Soldagem & Inspeção. 2018;23(4):474-484
https://doi.org/10.1590/0104-9224/SI2304.03
ISSN 1980-6973 (Online)
ISSN 0104-9224 (Print)

Technical Papers

A Multi-attribute Control Chart for Monitoring Friction Stir Welding Process Considering Small Sample Sizes

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Received: 22 Oct., 2018
Accepted: 31 Dec., 2018

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Abstract: Often, welding processes used in the industry affect the mechanical properties of materials and quality of a manufactured product. There is, however, an alternative process named Friction Stir Welding (FSW), which is an solid state welding process developed to weld light alloys without compromising their mechanical properties. It is of interest to monitor the performance of FSW process to detect loss of quality. In practice, superficial and internal defects can be found; they can be identified through simple visual inspection and through visual recognition on destructive testing respectively, both procedures represent inspection by attributes. Therefore a multi-attribute control chart is assessed to monitor the process. Commonly, multi-attribute control charts involve high sampling rates to ensure accurate monitoring. In this paper, a multi-attribute control chart is proposed, considering the use of empirical control limits, instead of the theoretical ones, in order to improve its accuracy and lessen the small sample sizes effect. The performance of proposed approaches is analyzed by means of Monte Carlo simulation. The results suggest that the performance of the empirical designs is better than the theoretical ones in all tested cases. Finally, the results of monitoring FSW process data are detailed.

Key-words: FSW; Attribute control chart; Categorical defects; Small sample sizes.

Um Gráfico de Controle de Múltiplos Atributos para Monitorar o Processo de Soldagem por Fricção Considerando Tamanhos de Amostra Pequenos

Resumo: Frequentemente, os processos de soldagem usados na indústria afetam as propriedades mecânicas dos materiais e como resultado a qualidade de um produto manufaturado. Existe, no entanto, um processo alternativo denominado Soldagem por Fricção e Mistura Mecânica (FSW - Friction Stir Welding), que é um processo de soldagem em estado sólido desenvolvido para soldar ligas leves sem comprometer suas propriedades mecânicas. Nesta perspectiva, é interessante monitorar o desempenho do processo FSW para detectar a deterioração na qualidade das juntas soldadas. Na prática, defeitos superficiais e internos podem ser encontrados; eles podem ser identificados por meio de inspeção visual simples, que não é muito precisa, e por meio do reconhecimento visual em testes destrutivos, respectivamente, ambos procedimentos representam inspeção por atributos. As características de qualidade categóricas relacionadas a defeitos internos precisam de atenção especial, portanto, um gráfico de controle de múltiplos atributos é avaliado para monitorar o processo. Comumente, os gráficos de controle de múltiplos atributos envolvem altas taxas de amostragem para garantir um monitoramento preciso e reduzir o risco de um produto defeituoso atingir o cliente. Neste trabalho, um gráfico de controle de múltiplos atributos é proposto para monitorar um processo de FSW, considerando o uso de limites de controle empírico, ao invés dos teóricos, a fim de melhorar sua precisão e diminuir o efeito de amostras de tamanho pequeno. O desempenho das abordagens propostas é analisado por meio da simulação de Monte Carlo. Os resultados sugerem que o desempenho dos desenhos empíricos é melhor que os teóricos em todos os casos testados. Finalmente, os resultados do monitoramento dos dados do processo FSW são detalhados.

Palavras-chave: FSW; Carta de controle por atributos; Defeitos categóricos; Amostras pequenas.

1. Introduction

The automotive industry has been searching for materials that allow it to reduce the vehicles' weight, with the aim of reducing fuel consumption and comply with the new safety regulations. Aluminum is used in the aerospace and automotive industries, and it represents a feasible alternative. In that sense, a greater understanding and knowledge of how this material can be welded conserving mechanical and metallurgical properties at welds is required. It is necessary to understand the mechanisms that produce such changes in order to comply with the desired performance of the welded joints and the quality standards established in the manufacturer's design bases. Conventional welding processes affect its mechanical properties; some problems are associated with the presence of a tenacious aluminum oxide, high thermal conductivity, high thermal expansion coefficient, solidification shrinkage and absorbed gases in molten aluminum (Ugrasen et al., 2018). Friction Stir Welding (FSW) is an advanced solid state welding process invented in 1991 by the Welding Institute of the United Kingdom. FSW process is a feasible option to weld aluminum alloys without compromising their mechanical properties, and it also brings improvements in production volume, lack of inputs, health and environmental problems (Dawes and Thomas, 1999).

