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Hydromechanical behavior of unsaturated soils: Interpretation of compression curves in terms of effective stress

John S. McCartney[1#], Fatemah Behbehani[1]

Article

Keywords
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Yielding
Suction hardening
Compacted soils

Abstract
This state-of-the-art paper on the hydromechanical behavior of unsaturated soils focuses on the interpretation of the compression curves of unsaturated soils in terms of effective stress, with the goal of understanding the relative impacts of suction on the effective stress, net yield stress, effective yield stress and slope of the virgin compression line (VCL) during a monotonic increase in net stress. A database of compression curves was compiled for both high and low plasticity fine-grained soils under a wide range of suctions, isotropic or oedometric stress states, drainage conditions (constant suction or constant water content) and preparation techniques (impact compaction, static compaction, consolidation from slurry). Most of the compression curves plotted in terms of effective stress revealed a consistent hardening response with increasing suction and a slight suction dependency on the slope VCL. Interpretation of the compression curves in terms of effective stress led to load-collapse curves with a similar shape for a wide range of soils. Most soils evaluated had a greater rate of increase in effective yield stress with suction than the rate of increase in suction stress with suction, implying that these compacted soils may be susceptible to collapse upon wetting. Inconsistent trends were observed in some studies, which were attributed partially to natural variability but also experimental issues and limitations on the range of conditions investigated. Accordingly, recommendations are provided for future studies on the compression curves of unsaturated soils to ensure that results can be clearly interpreted in terms of effective stress.

1. Introduction

Several studies have demonstrated that the effective stress principle can be used to interpret the shear strength of unsaturated soils so that a unified shear strength model can be applied to both saturated, unsaturated, and dry soils (Khalili et al., 2004; Lu & Likos, 2006; Lu et al., 2010). While early studies like Escario & Saez (1986) and Gan et al. (1988) observed that application of suctions greater than the air entry suction led to nonlinear increases in apparent cohesion, the shear strength values of these unsaturated soils plotted in terms of effective stress fell onto the same effective stress failure envelope as saturated and dry soils with negligible drained cohesion values. These observations were also confirmed for soils under high suction magnitudes by Alsherif & McCartney (2015), who performed shear strength tests on unsaturated soils at low degrees of saturation using the vapor flow technique and found that the effective stress principle was valid over the full range of suction. More recently, studies have demonstrated that the effective stress principle can be used to interpret the small-strain shear modulus of unsaturated soils (Khosravi & McCartney, 2012; Dong et al., 2016).

Despite the success in interpreting shear strength and shear modulus data in terms of effective stress, fewer studies have interpreted the volume change of unsaturated soils in terms of effective stress. This is perhaps due to the influence of early studies on the collapse of unsaturated soils during wetting. For example, Jennings & Burland (1962) performed collapse upon wetting tests on compacted soils and observed a decrease in volume during a reduction in effective stress (associated with a reduction in suction and increase in degree of saturation due to wetting). One of their conclusions was that the effective stress principle is not valid in unsaturated soils as soils should expand during a reduction in effective stress. Accordingly, many experimental studies on the compression response of unsaturated soils followed the double oedometer approach of Jennings & Knight (1957a), where the compression curves of saturated and as-compacted specimens were interpreted in terms of net

[#]Corresponding author. E-mail address: mccartney@ucsd.edu
[1]University of California, Department of Structural Engineering, San Diego, CA, USA.

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stress (the difference in total stress and pore air pressure) to predict the amount of collapse or swelling during wetting. Further, early theories for the volume change of unsaturated soils like the Matyas & Radhakrishna (1968) considered the volume changes due to suction (wetting or drying) and net stress to be independent.

More recent studies have refuted the concerns on the application of the effective stress principle to the interpretation of volume change data. Khalili et al. (2004) explained that collapse occurs due a reduction in the effective yield stress with wetting, resulting in an unstable stress state causing a reduction in volume even when the effective stress decreases. Khalili et al. (2004) noted that elasto-plastic constitutive models that use the effective stress principle can capture the collapse phenomena observed by Jennings & Burland (1962). Khalili et al. (2004) evaluated data from the literature to show that elastic strains are directly related to the effective stress state in unsaturated soils to prove the validity of the effective stress principle in capturing volume change behavior and noted that soils susceptible to collapse would have a greater increase in effective yield stress with suction than the increase in effective stress with suction.

The goal of this state-of-the-art study is to reinterpret data from the literature to better understand the role of effective stress in the volume change of unsaturated soils, with a specific focus on the volume change encountered during a monotonic increase in net stress. As many studies have presented the compression curves of unsaturated soils in terms of net stress, a reinterpretation of the data may help to isolate the effects of suction on the shape the compression curve, most importantly the yield stress and the slope of the virgin compression line (VCL). Accordingly, the main objective of this study is to understand relationships between the suction, yield stress, and effective stress from a database of compression curves of unsaturated high and low plasticity fine-grained soils under a wide range of suctions, isotropic or oedometric stress states, drainage conditions (constant suction or constant water content) and preparation techniques (impact compaction, static compaction, consolidation from slurry). The intention of this study is not to use the information from the database to calibrate different constitutive models, but instead to compare the relative rates of increase in yield stress and effective stress with increasing suction. It is important to note that although interpretation of shear strength data in terms of effective stress leads to a clear unification of trends among unsaturated, saturated, and dry soils, the same unification is not expected for the volume change of unsaturated soils as the suction and degree of saturation influence the effective stress as well as the yield stress and the slope of the VCL. As the reinterpretation of the compression curves in terms of effective stress requires more information than the interpretation in terms of net stress, a secondary goal of this study is to provide lessons learned on what information should be collected as part of future testing programs.

2. Background

The effective stress is a key variable that permits application of continuum mechanics principles to fluid-filled, deformable porous media (Bishop & Blight, 1963; Khalili et al., 2004). The effective stress definition in unsaturated soils was first proposed by Bishop (1959):

$$\sigma_{ij}^{'} = \left(\sigma_{ij} - u_a\delta_{ij}\right) + \chi\left(u_a - u_w\right)\delta_{ij} \tag{1}$$

where $\sigma_{ij}^{'}$ is the effective stress tensor, σ_{ij} is the total stress tensor, u_a and u_w are the pore air and water pressures, respectively, χ is the effective stress parameter, and δ_{ij} is the Kronecker delta. The difference between the total stress tensor and air pressure is referred to as the net stress tensor σ_{ij}^{net}, while the difference in pore air and water pressures is the matric suction ψ.

Many definitions for the effective stress parameter have been proposed, including χ equal to the degree of saturation (Bishop, 1959, Nuth & Laloui, 2008), χ equal to the effective saturation (Bolzon & Schrefler, 1995; Lu et al., 2010), or χ as a function of the ratio of the air entry suction to suction (Khalili & Khabbaz, 1998). Khalili & Khabbaz (1998) noted that the relationship between the effective stress parameter and degree of saturation may not be unique due to hydraulic hysteresis, which was demonstrated by Khalili & Zargarbashi (2010) in experiments involving multistage shearing tests after drying and wetting. On the other hand, Lu et al. (2010) noted the practical advantage of using the effective saturation as the effective stress parameter as it permitted incorporation of a soil-water retention curve (SWRC) model such as that of van Genuchten (1980) directly into the effective stress definition. Further, Khosravi & McCartney (2012) demonstrated that using a hysteretic SWRC model in the effective stress equation following the approach of Lu et al. (2010) was useful in interpreting variations in shear modulus during wetting and drying. Lu et al. (2010) defined the concept of suction stress to incorporate the impacts of all particle-interaction mechanisms in the definition of the effective stress, as follows:

$$\sigma_{ij}^{'} = \left(\sigma_{ij} - u_a\delta_{ij}\right) + \sigma_s\delta_{ij} \tag{2}$$

where σ_s is the suction stress. Mathematically the suction stress is equal to the product of the effective stress parameter and suction as noted in Equation 1, but physically it is meant to be a single variable that encompasses all interactions between particles due to capillarity, cementation, adsorptive forces, or van der Waals forces. The van Genuchten (1980) SWRC model is given as follows:

$$S_e = \left(\frac{S - S_{res}}{1 - S_{res}}\right) = \left[\frac{1}{1 + \left(\alpha_{vG}\psi\right)^{N_{vG}}}\right]^{\frac{1}{1-N_{vG}}} \tag{3}$$

where S_e is the effective saturation, S is the degree of saturation, S_{res} is the residual degree of saturation and α_{vG} and N_{vG} are fitting parameters.

