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Myths and misconceptions related to unsaturated soil mechanics

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Article

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Abstract
There have been three main pillars associated with the development of an applied engineering science for both saturated and unsaturated soil mechanics; namely, i) the synthesis of continuum mechanics theories of physical behavior, ii) the laboratory measurement of relevant soil properties, and iii) analyses that illustrate the solution of practical example problems. Geotechnical engineers have, however, been relatively slow in adopting unsaturated soil mechanics into geotechnical engineering practice. There have been several so-called "myths or misconceptions" that appear to have hindered the application of unsaturated soil mechanics. This paper attempts to describe and dispel what are deemed to be misconceptions related to the more general implementation of unsaturated soil mechanics into engineering practice. The so-called "myths" come from the acceptance of false information related to unsaturated soil behavior and a hesitancy to embrace changes to existing empirical protocols. Several misconceptions are identified in the paper that are related to: i) complexity of unsaturated soil mechanics theories, ii) inability to readily measure soil suctions *in-situ*, iii) the nonlinearity of unsaturated soil property functions, iv) permanency of soil suctions above the water table, v) difficulties associated with assessing ground surface moisture flux conditions, and vi) difficulties associated with numerical modeling that involves solving nonlinear partial differential equations. Each of the above-mentioned items are dealt with as myths or misconceptions in the sense of being impediments to the application of unsaturated soil mechanics in geotechnical engineering practice.

1. Introduction to debunking unsaturated soil mechanics myths and misconceptions

Soil mechanics became recognized as an important applied science following the study of saturated soil behavior near the middle of the 1900s. Soil mechanics became widely accepted through the publication of books such as *Theoretical Soil Mechanics* by Terzaghi (1943), *From Theory to Practice of Soil Mechanics* by Terzaghi & Peck (1967), *Fundamentals of Soil Mechanics* by Taylor (1948), and others. These books synthesized the general behavior of saturated soils for three main phenomenological processes; namely, i) flow of water through porous media, ii) volume change and distortion of soils, and iii) shear strength behavior. In the case of each physical behavior there were three so-called pillars of soil mechanics. These were: i) the synthesis of a theory of physical behavior, ii) the laboratory measurement of relevant soil properties, and iii) the presentation of relevant example problems. The three-pillar paradigm has well served the geotechnical community for over half a century.

Geotechnical engineers have, however, been relatively slow in embracing and implementing a similar consensus involving the three pillars for unsaturated soil mechanics. Unfortunately, geotechnical engineers seem to have "bought into" several misconceptions related to unsaturated soil mechanics. I will attempt to dispel these misconceptions and bring value-added benefits to the implementation of unsaturated soil mechanics in geotechnical engineering practice. The so-called "myths" are deemed to be the result of accepting misconceptions or false information along with a lack of desire to embrace change in existing protocols.

2. Describing misconceptions common in "unsaturated soil mechanics"

Along with all the outstanding past research on unsaturated soils, it seems inevitable that there should be a few inaccurate perceptions about unsaturated soil mechanics theory as it pertains to mainstream geotechnical practice. Finding a consensus on fundamental perceptions is important as engineers

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move towards the worldwide application of unsaturated soil mechanics in geotechnical engineering practice.

2.1 Myths & misconceptions

The application of unsaturated soil mechanics has been hindered through acceptance of one or more of the following false rationales. For example, it is often rationalized that;

1) unsaturated soil mechanics theories are excessively complex and hard to understand,
2) soil suctions need to be measured *in-situ*,
3) *in-situ* suctions are temporary and disappear following rainfall,
4) unsaturated soil properties are too costly and difficult to measure in the laboratory,
5) analysis of practical problems involves complex nonlinear mathematics, leading to serious modeling challenges and rendering it unpractical,
6) ground surface boundary conditions take the form of moisture fluxes that are related to highly variable and random weather conditions, and,
7) it is often rationalized that the present protocols are acceptable, "So why change? Aren't we doing fine?"

