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**A simple method for assessing the probability of liquefaction of tailings dams**

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**Case Study**



# **1. Introduction**

In recent years, significant accidents have occurred in tailings dams in Brazil, raising doubts about the safety of these structures and lack of trust in their performance (Morgenstern, 2018). The catastrophic failure of Fundão tailings dam, which occurred in Mariana, in December 2015, was one of Brazil's major environmental accidents, leaving a serious scar on the mining industry worldwide. A few years later, the world was again shocked by another tragedy, this time captured on the security video cameras showing the abrupt failure of the tailings dam at the Feijão mine, in Brumadinho, in January 2019, and the subsequent massive devastation caused by the uncontrolled flow of the saturated tailings. These two accidents were surely due to static liquefaction of the dams' tailings.

The accidents raised questions about the methodologies for analyzing dam stability in Brazil, as well as the acceptability of the risk by society.

Concepts of risk analysis can be applied to dams, but they are not yet widely used in Brazil. Espósito & Palmier (2013) applied two risk analysis methods to Brazilian dams and concluded that this type of analysis allows a better comprehension of dam behavior. At the same time, the use of probabilistic procedures has proved to be a valuable tool in assessing the safety of these structures. Regulatory guides usually specify a minimum acceptable factor of safety (*FS*) of 1.50 as the design criteria of dam structures. CDA (2013) indicates this value as the minimum adequate in static analyses of downstream slopes under steady state (long-term) conditions, as seen in Table 1. Australian National Committee on Large Dams (ANCOLD, 2022) presents, beyond the safety factor related to each case, the strength parameters that should be adopted, as displayed in Table 2.

In recent years, however, the use of probabilistic analysis of dam safety has been growing rapidly, as emphasized by CDA (2013). In Brazil, probabilistic analysis has been still limited to academic studies and ignored as a technical tool in the decision-making process of assessing the safety of existing structures.

The aim of this work is to contribute to a better understanding of the risk of failure of existing tailings dams. In this context, this paper presents a simple procedure for estimating the probability of liquefaction failure at the time of the accident, indicating that such analysis could have been of great help in anticipating these tragic occurrences.

# **2. Liquefaction probabilistic analysis**

Liquefaction may classically occur in saturated contractive loose sand deposits. Olson & Stark (2003a) suggests that a liquefaction analysis consists of three main tasks: (1) an analysis of flow failure susceptibility, (2) a triggering analysis, and (3) a post-triggering failure stability analysis.

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Loading Conditions	Minimum $FS$	Slope
End of construction, before the reservoir filling	1.30	Upstream, downstream
Long-term steady state	1.50	Downstream
Full or partial rapid drawdown	.20	Upstream

**Table 1.** Minimum Factor of Safety in static analyses recommended by CDA (2013).

#### **Table 2.** Minimum safety factors recommended by ANCOLD (2022).



Legend: <sup>a</sup>Related to the reliability on the residual parameters selection. *SF* = 1.0 can be applied in cases where conservative parameters are adopted. <sup>b</sup>Residual strength after several loading cycles or liquefied strength in the case of liquefiable materials.

The first step of a liquefaction analysis of tailings deposits is to investigate if the material is loose and contractive (i.e., susceptible to flow failure). Simple methodologies for assessing the soil's susceptibility to liquefaction have been described by Olson (2001) and Robertson (2010), with applications to practical cases.

As dam liquefaction usually occurs under undrained conditions, the key factor in computing the probability of liquefaction failure is to estimate the probability of undrained failure. Martin & McRoberts (1999) have proposed that the results of undrained analysis must be understood in the context of the potential triggers of undrained shear. These authors state that the stability margin against the shear failure of tailings dams is better expressed in terms of both the probability of undrained failure and a safety factor from a rigorous limit equilibrium method.

Martin & McRoberts (1999) suggest there are several potential triggers for undrained failure, including erosion, high impoundment rate, fast steepening of the downstream slope, seepage breakout at the dam's face, foundation and/or embankment movement, sudden reduction of mean effective stress, severe storm runoff, blockage and failure of spillways, earthquake, and seismic deformation.

Olson & Stark (2003a) stated that the strength parameter mostly used in stability analyses is the undrained stress ratio  $s_u / \sigma'$ <sub>v</sub>, which is the ratio of undrained strength  $(s_u)$  to initial vertical effective stress. This stress ratio parameter offers a rational way to account for effective stress variations, state parameter (or void ratio), and grain size characteristics (Olson & Stark, 2003b).