Early studies on the volume change of unsaturated soils focused on characterizing collapse during wetting at high net stresses (e.g., Jennings & Knight,1957b; Jennings & Burland, 1962; Matyas & Radhakrishna, 1968) or swelling during wetting at low net stresses (e.g., Blight, 1965; Brackley, 1973), and only few characterized the changes in yield stress of soils with increasing suction (e.g., Dudley, 1970). In the extension of the modified Cam-clay model to unsaturated soils that led to the development of the Barcelona Basic Model (BBM), Alonso et al. (1990) identified the need to include a yield surface that governs the increase in net yield stress with increasing suction which was referred to as the Load-Collapse (LC) curve. The development of the BBM led to several studies focused on the characterization of the yielding of unsaturated soils during compression, many of which are revisited in this study. Following the pioneering concepts in the BBM, several hydro-mechanical, elasto-plastic frameworks were developed to simulate the impacts of changes in net stress, effective stress and matric suction on the volume change of unsaturated soils. The ability of these frameworks to predict the hydro-mechanical behavior of unsaturated soil is influenced by the stress state definition. While the BBM and other elasto-plastic models used independent state variables (Alonso et al., 1990; Wheeler & Sivakumar, 1995; Bolzon et al. 1996; Cui et al., 1995; Sheng et al., 2004), other studies used the generalized effective stress concept (Loret & Khalili, 2002; Gallipoli et al., 2003; Wheeler et al., 2003; Tamagnini, 2004; Romero & Jommi, 2008; Khalili et al., 2008; Zhou et al., 2012a, b; Della Vecchia et al., 2013). An advantage of using the generalized effective stress concept is that a smaller number of material properties may be needed to simulate the complex volume change behavior of unsaturated soils, and the yield surface is always concave (Khalili et al., 2008). Further, it may be easier to consider the effects of hydraulic hysteresis in an elasto-plastic framework by incorporating the soil-water retention curve (SWRC) (Wheeler et al., 2003; Gallipoli et al., 2003; Tamagnini, 2004) or the air entry suction (Khalili et al., 2008) in the definition of the generalized effective stress concept. Sheng et al. (2008) was able to incorporate the effects of hysteresis in the SWRC into an elasto-plastic model that used independent stress state variables.

Regardless, a common feature of these elasto-plastic models is that they incorporate an LC curve acting as the yield limit transition from an elastic to an elasto-plastic volumetric soil response during compression of unsaturated soils. In general, most LC curves indicate an increase in either the net yield stress or effective yield stress with increasing suction, an effect commonly referred to as "suction hardening". Some studies like Uchaipichat & Khalili (2009) and Uchaipichat & Khalili (2009) observed suction hardening in the relationship between the effective yield stress and suction but a decrease (softening) in net yield stress with suction, although this observation may depend on the soils and compaction conditions being tested. Many analytical expressions have been proposed to characterize the LC curve for unsaturated soils have been proposed in the literature, and several were summarized by Mun et al. (2017). LC curves have been defined in the net stress vs. suction space (Alonso et al., 1990), effective stress vs. suction space (Salager et al., 2008; Tourchi & Hamidi, 2015), effective stress vs. degree of saturation space (Gallipoli et al., 2003; Romero & Jommi, 2008), effective stress vs. modified suction space (i.e., suction multiplied by the porosity) (Wheeler et al., 2003), and effective stress vs. effective saturation space (Zhou et al., 2012b). In several constitutive models, the shape of the LC curves is linked with the shape of the VCL for the unsaturated soil, which has been assumed to be a function of suction (Alonso et al., 1990) or effective saturation (Zhou et al., 2012b). However, Wheeler et al. (2002) noted issues with fitting the VCL of the BBM to data, and Mun & McCartney (2017) noted that the links between the LC curve and VCL may not be valid when predicting pressurized saturation of soils at high stresses.

3. General compression response of unsaturated soils

Before investigating the experimental compression curves from the literature, it is important to review the general compression curves of an unsaturated soil under different suctions in terms of effective stress. The schematic shown in Figure 1a shows hypothetical compression curves for different compacted soil specimens during constant suction compression tests with drained air and water. Several features can be noted from this figure that are expected when interpreting compression curves in terms of effective stress. First, application of suction to the soil will increase the effective stress, which will lead to an elastic contraction. This will also lead to a shift in the starting point of the compression curves. Loading the soil to higher stresses will follow an elastic relationship until reaching the yield stress, which depends on the suction and the shape of the LC curve. After this point, greater deformations will be noted as the soil deforms along a VCL that may depend on the suction or degree of saturation (Alonso et al., 1990; Zhou et al., 2012b). Actual compression curves are expected to be more nonlinear than the idealized curves in Figure 1a so the slope of the VCL will change with increasing normal stress. At high stresses, the compression curves for unsaturated specimens will converge with that of a saturated specimen after all pore air has been expelled or dissolved into the pore water. Mun & McCartney (2017) noted that greater pressures are required to reach this point of pressurized saturation for specimens with lower initial degrees of saturation. During compression, the degree of saturation will increase as the volume of voids decreases, as shown in Figure 1b. During the increase in degree of saturation, the suction stress will change. Further, the shape of the SWRC of the soil will change, leading to an increase in the air entry suction of

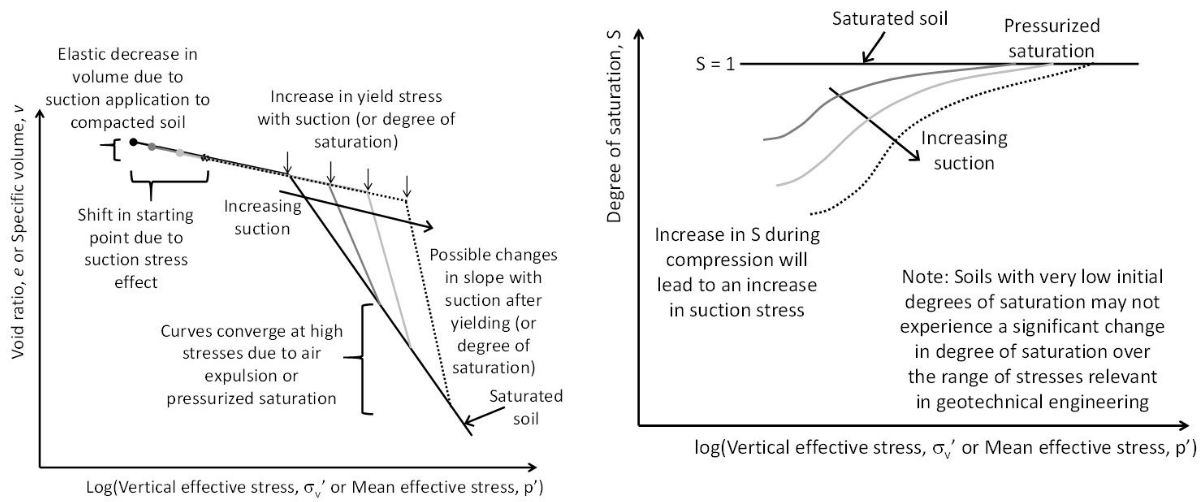


Figure 1. Hypothetical hydro-mechanical behavior of unsaturated soils during drained compression with constant suction: (a) Compression curves in terms of effective stress; (b) Increases in degree of saturation during drained compression.

the soil. This may lead to difficulties in using the effective stress definition of Khalili & Khabbaz (1998) to consider compression of unsaturated soils to high stresses. At the same time, it should be noted that during compression of relatively dry soils under high suctions, negligible changes in degree of saturation can be expected over the range of stresses representative of geotechnical problems.

Some additional observations can be drawn from the typical shapes of the compression curves of unsaturated soils interpreted in terms of net stress. Jennings & Knight (1957b) observed that the net stress compression curve for collapsible soils will typically plot above the curve for a saturated soil, while the net stress compression curve for an expansive soil will plot below the curve for a saturated soil. These observations may help interpret the range of compression curve shapes in the database.

Despite the observation of suction hardening observed by many studies, the fundamental cause behind the increase in yield stress with suction is not well understood, as it is independent from the interparticle connections that affect the effective stress state. Suction hardening could be due to a stiffening of the soil structure associated with the formation of the unsaturated soil and the distribution in water throughout the specimen associated with different degrees of saturation. Relatively few studies have studied the effects of soil structure, which could be achieved by comparing the LC curves for soils prepared using compaction with those consolidated from a slurry. While several studies have investigated the volume change behavior of soils consolidated from slurry (Fleureau et al., 1993; Dong et al., 2020), only the volume change due to changes in suction was monitored in these studies, and the net stress was not increased monotonically to evaluate the effects of suction on the yield stress. In addition, few studies have isolated the impact of soil structure associated with compaction from the effects of suction on the yield stress. For example, Mun & McCartney (2017) performed drained compression tests on soils compacted to the same initial density but different compaction water contents. The observed yield stresses could be affected by both the soil structure from compaction at different water contents (dry and wet of optimum) or by the applied suction. It was not possible to investigate these issues in this study with the data available in the literature, but they are interesting topics for future research.

4. Database and methodology

A database of compression curves from the literature for different fine-grained soils was collected to evaluate the impacts of suction on the yield stress and effective stress (or suction stress). Several studies have investigated the volume change behavior of unsaturated soils but focused on the impact of suction application on the shrinkage response of soils (Fleureau et al., 1993; Dong et al., 2020) or the transient process of wetting leading to collapse (Kato & Kawai, 2000; Pereira & Fredlund, 2000; Sun et al., 2000, 2007), so these studies were not included in the database. A total of 25 studies were identified who presented the results from compression curves on unsaturated soils. The details of these studies are summarized in Table 1, including the soil type according to the unified soil classification scheme (USCS), the suction control method, the suction range, the drainage conditions, stress state, the method to estimate the degree of saturation if available, and the stress state used to report the compression curves.

The approach followed in this study was to reinterpret the compression curves in terms of effective stress, then to investigate linkages between the suction and parameters

Table 1. Summary of compression studies on unsaturated soils in the literature.