I will attempt to dispel each of the above-mentioned concerns as myths (i.e., unsubstantiated, indefensible false perceptions). The hope is that greater benefits might be accrued through bringing unsaturated soil mechanics into the practice of geotechnical engineering. Section 2 of this article describes the context and variables associated with unsaturated soils while the remaining parts of the article are devoted to dispelling myths and misconceptions amongst geotechnical engineers.

2.2 Pillars of unsaturated soil mechanics

There have been three so-called pillars associated with the application of saturated soil mechanics. Likewise, there are three pillars for unsaturated soil mechanics (See Figure 1). The pillars are labelled as: i) theory, ii) laboratory testing, and iii) numerical modeling. The pillar "theory" stands for a synthesis of research findings (and hopefully a consensus) that have been presented at research conference and through journal publications. "Theory" contains a distillation of protocols that appear to be acceptable for prudent geotechnical engineering practice. Protocols constitute a living, ongoing statement that may evolve as new research is published.

The *First International Conference on Unsaturated Soils* was held in Paris, France, in 1995. Since that time there has been a proliferation of regional and international conferences focusing on unsaturated soils research. The result has been the emergence of a theoretical context that treats unsaturated soil mechanics as a continuum mechanics type extension of the framework that has proven to be so successful for saturated soil mechanics.

Geotechnical engineers are known for their practice of retrieving small (undisturbed) soil samples from boreholes and then measuring relevant physical soil properties in the laboratory (e.g., coefficient of permeability, coefficient of volume change, shear strength). This procedure has proven to be acceptable for the application of saturated soil mechanics; however, the cost of using the same approach for testing unsaturated soils has proven to be far too high, being estimated to be in the order of 10 times as expensive as testing for saturated soil properties (Fredlund & Rahardjo, 1993).

The solution to the costly laboratory testing of unsaturated soils dilemma has involved the measurement of one or more alternate soil property functions that can then be used to estimate required unsaturated soil property functions, USPFs. The most common proposed procedure has been to measure the soil-water characteristic curve, SWCC, and the shrinkage curve, SC, in the laboratory. These two USPFs can then be used to calculate all volume-mass versus soil suction relationships for the soil (under drying soil conditions). The USPFs can cover the entire range of possible soil suctions (i.e., from a fraction of 1 kPa to a maximum value of 1,000,000 kPa). The USPFs are then used to estimate the i) unsaturated soils

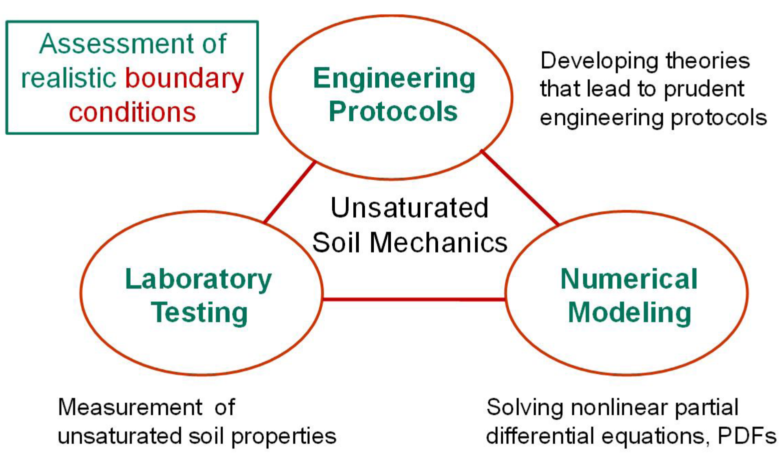

Figure 1. Basic pillars for implementation of unsaturated soil mechanics (Fredlund, 2017).

permeability function, ii) water storage function, iii) shear strength function, and iv) other unsaturated soil property functions. The cost of performing the combined SWCC and SC tests in the laboratory is generally less than $1500 (CAD).

Generally, the cost associated with measuring the SWCC and SC can be shown to yield considerable cost-benefit to the client. The functions are called "estimated" unsaturated soil property functions but are of satisfactory accuracy for most geotechnical engineering applications. Certainly, this approach becomes a significant improvement over not analyzing the unsaturated portion of the soil profile (Fredlund & Houston, 2009).