FSW process uses a non-consumable rotating tool with a probe and shoulder which goes forward through the material to be joined (Muthu Krishnan et al., 2018). This action causes plastic deformation because of the friction heat; thus, welded joints can present superficial and internal defects. The ISO states that all welds shall undergo inspection for conformance (International Organization for Standardization, 2011). In fact several defects associated to the FSW process are identified, obtaining categorical quality characteristics. Internal defects can be detected by means of X-ray examination and recognition on destructive testing, both in a visual context. For instance, some of those categorical defects in FSW are wormhole, kissing bond and hooking, among others. To avoid problems or accidents related to defective pieces reaching final clients, it is of interest to monitor the welded joints quality adequately.

Monitoring and controlling the FSW process quality is possible by means of Statistical Process Control (SPC) which comprises a powerful technique called control chart. Several approaches have been proposed to control quality characteristics such as continuous, discrete, univariate and multivariate. For instance, the multi-attribute control chart is capable of monitoring more than one categorical quality characteristic simultaneously. It can be used in processes where several categorical defects can appear, or not, in a treated piece; commonly, the defect identification is through visual inspection. Recently, multi-attribute charts have been receiving more attention. The current widespread applications of attribute charts are due to the simplicity in handling attribute quality characteristics, the capability of checking multiple quality requirements and the prevalence of count data in many industries and manufacturing sectors (Haridy et al., 2014). Moreover, recent approaches consider attribute control charts to monitor process means instead of variable control charts (Melo et al., 2017; Quinino et al., 2017; Leoni and Costa, 2018). In the control charts field, many approaches, generalizations and modifications have been proposed. For example, the adaptive control charts consider a variable sample size or a variable sample size frequency, or both (Mahadik and Shirke, 2011; Seif et al., 2011; Faraz et al., 2012). In control charts with sequential sampling, the sampling is made in steps according to the location of the statistic values (Khoo et al., 2010; Irianto and Juliani, 2010; Costa and Machado, 2011). There is another type of control chart named synthetic, which combines a classic control chart and the monitoring of a random variable, namely, the number of inspected samples among two consecutive out-of-control signals (Ghute and Shirke, 2008; Khoo et al., 2013).

Most of control charts approaches require a large sample size since they are defined by impractical statistical assumptions, such as charts proposed by Li et al. (2014), Cozzucoli (2009), Chiu and Kuo (2008) and others. However, there are processes where collecting large sample sizes is difficult, e.g. when the production standard is low, or when inspection requires destructive tests; such as the FSW process in which monitoring is based on high cost inspection. Consequently, a multi-attribute control chart with no high requirements on sample size must be selected. Some researchers have been focused in the problem of having a small sample size to develop and apply the attribute and multi-attribute control chart. For example, Aebtarm and Bouguila (2011) present a review of methods employed to improve the sensitivity of attribute C control chart and inspection cost. A useful approach is the one proposed by Mukhopadhyay (2008). This work details that the multi-attribute D^2 control chart does not use the chi-squared distribution; this implies that a minimum expected frequency is not necessary to ensure its performance. The author states that this chart does not require large sample sizes to operate efficiently in statistical terms. Nevertheless, this approach is based on an asymptotic approximation to Multinormal distribution, which could be inefficient to certain sample sizes, in fact, small sample sizes, causing inaccurate monitoring due to chart power loss; it increases the risk of a defective product reaching the customer.

In order to lessen the small sample size effect, in this paper an approach based in empirical distribution for selecting the control limits of the D^2 chart is proposed. Therefore, the chart designs suitable for the FSW process were obtained by Monte Carlo simulation. A comparison between theoretical and empirical control limits was accomplished. The results suggest that the performance of the empirical designs is better than the theoretical ones in all tested cases. Additionally, a procedure to select empirical control limits is provided. Results show that the proposed multi-attribute control chart represents a reliable option to monitor an FSW process when defects must be identified by means of simple visual recognition on destructive testing. The consideration of small samples translates into low inspection costs and it also reduces the risk of a defective product reaching the customer.

2. The Multi-attribute D^2 Control Chart

The control chart proposed by Mukhopadhyay (2008) is able to monitor more than one attribute simultaneously. It is based on a generalization of the Mahalanobis distance and the multinomial distribution. This chart is suitable to monitor processes where a produced unit can be classified on several excluding nonconforming categories or defects.

Assume that in a production process, a finished piece can be classified in one and only one of $K-1$ nonconforming categories (categorical defects) and 1 conforming category (nondefective pieces). Let p_{ij} be the independent observed proportion of items of the sampling time i in the category j ($i = 1, 2, \ldots$ and $j = 1, 2 \ldots k$) in a sample of size N_i, so, $\left(p_{i1}, p_{i2}, p_{i3}, \ldots, p_{ik}\right) = p_i^T$ is the vector of proportions observed from the process. Suppose $\overline{p}^T = \left[\overline{p}_1, \ \overline{p}_2, \ ..., \ \overline{p}_k\right]$ denotes the vector of proportions of an in-control process; $\overline{p}^T$ can be estimated either by means of a historical database or arbitrarily specified. Then p_i^T has a multinomial distribution with parameters $\overline{p}^T$. In the rest of the paper $\overline{p}^T$ will be called the target proportions vector, or shortly, the target vector. It is clear that the target vector will be fixed to represent a production process having high quality, this is, small proportion values in nonconforming categories and a greater proportion value in the conforming one.

