respective state variable used in the analysis. Transformed values  $x_i$  have a mean equal to 0.5. In equation (3),  $m$  is the mean of  $X_i$  and  $s$  the standard deviation.

## RESULTS AND DISCUSSION

Soil water content and temperature data used in this study, collected along the 84 point transect, are shown in Figure 2. The temperature data reflect visually the effect of the treatments on the average soil temperature of the surface layer (3 to 9 cm), treatments  $T_1$  and  $T_2$  presenting much lower temperatures (overall average of 23.2°C) due to the presence of the mulch (trash = tips + straw, 127 kg.ha<sup>-1</sup> of dry matter);  $T_3$  with bare soil surface presenting an average of 30.1°C; and  $T_4$ , the burned treatment, with an average of 28.3°C. The sharp temperature differences are due to the fact that they were measured two weeks after harvest of the first crop, when the ratoon crop was starting to sprout, so that the field was completely exposed to sunshine on November 20, 1998, a late spring day, after a sequence of 6 days without rainfall.

Soil water content data, collected at the same day, present an inverse relation, the mulched treatments  $T_1$  and  $T_2$  show relatively higher water contents in relation to the bare  $T_3$

and the burned  $T_4$  treatments. This is demonstrated in Figure 3, which presents a correlation ( $R^2 = 0.4491$ , significant at the level of 5 %) between soil temperature  $T$  and soil water content  $\theta$ . The negative slope of the relation expresses the inverse relation between  $T$  and  $\theta$ .

The crosscorrelogram of Figure 4 shows the strong spatial dependence of the variables  $T$  and  $\theta$ , which is significant at the 5 % probability level, up to 7 or more lags. The semivariograms (gamma  $h$ ) presented in Figures 5 and 6 show also a clear spatial dependence of both variables, since values of gamma increase with the tendency of reaching the variance ( $s^2$ ), which represent the variability of the total population disregarding their locations.

The state-space analysis applied to soil water content and temperature, is presented in Figures 7 and 8 respectively, after transforming the data according to equation (3). The obtained matrix coefficients were:

$$\theta_i = 0.881 \theta_{i-1} + 0.1148 T_{i-1} + W_{\theta i} \quad (1b)$$

$$T_i = 0.0615 \theta_{i-1} + 0.9272 T_{i-1} + W_{T i}$$

The shaded area of Figures 7 and 8 represent the fiducial limits considering  $\pm$  one standard deviation ( $s$ ). For  $\theta$  and  $T$  the average values of  $s$  given by the ASTSA program were 0.0937 and 0.0732, respectively.

Analysing equation (1b), it can be seen that  $\theta$  at location  $i-1$  contributes with 88.1% to the estimate of  $\theta$  in  $i$ , while  $T$  at  $i-1$  contributes with 11.5%, showing that the contribution of  $\theta$  of the first neighbour is more significant than that of  $T$ . Morkoc et al. (1985) verified a contribution of  $\theta_{i-1}$  and  $T_{i-1}$  of 98.9 and 0.3%, respectively, on the estimation of  $\theta_i$ .

For the case of temperature estimation (Figure 8), equation (1b) shows that  $\theta_{i-1}$  contributes with 6.2% in the estimate of the temperature at point  $i$ . On the other hand,  $T_{i-1}$  contributes with 92.7%. Morkoc et al. (1985) found, in this case, contributions of 11.0 and 96.4%. It is important to note that these authors did not transform their data according to equation (3), and that for their case, it is more difficult to estimate the contributions of each variable.

The above shown state-space analysis is the first performed on soil spatial data in Brazil. One objective is its introduction into the Brazilian literature and, as already said, contribute for a better understanding of the relation between  $\theta$  and  $T$ . Figures 9 and 10 indicate how well the state-space model estimates the variables under

study. This can also be seen in the already discussed Figures 7 and 8, in which most of the observed values fall inside the fiducial limit of  $\pm$  one  $s$ .

Data show clearly, mainly Figure 3, the inverse relation between  $T$  and  $\theta$ . Added to (Figures 4, 5 and 6) this, there is a strong spatial dependence, which justifies the application of the state-space methodology. Therefore, soil water contents could be well estimated from neighbour values, including information on temperature. Since temperature measurements are easier and quicker than those of soil water content, the analysis here presented suggests that the one measurement could, in many situations, replace the other. Relating soil properties at sites  $i$  to properties at sites  $i-h$  is also of practical importance, mainly to farmers. In this study, the lag of 1m is small for practical purposes, it is however very important to better understand how far one property is affected by its neighbour, and so recognise management measures that would lead to increased yield. Precision agriculture is one of the recent fields very interested in these aspects.