This parameter may be defined in undrained triaxial tests in the laboratory or estimated from field piezocone (CPTu) tests results (Olson & Stark, 2002, 2003a; Sadrekarimi, 2014; Robertson, 2010).

After defining the parameters needed for the analysis, the probability of undrained failure can be obtained in several ways. In this paper, Spencer's limit equilibrium method and the FOSM probabilistic method (First Order Second Moment, described by Christian et al., 1992) were adopted, for being practical and suitable in previous applications (Dell'Avanzi & Sayão, 1998; Sayão et al., 2012; Vecci & Sayão, 2019; Yokozawa et al., 2019).

The Spencer limit equilibrium method (Spencer, 1967) was chosen for this analysis because it is a widely used rigorous method. In this scenario, the rupture surface is free and can be either circular or non-circular, satisfying the forces and moments equations of equilibrium. The main hypothesis adopted in this method is the parallel inter-slice forces, which means that the forces between the slices can be replaced by a resultant  $Q_i$  inclined  $\delta$  to the horizontal (Spencer, 1967).

Christian et al. (1992) and Sayão et al. (2012) indicate that the partial derivatives of the safety factor function in the FOSM method may be obtained by a small variation of each random variable. They suggested that each variable should be distinctly subjected to an increase of 10% on its average value while keeping all other variables fixed. In this way, the resulting variation of the safety factor is adequately assessed. In order to estimate this probability of failure by the FOSM method, the average and standard deviation values of each random variable must be previously obtained, from a proper number of reliable field or lab test results. This requirement may impose a serious difficulty to use probabilistic methods in smaller, low-budget projects.

In the tailings dam's stability analyses herein presented, the ratio  $s_u / \sigma'$ , was considered the only random variable, due to its notorious higher relevance.

If the tailings are susceptible to liquefaction, the probability of liquefaction may be given by Equation 1:

$$
P_L = P_{FOSM} * P_t \tag{1}
$$

where  $P_L$  is the probability of liquefaction,  $P_{FOSM}$  is the probability obtained by the FOSM method, and  $P_t$  is the





**Legend:** Note 1. Definitions for the population at risk: None – There is no identifiable population at risk, so there is no possibility of loss of life other than through unexpected misfortune; Temporary – People are only temporarily present in dam-breach inundation zone (e.g., in cottages for seasonal use, passing through transportation routes, or participating in recreational activities); Permanent – The population at risk is usually located in the dam-breach inundation zone (e.g., as permanent residents): three consequence risk classes (high, very high, extreme) are proposed to allow for more detailed estimates of potential loss of life (to assist in decision-making if appropriate analysis is carried out). Note 2. Implications for loss of life unspecified – the appropriate level of safety required at a dam where people are momentarily at risk depends on many conditions: the number of people, the exposure time, the nature of their activity, etc. A higher class may be appropriate, depending on the conditions. The design flood requirement, for example, could not be higher if the temporary population is likely to be absent during the flood season.

probability of occurrence of a trigger, as indicated by D'Hyppolito (2023).

After careful analysis of three Brazilian tailings dams, D'Hyppolito (2023) concludes that the probability of trigger for liquefaction in Brazilian tailings dams may be taken 1.0, due to frequent problems, such as insufficient geotechnical investigation, fast tailings disposal, malfunction of drainage systems (Ávila et al., 2021) and divergences with design parameters.

CDA (2013) suggests that the probability of liquefaction be obtained from Equation 1 and compared to reference values, which depend on the consequences of a potential dam failure (Table 3). This CDA classification table regulates the necessary safety criteria in terms of design parameters and proposes the risk assessment for dam safety considering the approach shown in Figure 1, with life safety risk guidelines consistent with values used in other hazardous industries and with the principle that risks should be made as low as reasonably practicable (ALARP). In Figure 1, the probability of more than N fatalities may be considered equal to the probability of liquefaction obtained from Equation 1.

Recently, the Australian National Committee on Large Dams released a review of the guidelines on risk assessment (ANCOLD, 2022), presenting a chart to evaluate the societal risks for existing dams, as shown in the Figure 2. It is interesting to note the similarities between CDA and ANCOLD charts.