For multinomial data, a generalized Mahalanobis distance is defined in Equation 1:

$$D_i^2 = (p_i - \overline{p})^T \ \Sigma_i^{-1} \left(p_i - \overline{p}\right) = \sum_{j=1}^{k} \frac{N_i (p_{ij} - \overline{p}_j)^2}{\overline{p}_j} \tag{1}$$

where Σ_i is the variance-covariance matrix of vector p_i and is equal to $N_i^{-1} \Sigma$ and $\Sigma = \left[\sigma_{ij}\right], \sigma_{ij} = \begin{cases} \overline{p}_i \left(1 - \overline{p}_i\right) for\, i = j \\ -\overline{p}_i \overline{p}_j \quad for\, i \neq j \end{cases}$.

Since the covariance structure indicates an increment in a multinomial vector component requires a decrement in any other component; this result in negative covariance.

Then, in a particular point in time, D_i^2 measures the distance between the observed and the target vector. UCL_i, the upper control limit for D_i^2 can be computed with Equation 2 using the theorem provided by Mardia et al. (1979),

$$UCL_i = \left[\frac{\left(N_i \left(K-1\right)\right)}{\left(N_i - K + 2\right)}\right] F_{K-1, N_i - K + 2, \alpha} \tag{2}$$

where the quantity $F_{K-1, N_i - K + 2, \alpha}$ is the quantile α of the *F* distribution with ($K-1$) and $N_i - K + 2$ degrees of freedom respectively (for details see Mukhopadhyay, 2008).

2.1. Distributional assumption discussion

Even though Mukhopadhyay (2008) suggests that the D^2 control chart can be employed with small samples, it is reasonable to think that the chart can be deficient when the sample size is small. The expression to compute the control limit is based on an asymptotic approximation. Mardia et al. establish that if $\mathbf{x} : \ N_p(\mu, \Sigma)$, then $m\left(\mathbf{x} - \mu\right) \ \mathbf{M}\left(\mathbf{x} - \mu\right) : \ T^2\left(p, m\right)$, which is true only if the $\mathbf{x}$ vector follows a ***Multinormal distribution*** (Mardia et al., 1979). On the other hand, the D^2 statistic defined in (1), is a generalization to the multinomial case, and in this case, p_i has a multinomial distribution with $\overline{p}^T$ parameter Mukhopadhyay (2008). Although the distribution of p_i can be approximated to a Multinormal distribution, such approximation is asymptotic regarding the sample size.

Therefore, if the sample is insufficient, this assumption will not be met and the efficiency of the control chart D^2 will be affected.

Therefore, considering that process quality can be determined by presence or absence of defects, multinomial distribution is used to model the FWS process quality by means of proportions vector. Note that proportions vector includes information about failure mode. This vector is a sufficient statistic due to contain all information about the real proportion of defects. The covariance matrix is obtained from this vector, as mentioned.

3. Empirical Multi-attribute D^2 Control Chart

In order to improve the D^2 chart efficiency on small sample sizes, a general procedure for selecting empirical control limits instead of the theoretical ones is developed considering two essential parts as follows:

Part I. *Empirical distribution*

It is necessary to set: categories *k* (defects number plus 1), sample size *n* (desired sample size, $n > k$) and target vector $\overline{p}$.

In practice, if target vector $\overline{p}^T$ is arbitrary selected, case (a) (above described) must be used; so that p_0 value should be set, for example, according to the company quality standards. Then, practitioners can apply (3) to generate the target vector. Otherwise, target vector can be estimated from a process data base, it must be represent in-control process (case (b), above labeled). However, it is required a process under stable operating conditions in order to be monitored. It is worth mentioning that when process is unstable, statistical methods like Design of Experiments, Analysis of Variance, regression among others, can help to establish relationship between input variables and defects; so, designed requirements could be fulfilled (Qiu, 2014).

Then, for given *k*, sample size *n* and target vector $\overline{p}^T$:

1. Obtaining multinomial vectors. By means of Monte Carlo simulation, random multinomial vectors must be obtained with the $\overline{p}^T$ parameter;
2. Calculating D^2. For each random vector the D^2 statistic is computed by Equation 1;
3. Pre-selecting control limits (PCL). Percentiles of the D^2 calculated values are obtained, in which the limit control is chosen (for example, 0.90 to 0.99 with 0.01 shifts).