Study	Soil Type	Preparation Technique	Suction Control/ Drainage	Suction Range (kPa)	Drainage	Compression	Degree of Saturation Estimate	Reported Stress State
Wheeler & Sivakumar (1995)	Kaolinite (CL)	Static compaction to 400 kPa	Axis translation	100-300	Drained	Oedometer	Outflow	Net
Sharma (1998)	Bentonite-Kaolin (CH)	Static compaction to 400 kPa	Axis translation	100-300	Drained	Oedometer	Outflow	Net
Maâtouk et al. (1995)	Silt (ML)	Tamped	Axis translation	0-600	Drained	Isotropic	None	Net
Rampino et al. (1999)	Decomposed granite (ML)	Compacted	Axis translation	0-400	Drained	Isotropic/ Oedometer	Outflow	Net
Al-Mukhtar et al. (1999)	Na-Laponite (CH)	Compacted	Vapor equilibrium	0-298000	Drained	Oedometer	CWC assumed	Net
Cunningham et al. (2003)	Silty Clay (CL)	Static compaction to 200 kPa	Tensiometer/ Vapor equilibrium	0-1000	Drained	Isotropic	CWC assumed	Net
Lloret et al. (2003)	Bentonite (CH)	Static compaction to 20 MPa	Vapor equilibrium	0-500000	Drained	Oedometer	CWC assumed	Net
Cuisinier & Masrouri (2005)	Silt-Bentonite (ML)	Compacted	Vapor equilibrium/ Osmotic	0-39700	Drained	Oedometer	CWC assumed	Net
Villar (2005)	MX80 Bentonite (CH)	Static compaction	Vapor equilibrium	0-1400	Drained	Oedometer	CWC assumed	Net
Geiser et al. (2006)	Sion Silt (ML)	Slurry consolidated	Axis translation	0-280	Drained	Isotropic	Outflow	Net/ Effective
Jotisankasa et al. (2007)	Silt-Clay (CL)	Compacted	Tensiometer/ Filter Paper	0-31800	CWC	Oedometer	CWC	Net
Thu et al. (2007)	Coarse Kaolinite (ML)	Compacted	Axis translation	0-300	Drained	Isotropic	SWRC	Net
Casini (2008)	Jossigny Silt (ML)	Compacted	Axis translation	0-200	Drained	Oedometer	SWRC	Net/ Effective
Salager et al. (2008)	Sion Silt (ML)	Slurry consolidated	Axis translation	0-300	Drained	Isotropic/ Oedometer	SWRC	Net/ Effective
Uchaipichat & Khalili (2009)	Bourke Silt (ML)	Compacted	Axis translation	0-300	Drained	Isotropic	None	Net/ Effective
Sun et al. (2010)	Pearl Clay (CL)	Compacted	Axis translation	100-150	Drained	Isotropic	Outflow	Net
Tang & Cui (2010)	MX80 Bentonite (CH)	Static compaction to 40 MPa	Vapor equilibrium	0-110000	Drained	Isotropic	SWRC	Net
Uchaipichat (2010)	Kaolinite	Compacted	Axis translation	0-300	Drained	Isotropic	None	Effective
Ye et al. (2012)	GMZ01 Bentonite (CH)	Compacted	Vapor equilibrium	0-110000	Drained	Oedometer	None	Net
Coccia & McCartney (2016)	Bonny Silt (ML)	Compacted to e=0.75 to 0.82	Axis translation	0-40	Drained	Isotropic	Outflow	Effective
Khosravi et al. (2016)	Bonny Silt (ML)	Compacted to e=0.69	Axis translation	0-55	Drained	Isotropic	Outflow	Net/ Effective
Mun & McCartney (2017)	Boulder Clay (CL)	Compacted to e=0.51	Axis translation at compacted	0-150	Drained	Isotropic	Outflow	Effective
Khosravi et al. (2018)	Bonny Silt (ML)	Compacted to e=0.85	Axis translation	0-100	Drained	Isotropic	Outflow	Net
Li et al. (2018)	Boughrara clay (CH)	Compacted	Vapor equilibrium	200-8000	Drained	Oedometer	Electrical Res.	Net
Haeri et al. (2019)	Loess (CL)	Compacted	Axis translation	0-400	Drained	Oedometer	SWRC	Net

representing the shapes of the compression curves. While some of these studies provided a comprehensive evaluation of the evolution in key variables during volume change (net stress, void ratio, suction, degree of saturation) along with measurement of a soil-water retention curve (SWRC), others provided less information and required careful interpretation. Most of the tests were performed in constant suction conditions (drained air and water), while one was performed in constant water content (CWC) conditions (drained air but undrained water). However, in some of the high suction tests, constant water content conditions could be assumed due to the small changes in water content during compression. The compression curves presented by Sivakumar (1993), which were also presented by Wheeler & Sivakumar (1995), Sharma (1998), Thu et al. (2007), Coccia & McCartney (2016), and Mun & McCartney (2017) were all obtained using constant rate of strain testing, while the other studies in Table 1 were obtained using incremental loading. The term yield stress is used to represent the mean apparent preconsolidation stress obtained from isotropic compression tests or the vertical apparent preconsolidation stress obtained from oedometer tests. In some studies, the variation in degree of saturation during compression was obtained and reported from outflow measurements or electrical resistivity sensors, but in other studies no information on the degree of saturation during compression was provided. In some of these cases, a constant degree of saturation was assumed based on the SWRC, while in other cases the degree of saturation was estimated from phase relationships using the initial gravimetric water content and the measured void ratio based on a constant water content assumption. Because the degree of saturation was estimated in some of the studies, the results presented in this study are only useful for assessment of general trends and should not be used to calibrate constitutive models. When defining the effective stress using Equation 1, the effective stress parameter was assumed to be the degree of saturation as this was the most consistently available piece of information from most of the studies considered in the database. An implication of this assumption is that the suction stress in Equation 2 is equal to the product of the degree of saturation and suction. As most of the soils evaluated in this study have a small residual degree of saturation, the effective saturation and degree of saturation are similar. As the degree of saturation may evolve during compression, the suction stress at the point of yielding was used to define the trend with increasing suction. The air entry suction is provided for as many of the soils as possible which permits evaluation of the effective stress using the approach of Khalili & Khabbaz (1998), but this was not used in the analysis as the air entry suction may evolve during compression. If the actual yield stress was not identified in the relevant study, the yield stress was defined using Casagrande's method of finding the intersection between tangent lines fit to the initial and final parts of the compression curve.

5. Results

Selected compression curves from the database are presented in Figures 2 through 16 in terms of both net stress and effective stress, with results presented in the order of publication of the study. Wheeler & Sivakumar (1995) was one of the earliest studies to present the results of compression curves for compacted kaolinite clay with information on the degree of saturation evolution during compression. The compression curves shown in Figures 2a and 2b in terms of net stress and effective stress, respectively, reflect a clear rightward shift in the compression curves after reinterpretation using the effective stress principle. A clear evolution in the yield stress with increasing suction is observed, even though the specimens all had different initial void ratios and that yielding was observed shortly after the beginning of compression for the different specimens. Similar trends were observed for the results from compacted kaolinite-bentonite mixtures presented by Sharma (1998) in Figure 3. Although not shown, the results from Maâtouk et al. (1995) showed suction hardening in a compacted silt over a wider range of suction than evaluated in previous studies. The results from Rampino et al. (1999) for compacted decomposed granite

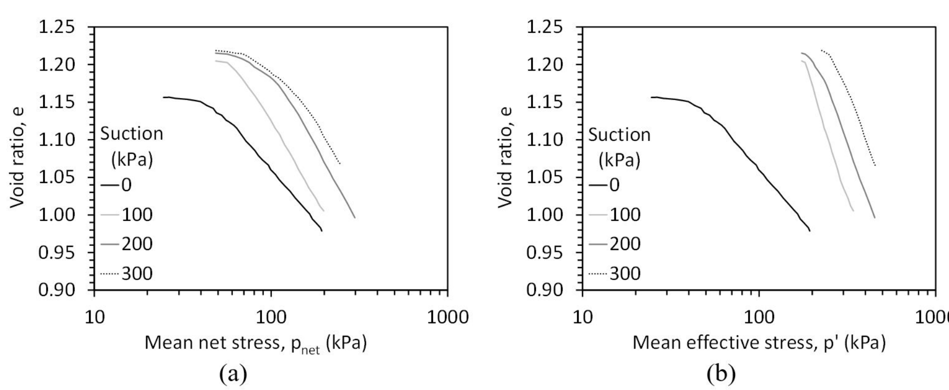
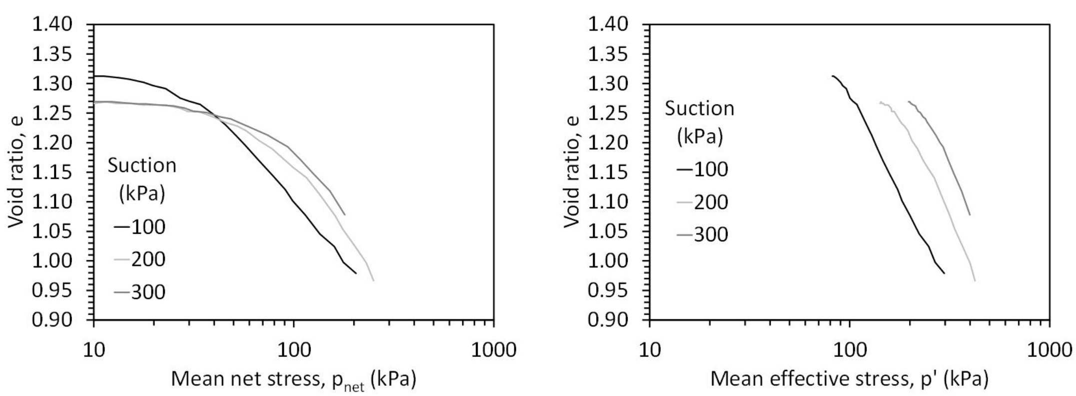


Figure 2. Compression curves from Sivakumar (1993) and Wheeler & Sivakumar (1995): (a) Reported curves in terms of net stress; (b) Reinterpreted curves in terms of effective stress.