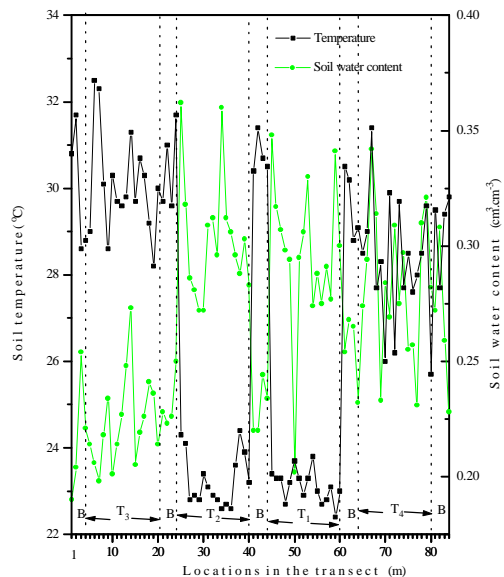


Figure 2 - Soil temperature and water content distributions, meter by meter, along the 84 point transect, at noon (11:00 - 12:00) on Nov. 20, 1998. B=border;  $T_1$  and  $T_2$ = trash mulching;  $T_3$ = bare soil;  $T_4$ = burned trash.

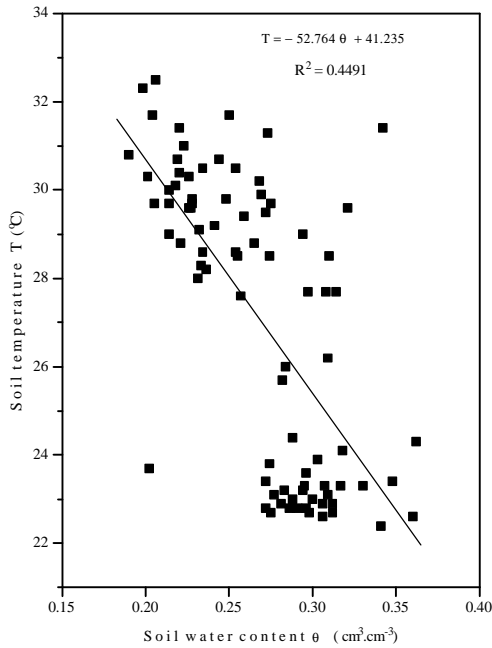


Figure 3 - Correlation between soil temperature and water content data of Figure 2.

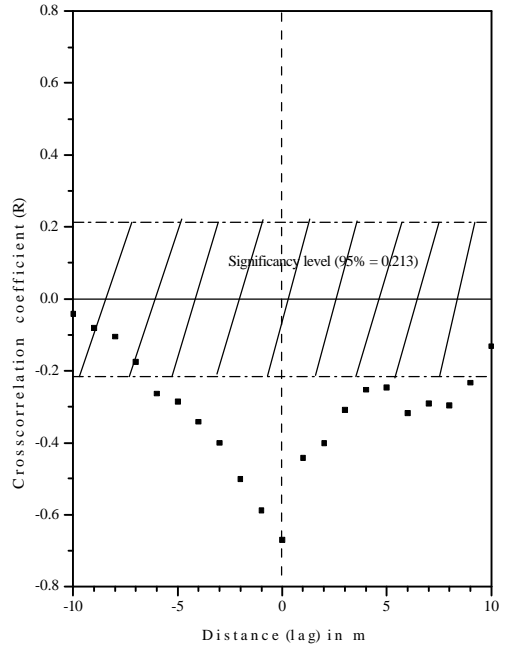


Figure 4 - Crosscorrelogram between soil temperature and water content data of Figure 2.

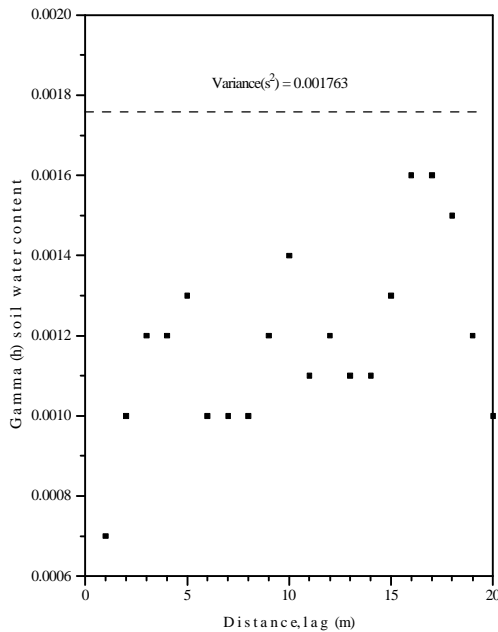


Figure 5 - Soil water content semivariogram, estimated using data of Figure 2.

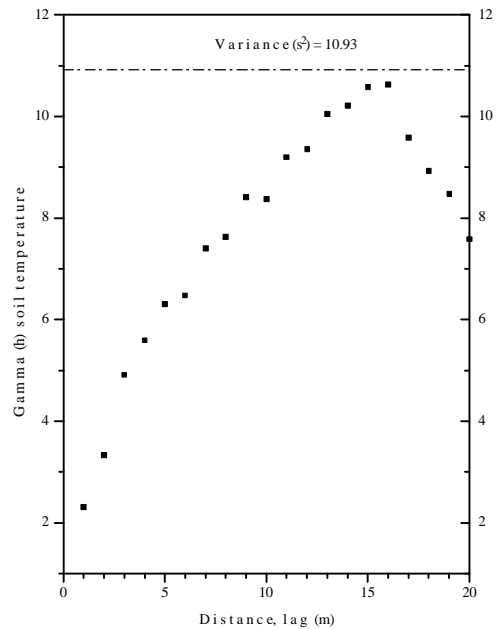


Figure 6 - Soil temperature semivariogram, estimated using data of Figure 2.