Part II. *Control limit selection*

For selecting the control limit, it is necessary to assess the performance of each PCL, assuming different variations on the observed vector p_i. The performance measurement commonly used to evaluate the control chart efficiency is the Average Run Length (*ARL*). It is assumed that the process starts in-control, i.e. $p_i = \overline{p}$, and sometime after, a process shift occurs ($p_i \neq \overline{p}$). It is assumed that the increase in p_i occurs between sampling times. It is also assumed that the items produced are independent, and that when the process is in the out-of-control state, it remains in this state until there is an intervention to bring it back to the in-control state. The *ARL* is defined as the average number of samples collected from the process until an out-of-control signal occurs. When the process is in-control, an out-of-control signal is a false alarm, so, it is desirable that the *ARL* is large, in order to have a low false alarm rate, and it is denoted by ARL_0. Moreover, when the process is out-of-control, the *ARL* should be small to provide a fast detection of the quality changes. In this case, it is denoted by ARL_1.

1. *Out-of-control* scenarios. To simulate *out-of-control* possible scenarios, multinomial random vectors were generated with the p_1 ($p_1 \neq \overline{p}$) parameter. At least two cases were able to be considered for the simulation.

(a) All defects may occur with the same probability.

(b) Each defect has different probability of occurrence.

The control chart designs were obtained considering (a), for this the target vector $\overline{p}^T$ was established in Equation 3:

$$\overline{p}^T = \left[\left(p_0 \right), \left(1 - p_0 / k \right), \ldots, \left(1 - p_0 / k \right) \right] \quad (3)$$

$p_0 = \{0.90, 0.95, 0.99\}$ are selected, and p_1^T is calculated by Equation 4:

$$p_1^T = \left[\left(\overline{p}_1 - \rho \right), \left(\overline{p}_2 + \rho / k \right), \ldots, \left(\overline{p}_k + \rho / k \right) \right] \quad (4)$$

where ρ represents a variation in the process quality, in this case magnitude shifts $\rho = \{0.00, 0.01, 0.02 \ldots 0.40\}$ are assumed. The particular case $\rho = 0$, corresponds to an in-control process and is employed to calculate the ARL_0.

The *p* vectors were simulated from binomial independent distributions, through scaling *p* to sum 1 and satisfy the multinomial law (Kachitvichyanukul and Schmeiser, 1988). In that sense, binomial random data generation depends on sample size and the proportions vector. Note that covariance matrix is obtained from *p* as it was mentioned in section 2.

2. Calculating D^2. By means of Equation 1, the D^2 statistic is computed for random vectors corresponding to each *out-of-control* scenario. Note that D^2 statistic is based on Mahalanobis distance, which considers covariance matrix of the multinomial distribution.
3. *ARL* estimation. An implementation of the control chart must be simulated, for this:

   3.1 D^2 values are compared with the corresponding control limit. From this comparison, a vector with 0 and 1 elements will be obtained, where 0 means $D^2 < PCL$ and 1 means that $D^2 \geq PCL$.

   3.2 Run lengths are obtained, a run length is the sampling quantity until an out-of-control signal occurs ($D^2 \geq PCL$).

   3.3 Finally, the *ARL* is calculated as the average of the run lengths calculated in the previous step. Resulting in an estimation of the *ARL* for each *out-of-control* scenario, so that, for each given *k, n* and $\overline{p}^T$ values, *ARL* values curves are drawn, which correspond to each ρ and PCL.

4. Selecting empirical control limits (ECL). In each case, for selecting the empirical control limit, the ARL curves of each PCL are compared. The PCL whose ARL curve approximates more rapidly to 1 as ρ increases and presents an $ARL_0 \geq 200$ (or the bigger ARL_0 value instead) is selected. Thus, the selected control limit is the one that shows better performance in ARL terms. For attribute control charts a reasonable criterion to select designs is to consider $ARL_0 \geq 200$ (Araújo Rodrigues et al., 2011). However, it was decided to include designs with lower ARL_0 values when the requirement was not satisfactory.

By following the proposed methodology: considering two different categorical defects in the FSW process, also one additional category considered the fact that defects can be found simultaneously in the same piece, and a conforming category of non-defective pieces, hence, the control chart design with $k = 4$ is obtained, results are shown in Table 1.

4. Monitoring the FSW Process

4.1. Friction Stir Welding process description

FSW is a joining technique in solid state developed mainly to weld light alloys. The process requires the use of a cylindrical tool of hard material, which is introduced into the pieces, forging and stirring the material until the weld is done. During this process, the material is plasticized without achieving the melting point by the rotation and advancing of the tool, generating a refined microstructure with improved mechanical properties; Figure 1 shows the FSW process.