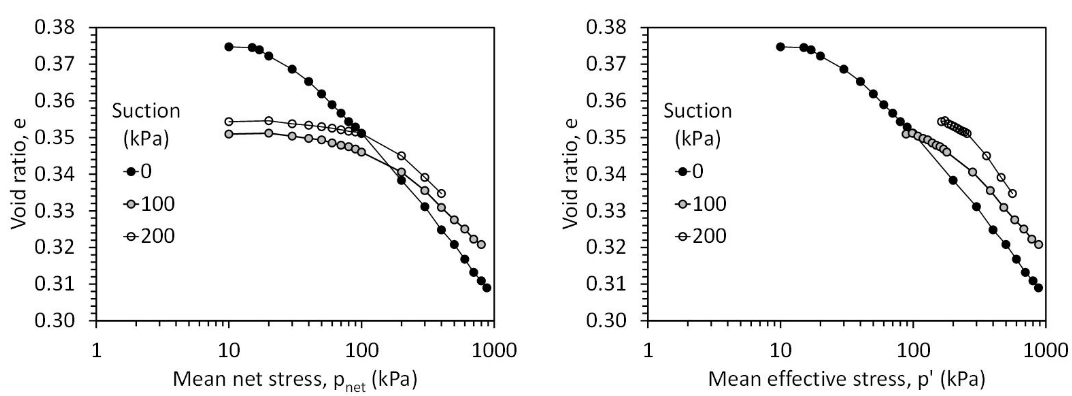
in Figure 4 show how application of the effective stress differentiates the curves at elevated suctions. Comparison of the results in Figures 2 through 4 indicates that the effective stress VCLs for the unsaturated soils are approximately parallel for the range of effectives stresses are applied. A comment from these figures is that if the volume changes during application of suction to the specimens had been tracked in these two studies, it may have been possible to define the initial elastic slope of the effective stress compression curves and better identify the yield stress.

The study by Lloret et al. (2003) was one of the first to investigate the compression curves of compacted high plasticity clay over a wide range of suction values. Although the study by Al-Mukhtar et al. (1999) also evaluated the compression response of high plasticity clays in the high suction range (greater than 1000 kPa), the range of net stresses applied were not sufficient to cause yielding of the soil. The results from Lloret et al. (2003) in Figures 5a and 5b show that the application of high suctions to the high plasticity clay led to a reduction in the initial void ratio, but that the shift of the curves led to VCLs that are clearly parallel. Although yielding was not apparent in the highest suction tests, the data points are still approximately parallel to the lower suction tests. Cunningham et al. (2003) also applied a wide range of suctions to a compacted silty clay. Although the specimens evaluated had a range of initial void ratios, application of the effective stress principle showed interesting results in Figures 6a and 6b. All the compression curves for unsaturated soils in effective stress space were collocated with the compression curve for saturated soil, different from the hypothetical curves in Figure 1a. In this case, yielding seemed to occur as soon as the compression curves for the unsaturated soils approached the saturated compression curve. This interesting observation could have been due to the much higher initial void ratio of the saturated specimen, and to the limited range of net stresses used to compress the unsaturated soils. Cuisinier & Masrouri (2005) evaluated the compression curves of a high plasticity clay over a wide range of suctions as shown in Figures 7a and 7b. Similar to the curves of Lloret et al. (2003), the compression curves in effective stress space were approximately parallel to each other but were steeper than the curve for saturated soil, which



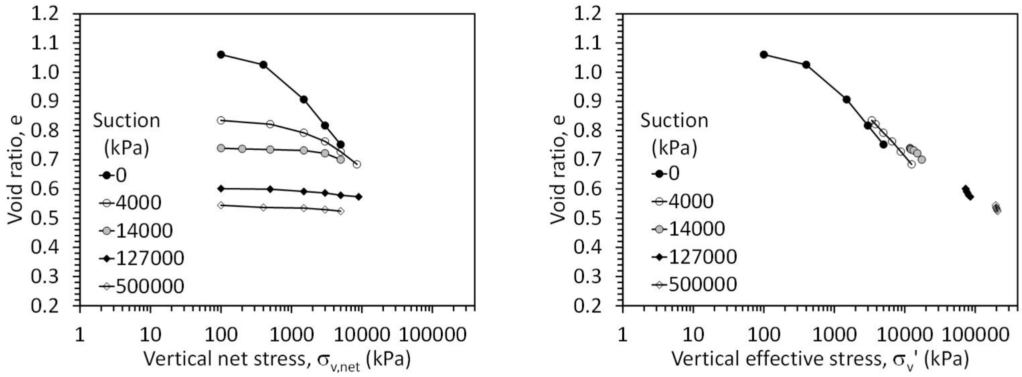
(a) (b)

Figure 3. Compression curves from Sharma (1998): (a) Reported curves in terms of net stress; (b) Reinterpreted curves in terms of effective stress.



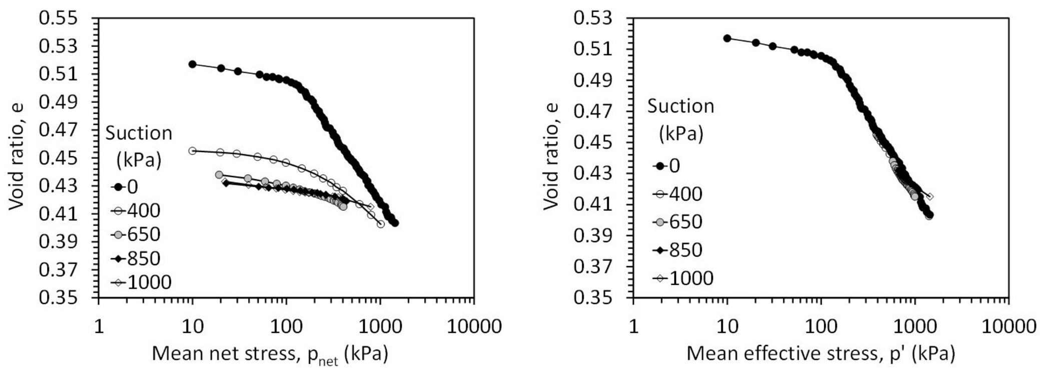
(a) (b)

Figure 4. Compression curves from Rampino et al. (1999): (a) Reported curves in terms of net stress; (b) Reinterpreted curves in terms of effective stress.



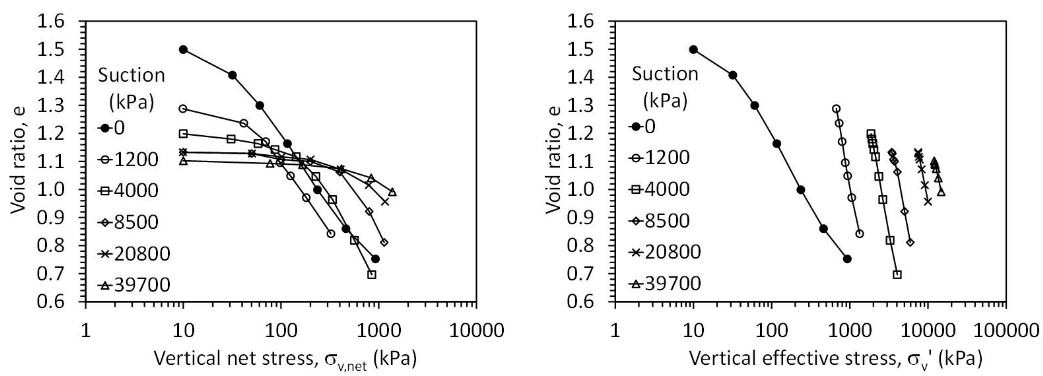
(a) (b)

Figure 5. Compression curves from Lloret et al. (2003): (a) Reported curves in terms of net stress; (b) Reinterpreted curves in terms of effective stress.



(a) (b)

Figure 6. Compression curves from Cunningham et al. (2003): (a) Reported curves in terms of net stress; (b) Reinterpreted curves in terms of effective stress.

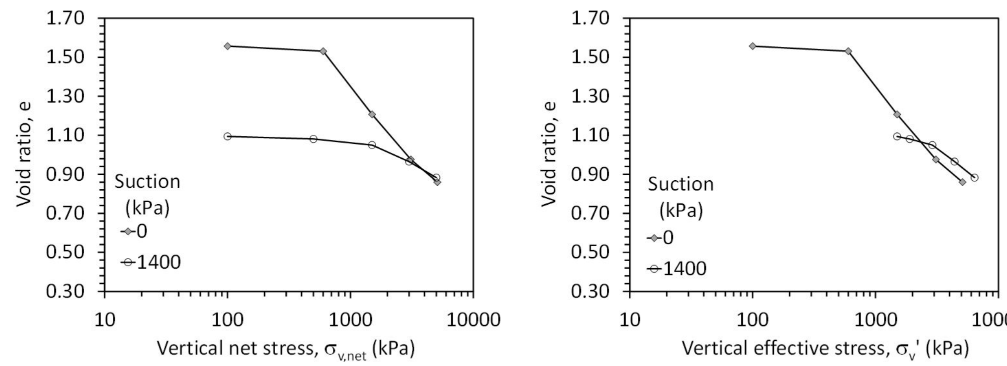


(a) (b)

Figure 7. Compression curves from Cuisinier & Masrouri (2005): (a) Reported curves in terms of net stress; (b) Reinterpreted curves in terms of effective stress.