**Figure 1.** Risk assessment for dams, as a function of number of fatalities (CDA, 2013).

#### **2.1 Fundão dam**

On the afternoon of November 5th, 2015, Fundão Dam started to collapse, in Mariana, an important iron mining center in Minas Gerais State, Brazil. This dam was built by the upstream method and was 110 m high, by the time of the accident. This event has been reported in detail by Morgenstern et al. (2016).

Considered as one of the Brazilian's worst environmental disasters, the failure of Fundão Dam resulted in massive damages and caused 19 deaths. Until January 2020, a total of R\$ 8.2 billion (or US\$ 1.8 billion) had been already disbursed in repairs and compensations, as reported by Renova (2022). The number of deaths would have been much higher if the accident had occurred at night, as most people would have been sleeping in houses devastated by the massive flow of saturated tailings released by the dam's failure.

The cross-section of Fundão Dam (Figure 3), reported by Morgenstern et al. (2016), has been considered in the analyses reported in this paper. Two types of slurry tailings may be noted, both delivered in separate pipelines to the Fundão impoundment. Sandy tailings, or simply Sands, are a mixture of sand-sized and finer silt particles and are relatively free-draining, but susceptible to liquefaction when loose and saturated. The slimes, on the other hand, are much finer and clay-like in nature - soft and compressible tailings, with low permeability.

In Figure 3, a mixture of sand and slimes can be noted, but the exact proportion of this material is not clear. A unique parameter, chosen from field piezocone (CPTu) test results, was adopted to represent the tailings.



Regarding previous studies related to similar tailings dams (Sayão et al., 2012; Braga, 2019), it was possible to observe that the variation of the undrained strength ratio is responsible for more than 90% of the failure probability. Additionally, Duncan (2000) indicates this parameter presents the most expressive variability of all soil's strength parameters, with a coefficient of variation reaching about 40%. Consequently, it may be suggested that adopting the undrained strength ratio as the only random variable is justified, considering the simplification of the proposed procedure, without compromising the accuracy of the result.

Regarding the in-situ tests performed on the dam, over 50 different expeditions were reported containing a wide range of tests, including field piezocone test (CPTu), standard penetration test (SPT), and permeability test. Morgenstern et al. (2016) report that at that time there was no CPTu or SPT made from the dam's crest until the bedrock foundation. After the failure, additional tests were performed in the Germano dam, similar to and close to Fundão dam.

The analysis of CPTu data showed that about 70% to 80% of the sandy tailings within 75 m of the dam crest were noted to be contractive, based on the liquefaction criterion proposed by Robertson (2010). At greater distances from 180 m, 95% of the sandy tailings are contractive. This indicates that the hydraulically discharged Fundão sandy tailings satisfied the contractiveness requirement for liquefaction flow sliding.

Additionally, a back analysis was carried out of a failure incident that occurred in September 2014. At that time, there was a local failure of the dam. The strength parameters resulting from this back analysis may be used to compare with the parameters from CPTu tests. From this analysis, Morgenstern et al. (2016) obtained a value of  $0.22$  for  $(s_u / \sigma'_v)_{peak}$ .

The average results for each undrained strength ratio in Fundão tailings are shown in Table 4, obtained from the results reported by Morgenstern et al. (2016).

The present work adopted as average value the result obtained in the back analysis from the 2014 incident. Since there was a located failure in this incident, it can be considered that the undrained strength was mobilized. Furthermore, this **Figure 2.** Societal risk assessment for existing dams (ANCOLD, 2022). value is similar to the CPTu results, as shown in Table 4.



**Figure 3.** Cross section of Fundão dam from Morgenstern et al. (2016).

The standard deviation from the results of CPTu tests may be regarded as evidence of the variability of the material at depth. It is important to consider the fact that various problems were reported in the tests performed in Fundão, which led to doubts about the reliability of the results and their representativeness related to all the material contained in the dam, as presented in Morgenstern et al. (2016). Nevertheless, the analysis of Morgenstern et al. (2016) results shows that it can be obtained a standard deviation of 0.03 considering all the CPTu results available in the area. Briefly, the adopted values are:  $(s_u / \sigma'_v)_{neak} = 0.22$  e standard deviation = 0.03.

#### **2.2 Feijão dam**

At 12:28 pm local time on January  $25<sup>th</sup>$ , 2019, the tailings dam B1 at Córrego do Feijão Iron Ore Mine (Vale, Brazil), in Brumadinho, Brazil, suffered a sudden failure, releasing  $12.7$  million  $m<sup>3</sup>$  of tailings, and resulting in a catastrophic mudflow that traveled rapidly downstream. The mudflow destroyed several structures (refectory, offices, homes, and a bridge). This dam failure is unique for having high-quality video images that provide insight into the failure mechanism (Robertson et al., 2019).