Table 1. Empirical D^2 chart and D^2 (conventional) chart designs for $k = 4$, several p_0 and n values; the desired proportion of non-defective pieces and sample size, respectively. *ECL* empirical control limit, UCL, upper control limit and ARL_0, the in-control Average Run Length are showed in columns 4-7.

Case	p_0	n	Empirical D^2 chart		D^2 chart	
			ECL	ARL_0	UCL	ARL_0
1	0.9	3	6.7531	100.7419	48630.1681	>6000
2	0.95		7.7992	163.3517	48630.1681	>6000
3	0.99		10.5054	222.1527	48630.1681	>6000
4	0.9	4	6.3086	115.2890	594.9972	>6000
5	0.95		6.7018	235.1376	594.9972	>6000
6	0.99		7.7284	200.8734	594.9972	>6000
7	0.9	5	6.1852	209.2582	147.2835	>6000
8	0.95		9.6456	180.3038	147.2835	>6000
9	0.99		12.1268	120.0488	147.2835	>6000
10	0.9	10	5.8951	186.9446	28.4662	>6000
11	0.95		6.0702	193.0958	28.4662	>6000
12	0.99		12.1324	298.0578	28.4662	>6000
13	0.9	15	5.2719	167.5292	19.8671	>6000
14	0.95		5.8577	211.1191	19.8671	>6000
15	0.99		8.9156	141.0744	19.8671	2045.3333
16	0.9	20	5.0341	172.2079	16.9730	>6000
17	0.95		6.0058	180.7009	16.9730	>6000
18	0.99		7.0819	189.2469	16.9730	>6000
19	0.9	25	4.8273	198.3758	15.5376	>6000
20	0.95		6.0084	187.1380	15.5376	>6000
21	0.99		9.7789	198.8774	15.5376	1864.1667
22	0.9	30	4.7144	188.0109	14.6832	>6000
23	0.95		5.2575	169.9843	14.6832	>6000
24	0.99		8.5490	207.3138	14.6832	1483.833

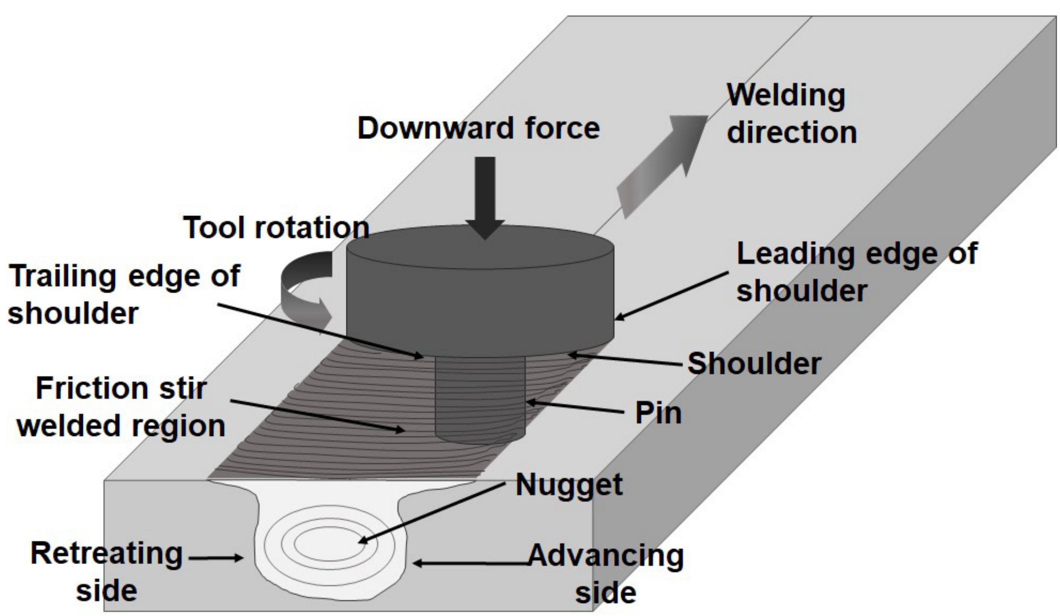

Figure 1. Schematic drawing of FSW process (Rajiv and Murray, 2007).

4.2. Quality inspection

In order to assess the quality of the joint, visual inspection (VI) is the first method employed. This step is executed to avoid unnecessary cost, and to determine whether the piece complies with the minimum quality. Some FSW defects can be detected using VI, such as flashing and surface cracks (American National Institute of Standards, 2010). However, a deeper analysis using destructive tests must be performed in order to identify defects that were not evident using non-destructive tests. For instance, among others, some detrimental defects in FSW are kissing bond and hooking (Lohwasser and Zhan, 2001). Those defects are related to heat generated by probe rotation velocity and the increment in advance velocity. This inhibits tool heat increasing, producing an inadequate consolidation in material (Morales-Bazaldúa, 2017). Therefore, considering that process quality can be determined by presence or absence of defects, multinomial distribution is used to model the FWS process quality by means of proportions vector, which includes information about failure mode. This vector is a sufficient statistic due to contain all information about the real proportion of defects.