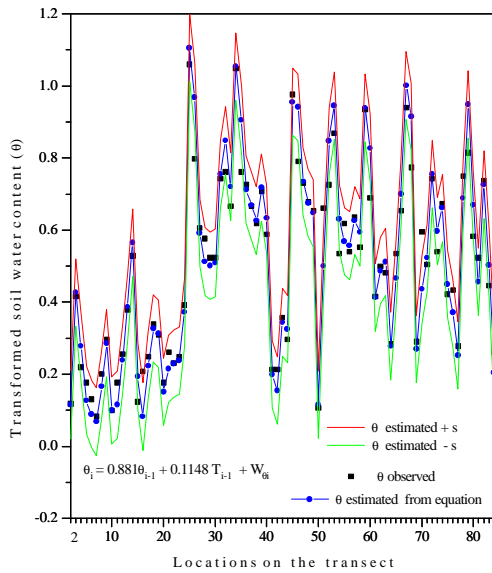


Figure 7 - State-space analysis of transformed (equation 3) soil water content  $\phi$  data of Figure 2.

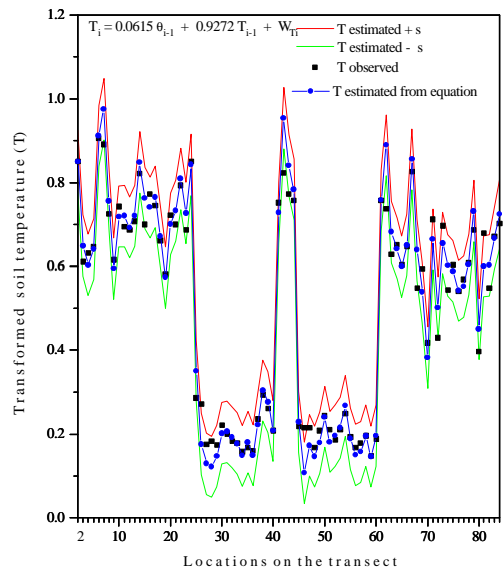


Figure 8 - State-space analysis of transformed (equation 3) soil temperature  $T$  data of Figure 2.

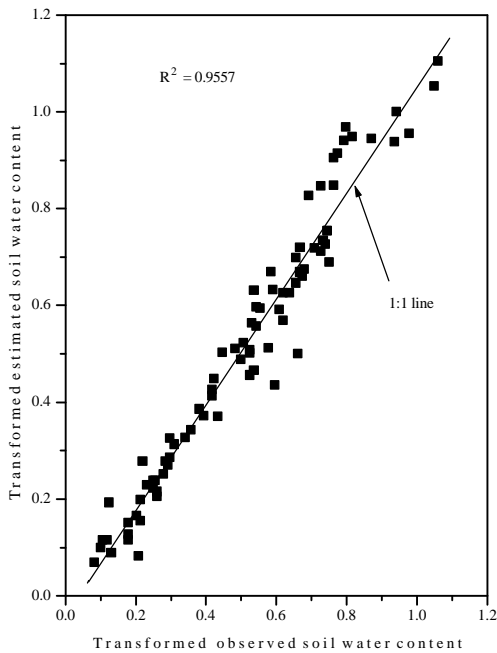


Figure 9 - Correlation between transformed (equation 3) estimated and observed soil water content data.

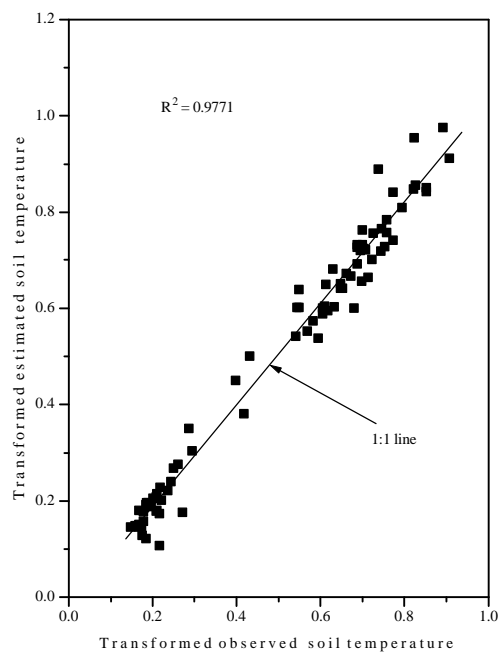


Figure 10 - Correlation between transformed (equation 3) estimated and observed soil temperature data.

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