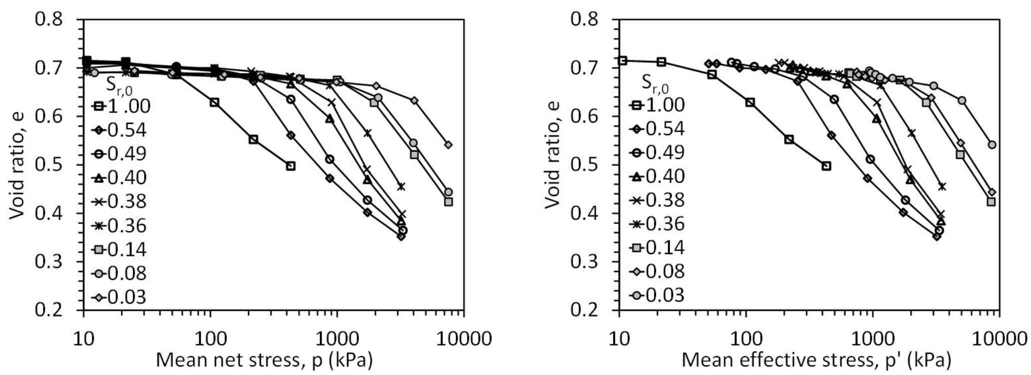
had a much larger initial void ratio. Although the points of yielding are clear in the net stress space in Figure 7a, they are not as apparent in effective stress space in Figure 7b. Villar (2005) presented drained compression curves on compacted MX80 bentonite performed after swelling from high suctions. The results shown in Figures 8a and 8b are similar to those of Lloret et al. (2003) although the point of yielding was more apparent in the high suction test.

The study of Jotisankasa et al. (2007) perhaps has the most comprehensive evaluation of suction hardening of a soil over the widest range of conditions. Constant water content tests were performed, in which case the suction changes during compression. Accordingly, the initial degrees of saturation for the specimens are shown in Figures 9a and 9b. The suction was tracked using a high capacity tensiometer for the tests with initial degrees of saturation greater than 0.36, and the filter paper method was used



(a) (b)

Figure 8. Compression curves from Villar (2005): (a) Reported curves in terms of net stress; (b) Reinterpreted curves in terms of effective stress



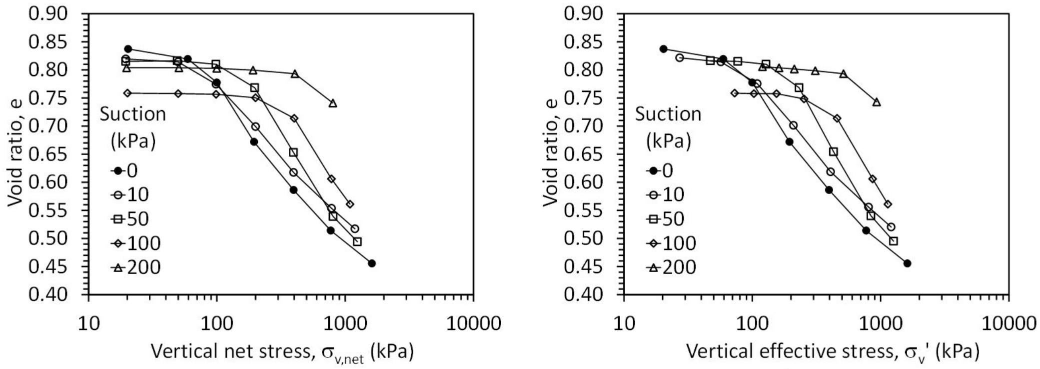
(a) (b)

Figure 9. Compression curves from Jotisankasa et al. (2007): (a) Reported curves in terms of net stress; (b) Reinterpreted curves in terms of effective stress.

for the lower initial degrees of saturation. Although application of the effective stress principle leads to a shift to the right for the compression curves for unsaturated soils, the shapes of the curves remain the same, which is a different observation from the results presented in the previous figures. Casini (2008) was one of the first studies to report compression curves in terms of effective stress, and although similar trends were observed to those of Jotisankasa et al. (2007), the initial void ratio of each specimen were slightly different.

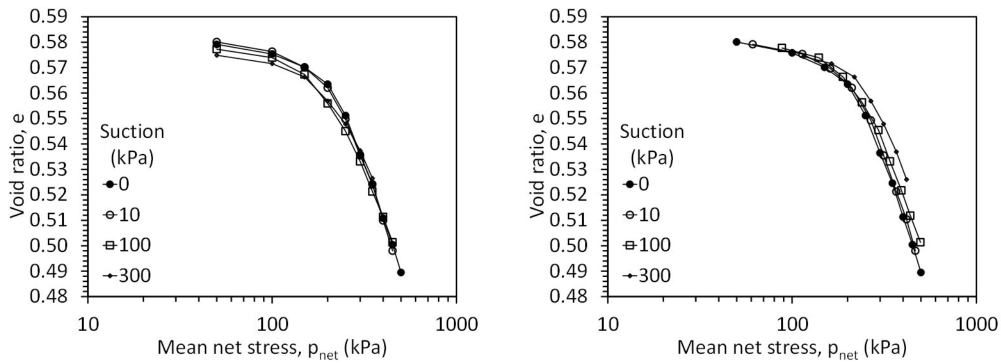
Although the results from Geiser et al. (2006) and Salager et al. (2008) on compacted Sion silt are not shown here, both studies presented compression curves in terms of effective stress only. Although the yield stress values in terms of net and effective stress were reported, neither study presented the evolution in degree of saturation during compression. These studies were the first to note that the yield stress in terms of effective stress should not change until the suction is greater than the air entry value. Uchaipichat & Khalili (2009) presented the compression curves for compacted Bourke silt in Figures 11a and 11b. An interesting observation from this study is that the net yield stress decreased with suction, while the effective yield stress increased with suction. The degree of saturation was not presented in this study, but the effective stress was defined using the initial air entry suction. This study also found that the yield stress does not change until the suction is greater than the air entry suction, which was confirmed by the test at a suction of 10 kPa which was below the air entry suction of 50 kPa. Similar observations in the yield stress were made by Uchaipichat (2010), although the compression curves were only presented in terms of effective stress. The compression curves from Thu et al. (2007) in Figure 12 are similar to those of Rampino et al. (1999), while the compression curves of Tang & Cui (2010) in Figure 13 are similar to those of Lloret et al. (2003) and Cuisinier & Masrouri (2005) except at the highest suction value. Ye et al. (2012) presented compression curves from GMZ bentonite, but only in terms of net stress without information on the degree of saturation.

Mun & McCartney (2017) presented compression curves for soils that were compacted to different initial water contents but at the same initial void ratio, as shown in Figure 14a and 14b. The initial suction in the specimen was measured using the tensiometer technique then applied using the axis translation technique. The shapes of the curves are similar to the hypothetical curves in Figure 1a, and high enough stresses were applied to reach pressurized



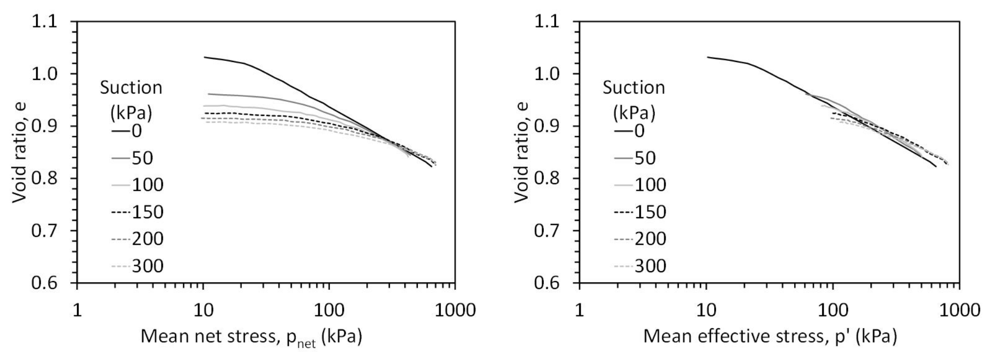
(a) (b)

Figure 10. Compression curves from Casini (2008): (a) Reported curves in terms of net stress; (b) Reported curves in terms of effective stress.



(a) (b)

Figure 11. Compression curves from Uchaipichat & Khalili (2009): (a) Reported curves in terms of net stress; (b) Reinterpreted curves in terms of effective stress.