4.3. Performance of the empirical multi-attribute D^2 control chart in FSW

For monitoring the FSW process, kissing bonding and hooking are important quality characteristics of the welded joints to be monitored. However, both defects can appear simultaneously in the same piece, so an additional category is considered in order to classify the pieces with both defects. In the process, the two different defects and the additional characteristic are equally likely to appear, and it is desirable to maintain the conforming pieces proportion at 0.90. Thus, a finished piece can be classified in one and only one of the $k = 4$ categories (including the non-defective pieces category). A $n = 5$ sample size is employed, since a destructive test inspection is used.

In order to investigate the effect of the sample size and compare the performance of both approaches (empirical and conventional), ARL values were obtained by means of Monte Carlo simulation. The results are shown in Figure 2; each dotted line represents the ARL values for different sample sizes and several quality shifts (ρ). Plots for $n < 10$ are not shown for D^2 chart because the ARL values exceed the number of computational cycles employed for the simulations (6000). The same happened for $n > 10$, but only for $\rho < 0.12$ in the worst case; for comparative purposes we decided to set these ARL values at 6000.

It can be observed in Figure 2a that in all cases, the D^2 chart performance is poor in *ARL* terms; the best efficiency is for $n = 30$ and the worst is for $n = 10$. All of the scenarios present too big ARL values (greater than 6000) for small ρ values. Besides, the performance is chaotic in different intervals of ρ in each case. Hence, for all sample sizes tested, the chart is completely impractical because of the huge insensitivity and instability in detecting quality shifts. In that sense, it is concluded that using the conventional D^2 control chart with small samples ($n \leq 30$) is not suitable for the studied FSW process and similar ones.

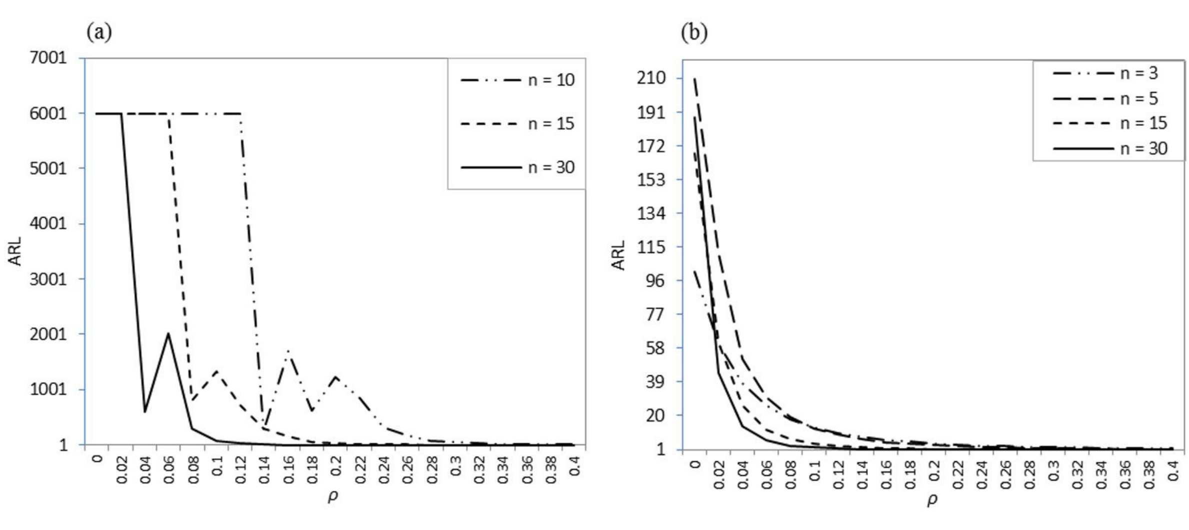

Figure 2. (a) ARL values for D^2 chart and (b) ARL values for empirical D^2 chart.

On the other hand, as it can be seen in Figure 2b, the empirical approach shows better performance, more sensitivity and stability as ρ increases, i.e. as the process quality decreases. The best efficiency is for $n = 30$ and the worst is for $n = 3$, but the performance is satisfactory in all tested circumstances. Thus, it can be concluded that by using empirical limits instead of theoretical ones the efficiency of the control chart for small sample sizes ($n \leq 30$) is improved. So, the D^2 chart with empirical limits is a reliable option for monitoring the studied FSW process. A procedure for monitoring the process with this approach is described below.