The third pillar of unsaturated soil mechanics involves the use of numerical modeling techniques for the solution of (nonlinear) partial differential equations that describe the physical behavior of an element of unsaturated soil. The past few decades have seen two disciplines rise to the occasion with valuable resources for solving unsaturated soils numerical modeling solutions. These are the computer software discipline along with high-speed computer hardware and the mathematics discipline. Together, these disciplines have provided practical modeling techniques that ensure the convergence of partial differential equations to a unique solution.

Also shown on Figure 1 is the importance of assessing climatic-dependent moisture flux boundary conditions. With only a few exceptions, most unsaturated soil problems are near ground surface problems. Therefore, soil-atmosphere interaction defines an important boundary condition for the solution of practical problems. Suffice it to say at this point that data from weather stations have become a valuable resource, providing an opportunity for extensive analysis of collected weather information.

2.3 Importance of reasonable assumptions

Moving a basic science to an applied science requires invoking a series of assumptions. The effect of various assumptions is usually studied by researchers and a decision is made on the most acceptable assumptions to invoke.

In the foreword to *Unsaturated Soil Mechanics in Engineering Practice*, Morgenstern (2012, p. xiii) wrote,

> A fundamental distinction between saturated and unsaturated soil behavior is the need to express the relationship in the latter between water content and soil suction, i.e., the soil-water characteristic curve. Since 1993, there has been an explosion of studies into the measurement of soil suction and the development of soil-water characteristic curves.

It is fair to say that the use of the soil-water characteristic curve (and shrinkage curve) has opened the way for an increase in the application of unsaturated soil mechanics. The laboratory testing protocols are not perfect but are a vast improvement over the previous omission of the unsaturated soil zone when performing an analysis. Research into the use of the SWCC for the estimation of USPFs has caught the imagination for geotechnical researchers around the world (Fredlund, 2017). Recent research conferences have shown this topic to be the single most popular area of unsaturated soil mechanics' research.

2.4 Boundaries of the unsaturated soil zone

An unsaturated soil profile (i.e., portion of soil above the water table), can be subdivided into three main zones, as shown in Figure 2. The zones are the: i) capillary zone immediately above the groundwater table where the voids are predominantly filled with water, ii) two-phase zone where the soil voids are filled with varying ratios of water and air, iii) dry zone where the voids of the soil mainly contain air (Blight, 2013; Fredlund, 2015; Houston, 2019; Rahardjo et al. 2019; Tarantino & El Mountassir, 2013; Vanapalli & Mohamed, 2006). It is important for geotechnical engineering purposes to recognize that the unsaturated zone commences immediately above the water table where the pore-water pressures become negative. The division between the capillary zone and the two-phase zone designates the air-

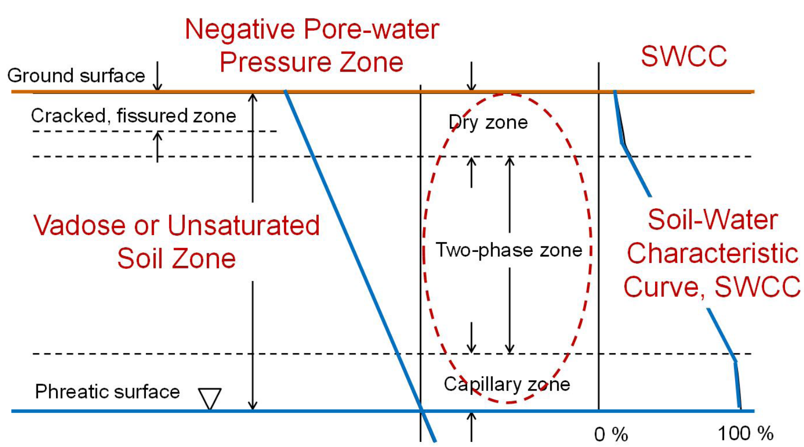

Figure 2. Subdivisions of the unsaturated soil zone (Fredlund, 2015).

entry value, AEV, of the soil. The AEV is likely the single most significant piece of soils information required by the geotechnical engineer. The AEV designates the point at which the soil begins to truly behave as an unsaturated soil. In other words, it is the point beyond which the soil properties are no longer constants but rather, take on the form of nonlinear functions of soil suction.