(a) (b)

Figure 12. Compression curves from Thu et al. (2007): (a) Reported curves in terms of net stress; (b) Reinterpreted curves in terms of effective stress.

saturation. The shapes of the net and effective stress curves are similar, consistent with the observations of Jotisankasa et al. (2007). The compression curves of Khosravi et al. (2018) are shown in Figure 15a and 15b for Bonny silt. This soil was also characterized by Coccia & McCartney (2016) and Khosravi et al. (2016), albeit for different compaction conditions. The effective compression curves in Figure 15b mimic those of the hypothetical compression curves in Figure 1a to an intermediate stress level. The compression curves from a compacted Loess reported by Haeri et al. (2019) are shown in Figure 16a and 16b. Similar to the results from Mun & McCartney (2017), they show a tendency to pressurized saturation at high stresses, especially for the lower suctions. Finally, Li et al. (2018) presented compression curves for a high plasticity clay over a range of suctions and used an innovative electrical resistivity technique to monitor the

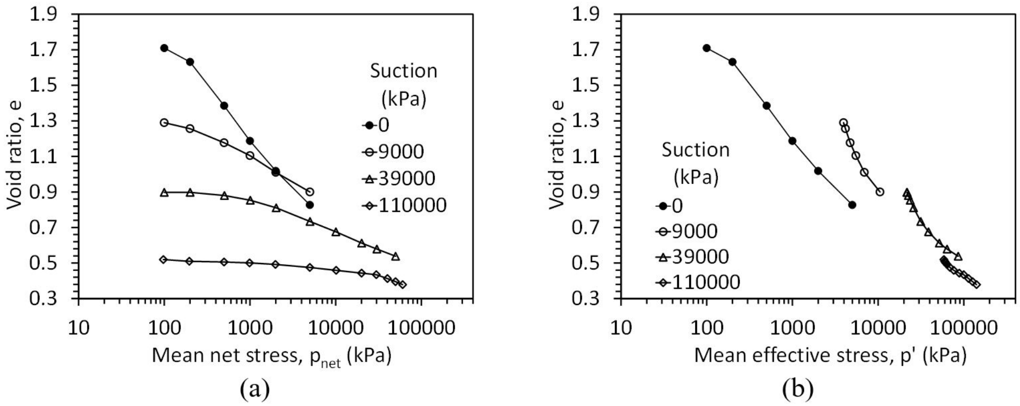


Figure 13. Compression curves from Tang & Cui (2010): (a) Reported curves in terms of net stress; (b) Reinterpreted curves in terms of effective stress.

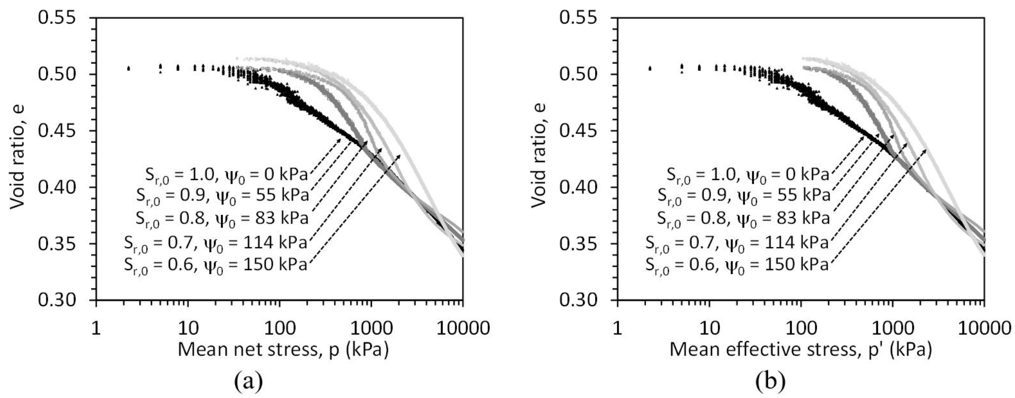


Figure 14. Compression curves from Mun & McCartney (2017): (a) Reinterpreted curves in terms of net stress; (b) Reported curves in terms of effective stress.

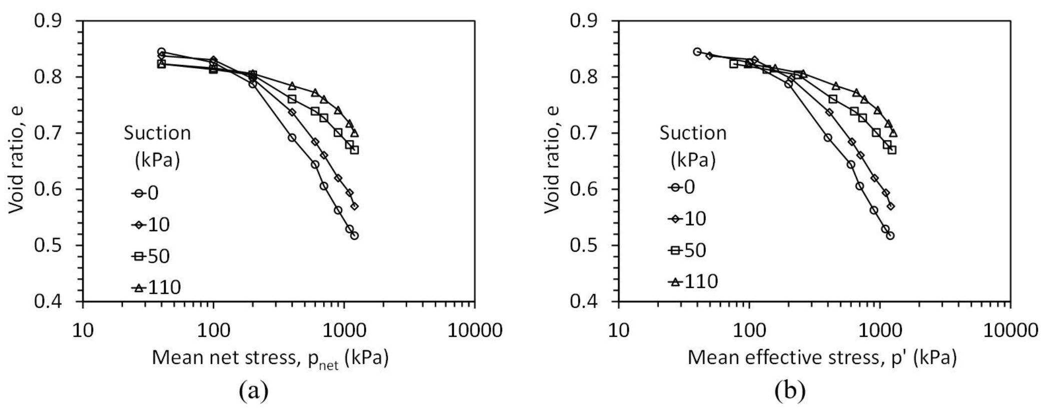


Figure 15. Compression curves from Khosravi et al. (2018): (a) Reported curves in terms of net stress; (b) Reinterpreted curves in terms of effective stress.

evolution in degree of saturation. They also tracked the change in void ratio during wetting or drying of compacted specimens to reach the target suction values.

6. Analysis

A plot of the suction versus the suction stress at yielding (i.e., the product of suction and the degree of saturation at yielding) for low suction magnitudes (below 1000 kPa) is shown in Figure 17a. Although all the curves intersect at the origin, this point was not included to better show the trends in the data. In general, most of the soils show a linear trend between suction and suction stress at yielding. The different slopes for these soils are due to the different shapes of the soil-water retention curves of the soils. As the suction and suction stress at higher suction magnitudes vary over a wider

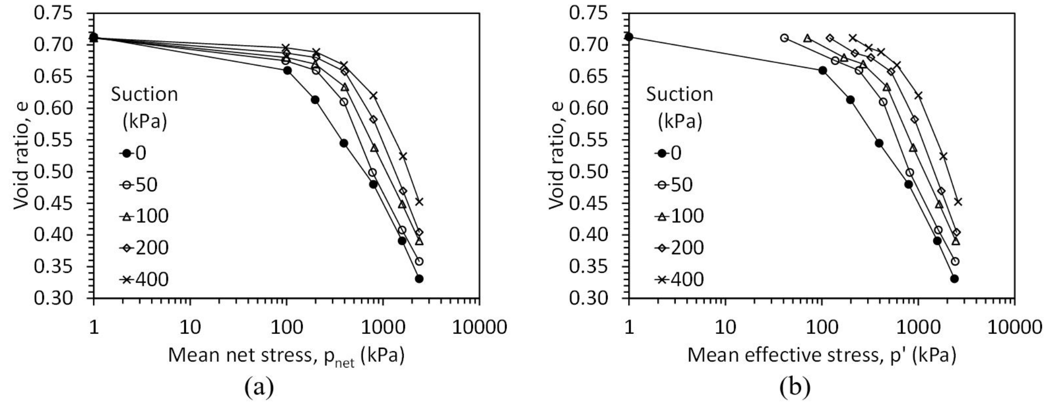


Figure 16. Compression curves from Haeri et al. (2019): (a) Reported curves in terms of net stress; (b) Reinterpreted curves in terms of effective stress.

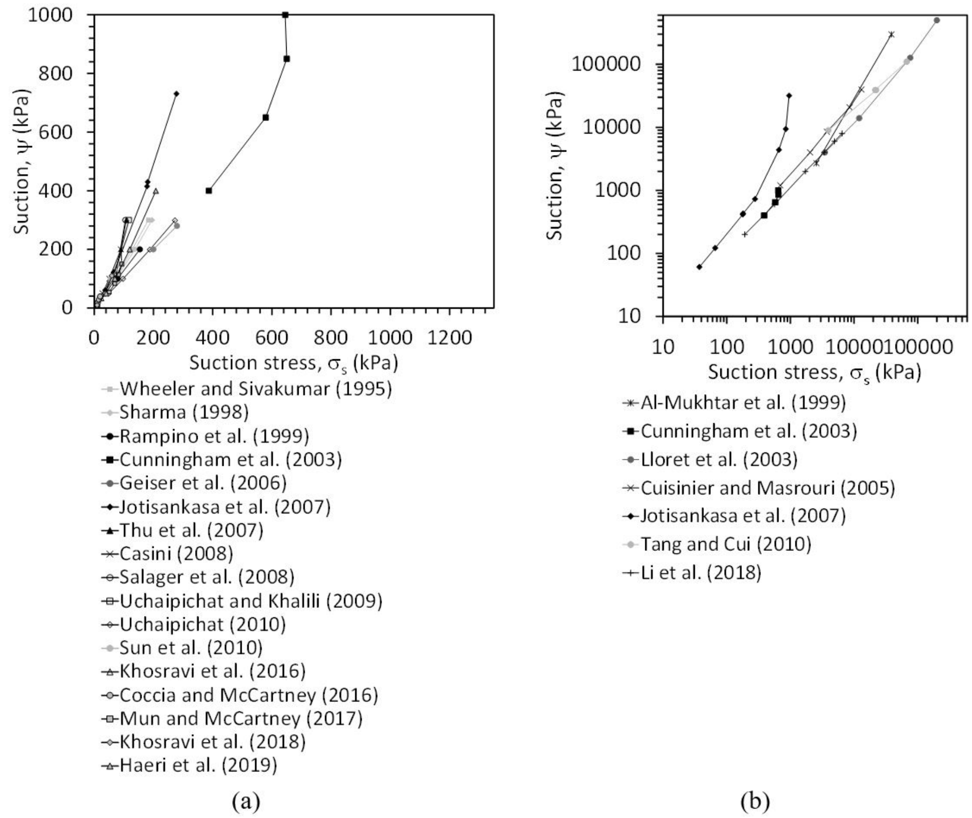
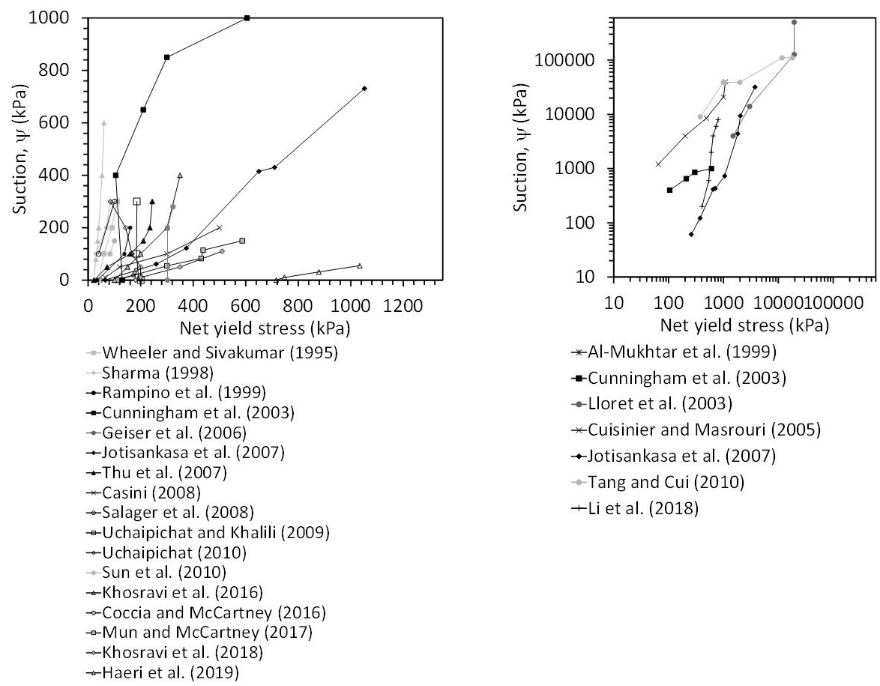