4.4. Monitoring procedure

1. Selecting the D^2 control chart design with empirical limits. According to the process characteristics, the design selection consists of: (i) selecting the most convenient design in Table 1 and (ii) applying the methodology described in Section 3. For example, design 7 of Table 1 with control limit $ECL = 6.1852$ is the most suitable for this analyzed FSW process.
2. Drawing the D^2 chart with empirical limits, employing the ECL value. Figure 3a shows the chart applied to FSW data.

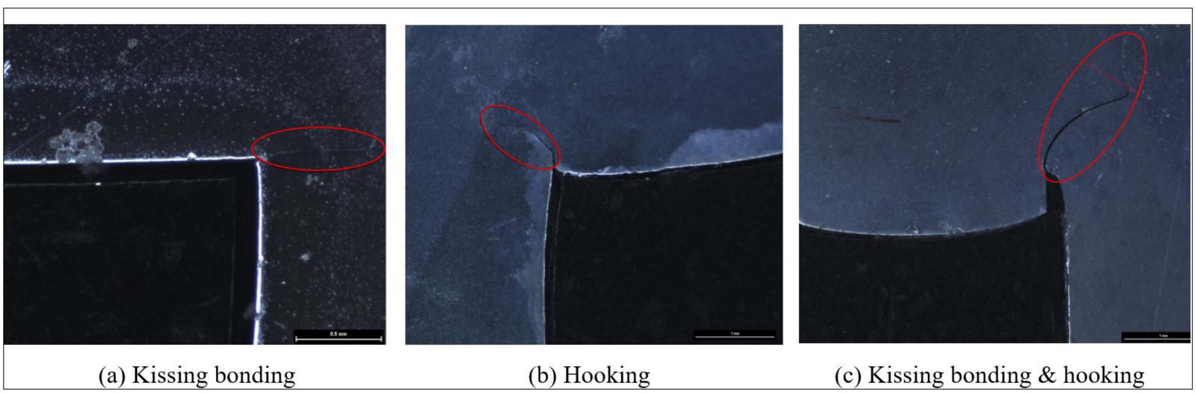

Figure 3. FSW defective pieces macrographs, picture shows pieces with the defect (a) kissing bonding; (b) hooking; and (c) kissing bonding and hooking, respectively.

3. Monitoring the process

3.1. In equidistant intervals of time, for example, each hour, a sample of 5 finished pieces is collected from the process.

3.2. Inspect the pieces in order to identify the presence of defects. The method for detecting samples defects consisted in metallographic analysis, transversal cutting of the sample, grinding and a three polishing steps (diamond paste of 16, 1.25 and 1 µm for 20 minutes each, colloidal silica of 0.01 µm for 30 minutes and 1 µm alumina for 3 minutes). The presence of these defects resulted in a discontinuity leading into a failure of the joint, in that sense any indication of this condition is considered defect. Then each piece is classified according to the presence or absence of defects the results are arranged in a vector. The categories must always be registered in the same order. The ith element of this vector represents the quantity of pieces classified for $i = 1,2,3,4$ categories. For instance, a dataset of 5 samples from the local FSW process is shown in Table 2. Also, the visual appearance of defective pieces is shown in Figure 3: (a) kissing bonding, (b) hooking and (c) kissing bond & hooking.

3.3. Calculate the observed proportion vector p_i^T by dividing the quantity of pieces classified in each category by sample size.

3.4. Calculate D^2 using Equation 1, and plot the resultant value in the empirical control chart. If the point is under the control limit ($D^2 < ECL$), the process is then declared in-control. Otherwise, if the point is above the control limit ($D^2 \geq ECL$), the process is declared out-of-control and it must be stopped to avoid more defective pieces. An examination must be performed to determine the reason of the quality deterioration. This assessment is made by means of observation of plotted points and lines of ECL in the chart, see Figure 4a. Table 2 shows D^2 values and recorded data from the FSW process.