At the time of the failure, Feijão dam was 86 m high, with a 720 m long crest. More details of the dam and the causes of its failure were reported by Robertson et al. (2019).

The main dam tragedy ever seen in Brazil has resulted in 270 deaths, severe damage to the environment and about US\$ 8 billion already spent by Vale for social and environmental repair. The cross-section of Feijão dam is presented in Figure 4. This was the highest and the most critical section, as indicated by Robertson et al. (2019) and Polícia Federal (Minas Gerais, 2019).

Like the Fundão case, there were two types of tailings, coarse and fine tailings, and the exact location/proportion of these materials is not clear. This paper adopted a unique parameter to the tailings in the analysis, and the parameter was chosen based on the CPTu tests, which reflect the real situation in the field. In Brumadinho's case, a larger amount of test data is available, and Robertson et al. (2019) presented a statistical analysis of the CPTu results, as shown in Figure 5. Considering the adoption of a single representative parameter, the option made was to consider the average value equivalent to the weighted average according to the frequency of each type of tailings, and the standard deviation adopted was obtained for the sand tailings due to the greater variability observed in this case. In this scenario, these are the selected parameters:  $(s_u / \sigma'_v)_{peak} = 0.27$  standard deviation = 0.12 Additionally, the value adopted is consistent with another analysis described in Braga (2019), Polícia Federal (Minas Gerais, 2019) and TUV SUD (2018).







**Figure 4.** Critical section from Feijão dam (Robertson et al., 2019).

## **3. Results**

#### **3.1 Fundão dam failure**

As presented, the average elected for  $(s_u / \sigma'_v)_{peak}$ was 0.22, with 0.05 for standard deviation. The other parameters were adopted equal to Morgenstern et al. (2016) because they have a small impact on the results. The stability



**Figure 5.** Statistical analysis of the peak undrained shear strength ratio (Robertson et al., 2019).

analysis performed for the average parameters using the Spencer method is presented in Figure 6. Table 5 exhibits the parameters adopted for the analysis.

The combination of the FOSM probabilistic method and the Spencer limit equilibrium method, considering that the random variable has a normal (Gaussian) distribution, resulting in 0.37 for the Reliability Index *β* (Christian et al., 1992) and the probability of failure  $P_{FOSM} = 36\%$ .

The calculated probability is quite high and inconsistent with any kind of engineering construction. Based on the results, the dam was not stable and could be subjected to an undrained collapse because of a trigger.

Regarding the occurrence of triggers, according to Morgenstern et al. (2016), the dam was showing (i) serious issues related to drainage; (ii) disobedience to the width of the beach designed, leading to a fine waste settlement in unforeseen places; (iii) small displacements through the instrumentation; and (iv) quick loading to heighten the structure. Those problems are known for their possibility to trigger the tailings liquefaction, as already commented.

Considering that the tailings were subjected to liquefaction and the triggers occurred, the Equation 1 shows that:  $P_{liquefaction} = 36\%$  The failure consequences can be classified as Extreme, according to Table 2 (CDA 2013), due to the resultant environmental impact, which was expressive and unreturnable.

The risk associated to the Fundão dam was way beyond the acceptable. In this scenario, measures should have been taken to mitigate this risk through probability reduction and consequence control.



**Figure 6.** Stability analysis of Fundão Dam.

Material name	Color	Unit Weight $(kN/m3)$	Cohesion (kPa)	$\phi$ (deg)	Vertical stress ratio (average)
$Dam - raisings$	Gray				-
Coarse tailings	Yellow	າາ	-		0.22
Slimes	Red	າາ ∠∠			0.22
Foundation	Brown	າາ ∸	40		-

**Table 5.** Parameters adopted in Fundão Dam analysis (D'Hyppolito, 2023).

#### **3.2 Feijão dam failure**

The stability analysis performed by the Spencer method with average parameters is presented in Figure 7. Table 6 exhibits the parameters adopted in the analysis. The average selected for  $(s_u / \sigma_v)_{peak}$  was 0.27 and 0.12 for standard deviation. These other parameters were adopted as indicated by Morgenstern et al. (2016) because they have a small impact on the results.