Figure 17. Suction vs. suction stress at the point of yielding: (a) Soils at low suctions (less than 1000 kPa); (b) Soils at high suctions.

range of values, the plot of suction versus the suction stress for high suctions is shown on a log-log scale in Figure 17b. The trends are more nonlinear in the high suction range, although the log-log slopes are very similar for many of the soils. For simplicity, the term suction is used interchangeably in Figures 18a and 18b even though the suction should be referred to as matric suction when capillarity is the dominant water retention mechanism, and the suction should be referred to as total suction when using Kelvin's law to calculate the suction from the relative humidity of the pore air.

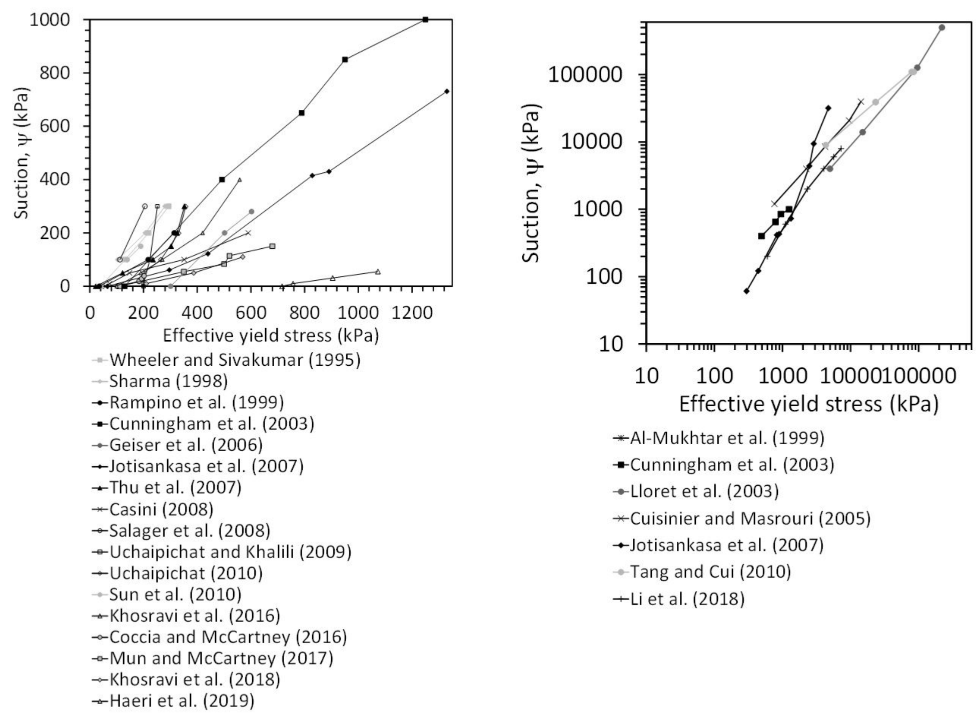
A plot of the suction versus the net yield stress is shown in Figure 18a. Different from the plot in Figure 17a, a much wider range of trends is observed in this figure. This demonstrates the variability in the net yield stress for different soils, which could be due to incorrect identification of the point of yielding from a relative flat compression curve plotted in terms of net stress. Similar conclusions can be drawn from the plot of suction versus net yield stress for soils tested at high suctions in Figure 18b, where a range of trends with both positive and negative intercepts is observed. However, when the results at low suctions are presented in terms of effective stress in Figure 19a, a more consistent behavior is noticed among the different soils, even though there is still greater variability than in the suction stress trends. The same can be said for the results at high suctions presented in terms of effective stress in Figure 19b, where the effective yield stress relationships fall into a tighter band.

A synthesis of the hydro-mechanical properties governing the shape of the SWRC and the compression curves for the unsaturated soils evaluated in the database is summarized in



(a) (b)

Figure 18. Suction vs. net yield stress: (a) Soils at low suctions (less than 1000 kPa); (b) Soils at high suctions.



(a) (b)

Figure 19. Suction vs. effective yield stress: (a) Soils at low suctions (less than 1000 kPa); (b) Soils at high suctions.

Table 2. This includes the parameters of the van Genuchten (1980) SWRC model fitted to the SWRC data presented in the studies, the estimated air entry suction, the slopes of the relationships between suction stress and suction, the slopes of the relationships between net and effective yield stresses and suction, and the slopes of the relationships between the VCL slope and suction. As the goal of this study is to understand the relative rates of increase in yield stress and effective stress with increasing suction, simple linear relationships were fitted to the yield stress and suction stress data. Although it is acknowledged that more advanced coupled relationships between suction, yield stress, and effective stress are available as summarized in the background sections, the simple linear relationships were found to provide a reasonable fit for most of the soils evaluated and the slopes summarized in Table 2 can be used for qualitative comparison purposes. The slopes in Table 1 were defined such that the suction is on the ordinate axis and the yield stress, suction stress, and VCL slope are on the abscissa. This definition was selected to be consistent with the way that the LC curve is typically presented, with the suction on the ordinate axis and the net yield stress on the abscissa. Following this definition, a small slope value means that the variable (suction stress, yield stress, or slope of the VCL) has a larger change with a unit change in suction than a large slope value. The R^2 values for the fitted relationships are typically greater than 0.95, with a minimum R^2 of 0.75 for the most nonlinear relationship. Other important hydromechanical variables noted in Figure 1 include the slopes of the recompression lines and the point of pressurized saturation, but insufficient information

Table 2. Summary of hydro-mechanical parameters of unsaturated soils from the literature.

Study	van Genuchten (1980) Drying Path SWRC Parameters			ψ_{aev} (kPa)	(Suction)/ (Suction Stress)	Net Yield Stress at ψ=0	(Suction)/ (Net Yield Stress)	(Suction)/ Effective Yield Stress)	(Suction)/ (Compression Curve Slope)
	N_{vG}	α_{vG} (kPa^{-1})	S_{res}						
Wheeler & Sivakumar (1995)	NR	NR	NR	NR	1.80	40	3.89	1.23	5E-04
Sharma (1998)	NR	NR	NR	NR	1.65	ND	3.41	1.12	9E-04
Maâtouk et al. (1995)	NR	NR	NR	NR	ID	20	16.9	ID	ID
Rampino et al. (1999)	NR	NR	NR	20	1.33	35	4.55	1.03	1E-05
Al-Mukhtar et al. (1999)	1.48	0.0002	0.00	ID	8.16	NY	NY	NY	-5E-07
Cunningham et al. (2003)	2.32	0.001	0.07	400	1.97	130	1.11	0.81	-2E-05
Lloret et al. (2003)	NR	NR	NR	3500	2.54	800	6.84	2.31	3E-07
Cuisinier & Masrouri (2005)	NR	NR	NR	NR	3.06	50	31.51	2.81	5E-06
Villar (2005)	1.80	0.00003	0.00	6000	ID	600	ID	ID	-1E-05
Geiser et al. (2006)	2.15	0.014	0.11	50	1.00	NY	3.86	0.80	ID
Jotisankasa et al. (2007)	1.51	0.007	0.05	45	2.77	65	0.87	0.66	9E-07
Thu et al. (2007)	2.80	0.009	0.17	60	4.04	22	1.20	0.94	-5E-05
Casini (2008)	2.04	0.079	0.37	5	2.34	59	0.41	0.35	7E-05
Salager et al. (2008)	2.28	0.009	0.12	50	6.00	110	3.33	2.15	3E-05
Uchaipichat & Khalili (2009)	3.15	0.025	0.03	50	4.95	200	-21.3	5.68	-7E-05
Sun et al. (2010)	1.29	0.020	0.00	40	1.48	ND	2.94	0.99	9E-06
Tang & Cui (2010)	1.95	0.00002	0.00	10000	1.59	200	5.48	1.26	-8E-07
Uchaipichat (2010)	1.49	0.00158	0.00	18	1.12	200	-2.23	1.50	1E-05
Ye et al. (2012)	NR	NR	NR	NR	ID	582	16.4	ID	ID
Coccia & McCartney (2016)	1.38	0.158	0.03	10	2.66	103	1.90	1.11	-3E-04
Khosravi et al. (2016)	2.60	0.019	0.06	10	1.54	716	0.16	0.14	-1E-03
Mun & McCartney (2017)	1.81	0.010	0.00	40	2.19	110	0.34	0.29	3E-04
Khosravi et al. (2018)	1.91	0.016	0.00	20	1.99	180	0.32	0.28	-2E-04
Li et al. (2018)	1.10	0.003	0.00	400	1.24	ND	21.1	1.18	-5E-06
Haeri et al. (2019)	3.00	0.100	0.00	10	2.10	100	1.60	0.93	2E-04

Note: NR = Not Reported; NY = No Yielding; ID = Insufficient Data.

was available to characterize these for the majority of the soils in the database.