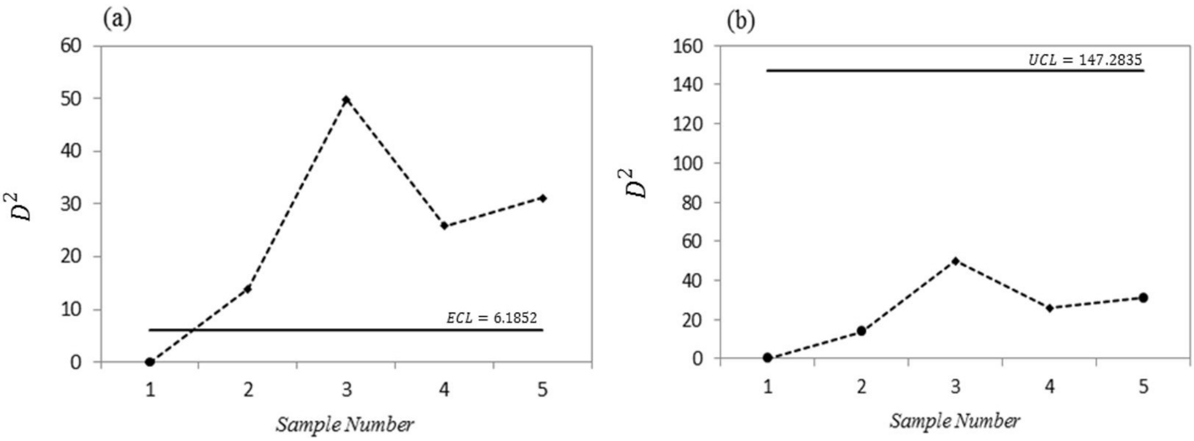

Figure 4. (a) Empirical limits D^2 control chart and (b) D^2 control chart.

Table 2. Data recorded from local FSW process, columns 2-4 correspond to the categorical defects of the process; column 5 is for non-defective pieces, D^2, test statistic, is showed in column 6.

Sample	Kissing bond	Hooking	Kissing bond & Hooking	Non-defectives	D^2
1	0	0	0	5	0.5556*
2	1	1	1	2	13.8889
3	3	0	0	2	49.8889
4	1	0	2	2	25.8889
5	1	1	2	1	31.2222

*Since all pieces are Non-defectives, this particular D^2 value shows quality improvement.

Data showed in Table 2 is preliminary for the chart implementation. For comparative and illustrative purposes, in Figures 4a and 4b, the D^2 control chart with empirical limits and the traditional D^2 chart are showed respectively. Note that the control limit of the conventional D^2 chart was calculated with Equation 2.

It can be observed in Figures 4a and b that the process starts in-control. On the second sampling interval a shift occurs, which is detected only by the empirical D^2 chart; since it is a preliminary phase, no action is applied and the process continues. However, it is important to highlight that the conventional D^2 chart does not detect any samples at all as out-of-control; instead the proposed D^2 chart with empirical limits indicates that the process is out-of-control through interval 2, which is a more precise representation of reality.

Hence, based on the obtained results, it is possible to assure that the designed empirical D^2 control chart can be implemented at the production line to monitor the FSW process with the considered characteristics and also, it is possible to analyze the capability of the process for manufacturing products with the desired welded joints quality; deeming the advantages of FSW process over the joints' mechanical properties. Therefore, the automotive industry may adopt the proposed control chart when the FSW process was used to weld aluminum pieces in a manufacturing process, which is a feasible alternative.

5. Conclusions

The automotive industry is searching for materials that allow it to obtain significant advantages; aluminum is used in automotive industries, representing a feasible alternative. Welding processes used in the industry affect the mechanical properties and quality of welded joints; FSW is the proposed process since it is an advanced solid state welding process.

The AWS standard states that visual inspection on destructive testing and X-ray examination must be used to identify internal defects, therefore a multi-attribute control chart has to be used to monitor the FSW process considering categorical quality characteristics.

Multi-attribute control charts require bigger sample sizes (due to asymptotical assumptions) for obtaining a good performance, which is not suitable for the FSW process since a destructive test inspection is used. Hence, in this study the D^2 multi-attribute control chart is used to assess the local FSW process, modifying the limit control in order to improve the efficiency and lessen the small sample sizes effect. An empirical control limit is proposed by using a Monte Carlo simulation, moreover the performance of the D^2 chart with different sample sizes ($n \leq 30$) was studied and compared with the performance of the conventional chart.

The results suggest that the conventional D^2 chart is deficient when $n \leq 30$, and therefore cannot be used to monitor the FSW process because it can lead into wrong conclusions about the state of the process. However, through the use of empirical control limits the enhancement of the chart is achieved in these cases. The proposed approach is a suitable option for the studied process as for similar ones, when small sample sizes are required or when it is necessary to reduce inspection costs.

It is important to highlight that designs showed in Table 2 are suitable for $k = 4$ categories. However, by following the described procedure in Section 3, it is possible to design empirical D^2 control charts for any categories (defects) $k \geq 2$.

Deeming the results, the proposed empirical D^2 control chart can be implemented at the production line to monitor the FSW process with the considered characteristics, considering the advantages of the FSW process over the joints' mechanical properties. In this sense, the empirical D^2 control chart is useful for the automotive industry when a FSW process was used to weld aluminum pieces, which is a feasible alternative.

Also for improving the proposed chart performance, it is possible to weight the more severe defects. In that way, if this defects occur then chart can signals an out of control state, immediately. Furthermore, it is necessary to investigate a method to identify the attributes that will more probably induce an out-of-control signal, as well as the use of a method for calculating a process capability index, which represents an opportunity of research.

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