Figure 2 corresponds to hydrostatic conditions of a homogeneous profile, for zero moisture flux at ground surface. Under equilibrium conditions the pore-water pressure is negative and varies linearly above the water table, regardless of the soil type. The equilibrium pore-water pressure line is referred to as the hydrostatic pressure line. While the equilibrium pore-water pressures are linear, the designation of the amount of water in the soil reveals two distinct zones; namely, i) the top of the capillary zone showing the air-entry value and, ii) the start of residual conditions. These two zones are associated with the soil-water characteristic curve, SWCC, and become the primary information that is required by the geotechnical engineer when defining unsaturated soil property functions. It is recognized that the SWCC is stress path dependent (i.e., hysteretic), but it is important to first visualize the broad general relationship that exists between field conditions and laboratory test conditions.

The linear (equilibrium) pore-water pressure line can deviate from hydrostatic conditions as a result of positive moisture fluxes (i.e., precipitation) or negative moisture fluxes (i.e., evaporation) at the ground surface as shown in Figure 3. In other words, the upper portion of the hydrostatic line responds to climatic conditions imposed at the ground surface. Historically, classic soil mechanics has focused mainly on imposing a "hydraulic head" or having a "zero moisture flux" boundary condition. For unsaturated soil conditions, it is necessary to accommodate varying ground surface moistures fluxes applied in a steady state and/or transient manner. Weather station information can be used to quantify ground surface moisture flux conditions.

The main change in going from saturated soil mechanics to unsaturated soil mechanics lies in the "sign" associated with the pore-water pressure. However, there are other complications that arise when considering negative pore-water pressures. The pressure can become extremely negative, reaching to a limiting value of 1,000,000 kPa under zero water content conditions.

The major difference between saturated and unsaturated soil mechanics lies in the fact that equal changes in total stresses and pore-water pressures do not produce the same physical response in an unsaturated soil (Fredlund & Morgenstern, 1977). Consequently, it becomes necessary to handle total stress changes independent from pore-water pressure changes, as shown in Figure 4. The independence of total stress changes and pore-water pressure changes is fundamental to understanding the difference between saturated and unsaturated soil behavior. Creating an acceptable and accurate theoretical visualization of unsaturated soil behavior constitutes the first step in applying unsaturated soil mechanics.

2.5 Myths to be dispelled

Before considering the application of unsaturated soil mechanics, let us address the myth regarding the permanency of *in-situ* soil suction. In other words, logic leads to the conclusion, "Why should geotechnical engineers care about unsaturated soil mechanics if there is no permanency associated with negative pore-water pressures?" This myth is closely related to the belief that it is also necessary to be able to measure suctions *in-situ* before being able to apply unsaturated soil mechanics in engineering practice. An attempt is made herein to show that this rationale cannot be justified or defended. There are significant value-added benefits to be