The combination of the FOSM probabilistic method and the Spencer limit equilibrium method, considering that the random variable has a normal (Gaussian) distribution, achieves 0.07 for the reliability index *β* (Christian et al., 1992) and the failure probability is 47%.

As presented by Robertson et al. (2019), (i) the dam was presenting significant issues related to drainage; (ii) there were layers of fine tailings in areas supposed to have tailings with larger particle size; (iii) the foundation data and information were very limited; and (iv) slow displacements were observed, consistent with a possible creep, through one of the instruments (InSar). In this scenario, the liquefaction triggers were present.

Considering the tailings were subjected to liquefaction and the triggers occurred, the use of Equation 1 results in  $P_{liquefaction} = 47\%$ . The failure consequences can be classified as Extreme, according to Table 2 (CDA, 2013), due to the associated numbers of deaths.

The risk associated to the Feijão dam was not satisfactory. In this scenario, actions should have been taken to mitigate this risk by reducing the probability of failure and controlling the consequences.

#### **4. Discussion**

Brazilian dams have specific problems demanding special caution, such as drainage systems (Ávila et al., 2021), quick loading, and lack of attention to the design conception elements, like the beach width (Martin et al., 2002). These problems may be seen frequently in tailing dams and open the chances for the occurrence of triggers.

Considering the information presented in this study, the most important element in the liquefaction risk analysis is the undrained probability of failure. The estimated probabilities in the two cases reviewed herein were -36% at the time of the Fundão dam (Mariana) failure, which collapsed in 2016, and 47% at the time of the Feijão dam (Brumadinho) failure, which collapsed in 2019.

These two cases discussed in this paper, Fundão and Feijão dams could be considered to present very high levels of associated consequences. For the Feijão dam, which caused over 200 deaths, the acceptance limit as suggested by CDA (2013) would be  $10^{-7}$  and tolerability of 10-5. In Fundão dam, in which much lower fatality level (19 deaths), the acceptance limit as suggested by CDA  $(2013)$  would be  $10^{-6}$  and tolerability of  $10^{-4}$ . The limits of tolerability proposed by ANCOLD (2022) are similar. However, it was not the worst case. As previously mentioned, the number of deaths in the Fundão case could have been

Material name	Color	Unit Weight $(kN/m3)$	Cohesion (kPa)	$\phi$ (deg)	Vertical stress ratio
Dam raisings	Yellow			36	$\overline{\phantom{0}}$
Dam raisings	Orange	28		36	0.22
Tailings	Gray	22	$\overline{\phantom{a}}$	-	0.27
Foundation	Green	20		30	$\overline{\phantom{0}}$

**Table 6.** Parameters adopted in Feijão dam analysis (D'Hyppolito, 2023).



**Figure 7.** Feijão dam stability analysis with average parameters, obtaining a safety factor of 1.03.

much higher, and similar to the Brumadinho case, had the accident occurred at night. The computed results for liquefaction failure probability were much higher than these reference values and would be unacceptable for any kind of engineering construction.

#### **5. Conclusions**

This paper presents a simple procedure for estimating the risk associated to tailings dams, using simple wellknown methods. The practical application of the proposed procedure was illustrated by two recent cases of dam failure, suggesting that the risk associated with the collapsed structures was higher than the acceptable level indicated by current standards. The results of the analysis presented in this paper indicate that in the case of the Fundão Dam, the estimated probability of failure was 36%, while in the case of the Feijão Dam, it was 47%. In both scenarios, the probabilities suggest a level of risk deemed intolerable for dam structures.

The probability of failure may be estimated with the procedure suggested in this paper and may be considered in the decision-making procedures, regarding the need for intervention and mitigation of the risk of accident and its consequences by triggering an Emergency Plan.

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## **Declaration of interest**

The authors have no conflicts of interest to declare. All co-authors have observed and affirmed the contents of the paper and there is no financial interest to report.

#### **Authors' contributions**

Lays Cristina de Souza D'Hyppolito: conceptualization, data curation, methodology, visualization, writing - original draft preparation. Alberto Jardim Sayão: formal analysis, funding acquisition, supervision, validation, writing reviewing and editing. Anna Laura Lopes Nunes: formal analysis, methodology, supervision, validation.

#### **Data availability**

The datasets generated and analyzed in the course of the current study are available from the corresponding author upon request.

#### **List of symbols and abbreviations**



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