A plot of the gradient in suction with suction stress versus the gradient in suction with effective yield stress is shown in Figure 20. If the two gradients fall onto the line of unity, a similar increase in both variables would occur with an increase in suction. Approximately half of the soils evaluated plot close to the line of unity. However, all but two of the soils plot below the line of unity, indicating that the soils exhibit a greater increase in effective yield stress with suction than the increase in suction stress with suction. The two points that plot above the line of unity were the two that had decreases in the net yield stress with increasing suction. According to Khalili et al. (2004), this indicates that the soils that plot below the line of unity will be susceptible to collapse upon wetting.

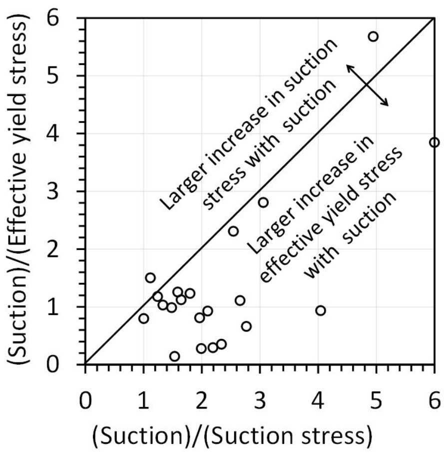
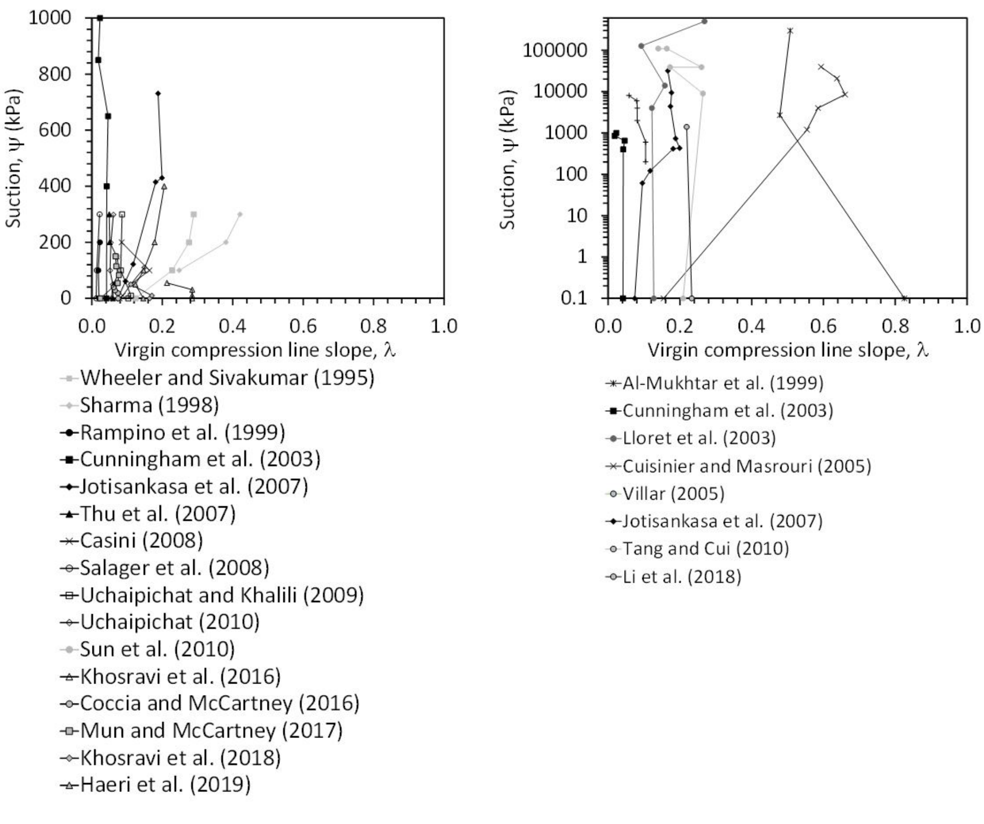


Figure 20. Slopes of suction vs. suction stress and suction vs. effective yield stress for different studies in the literature. Note: smaller slopes indicate greater increases in the variable with increasing suction.

Plots of the relationships between suction and the slope of the VCL for the soils tested at low and high suction ranges are shown in Figures 21a and 21b. A low suction, the slopes of the VCLs range from 0.01 to 0.42, while at high suctions much larger slopes of VCLs are noted ranging from 0.018 to 0.83. The abnormally large slopes of the VCLs were observed for some soils at high suctions because the compression curves at the highest stress ranges in these tests were still showing a nonlinear decrease after yielding and may not have yet stabilized at the actual VCL. The trends in the slopes of the VCLs with increasing suction are generally flat, indicating that suction does not have a major effect on the slope of the VCL for these soils. In other words, the VCL for the unsaturated soils is parallel to the VCL for saturated soil for the range of effective stresses



(a) (b)

Figure 21. Suction vs. slope of the virgin compression line: (a) Soils at low suctions (less than 1000 kPa); (b) Soils at high suctions (values at saturation shown at a suction of 0.1 kPa).

investigated in these studies. As summarized in the last column of Table 2, most of the soils have a small positive rate of increase in the slope of the VCL with suction with a magnitude less than 2×10^{-4}. Three soils have larger positive rates of increase in the slope of the VCL with suction while two soils showed large negative rates of increase in the slope of the VCL with suction. A positive rate of increase reflects a trend like that shown in Figure 1a, where the VCLs for unsaturated soils converge with the VCL for saturated soil, while a negative rate of increase reflects a diverging trend. It is expected that if higher magnitudes of net stress had been applied that a positive rate of increase would eventually be observed for all the unsaturated soils, similar to the trends in the compression curves observed by Jotisankasa et al. (2007) and Mun & McCartney (2017) in Figures 9 and 14, respectively.

7. Conclusion and recommendations for future testing

This state-of-the-art paper presents the results from the analysis of a database of compression curves for unsaturated soils with the goal of understanding trends in the effective stress (or suction stress), yield stress, and slope of the VCL with increasing suction. Suction hardening was observed in nearly all the soils evaluated, and the effective yield stress curves were found to have more uniform shapes when compared with the net yield stress curves. The effective yield stresses generally showed a linearly increasing trend with increasing suction. Similar behavior was noted for soils tested at both high and low suctions, although the soils at high suctions often were not compressed to high enough net stresses to fully exhibit yielding. For all but two of the soils, the rate of increase in the effective yield stress with increasing suction was greater than the rate of increase in the suction stress with increasing suction. However, the rate of increase in the effective yield stress with increasing suction was only significantly greater for about half of the soils evaluated. The slopes of the VCLs for unsaturated soils were typically steeper than the slopes of the VCLs for saturated soils, with a few soils showing the opposite trend due to lower net stresses applied.

The observations from this study emphasize the importance of tracking the degree of saturation during compression of unsaturated soils, as this is a critical component to calculating the effective stress. A related recommendation is that all studies focused on the unsaturated behavior of soils should present the soil-water retention curve along with any scanning paths that may occur during testing. The observations also emphasize the importance of tracking changes in volume of unsaturated soils during initial suction application, as this may provide insight into the elastic behavior of the soil before yielding occurs during application of net stresses. Inconsistent comparisons between the compression curves for saturated and unsaturated soils may have occurred because of large differences between the void ratios at the beginning of compression, which may have been caused by uncertainty in the volume changes occurring during suction application. The observations from this study indicate that it is critical to apply sufficiently high stresses to unsaturated soils to induce yielding, especially for heavily compacted soils or soils at high suction magnitudes. Future areas of research on this topic include characterizing the yielding behavior of unsaturated soils prepared using slurry consolidation to better understand the role of the soil structure induced by compaction and studying fundamental linkages between the yield stress and slope of the VCL.

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Declaration of interest

The authors have no conflicts of interest.

Authors' contributions

John S. McCartney: conceptualization, methodology, writing – original draft preparation, data curation. Fatemah Behbehani: investigation, data curation, reviewing and editing.

List of symbols

e	Void ratio.
N_{vG}	Parameter of the van Genuchten (1980) SWRC model.
p'	Mean effective stress.
p_{net}	Mean net stress.
S	Degree of saturation.
S_e	Effective saturation.
S_{res}	Residual degree of saturation.
v	Specific volume.
α_{vG}	Parameter of the van Genuchten (1980) SWRC model.
σ_s	Suction stress.
σ_v'	Vertical effective stress.
$\sigma_{v,net}$	Vertical net stress.
χ	Effective stress parameter.
ψ	Suction.

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