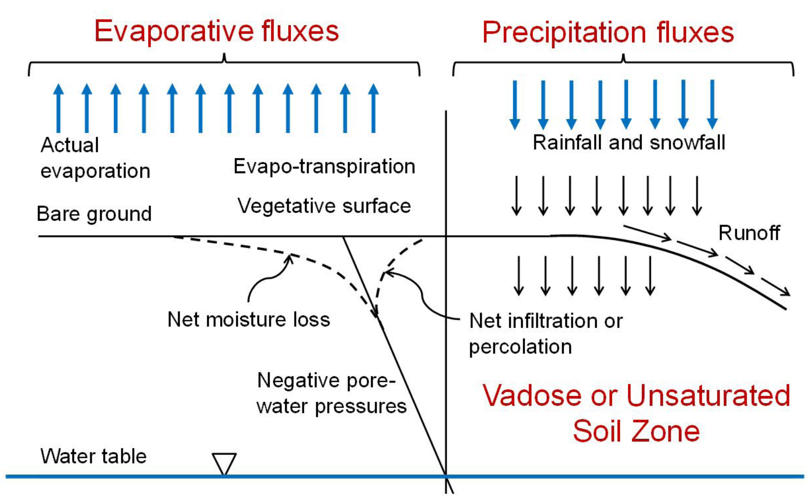

Figure 3. Components of moisture movement associated with net moisture flux conditions at ground surface (Fredlund, 2015).

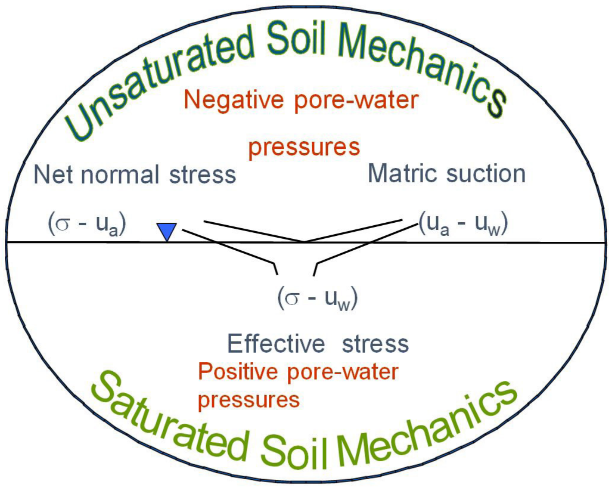

Figure 4. Pictorial visualization of the stress state for saturated/unsaturated soil systems.

gained through the application of unsaturated soil mechanics for near ground surface problems.

3. Misconceptions surrounding the permanency of soil suctions

There is one question that commonly arises during discussion sessions at unsaturated soils research conferences. The question goes something like this: "*In-situ suction are extremely transient, disappearing as soon as there is a rainfall and moisture infiltrates from the ground surface. Therefore, why are we interested in conditions other than saturated soil conditions?*" This sounds like a reasonable question; however, the answer is not quite that simple. In fact, the opposite conclusion might be more realistic. It might be more realistic to ask, *"What conditions need to be met in order for in-situ soil suctions to disappear following heavy or prolonged rainstorm conditions?"*

Before discussing the issue of soil suction permanency, let us first address concerns related to the measurement of *in-situ* soil suction. Negative aspersions regarding the application of unsaturated soil mechanics are often couched in questions that go something like this, *"Why should we concern ourselves with unsaturated soil mechanics when we have no easy-to-use methods to measure suction in the field?"*

3.1 If only it were possible to measure soil suctions in the field!

Tensiometers were manufactured in the early 1900s as a device that would extend the measurement range of piezometers into the negative pore-water pressure range. Over the years the tensiometers have undergone a series of minor refinements such as those illustrated in Figure 5.

Tensiometers provide a direct measure of negative pore-water pressure but have a measurement range limited to approximately 90% of one atmosphere, sustainable for a limited time (e.g., one day). The major shortcoming of tensiometers is related to possible cavitation of the fluid in the measurement system of the instrument.

Other proposed suction device designs have been proposed but also have limitations. For example, pre-pressurized, high suction range devices are mainly suitable for laboratory usage and thermal conductivity heat-dissipation sensors require calibration and lack desired accuracy. It is fair to say that there has not been a "game-changing" device that fully meets the desired requirements for geotechnical engineering applications.

The concept of axis-translation testing of unsaturated soils has worked well in the laboratory for a wide range of research studies by scaling up the ambient air pressure such that water in the measurement system never gets into the absolute negative pore-water pressure range. While axis-translation has been hailed a success for laboratory testing, the technique cannot be translated to field conditions.

Before despairing over the difficulties related to measuring *in-situ* suction, let us consider the following question, "*Is it necessary to be able to measure suctions in the field?*" The primary purpose for measuring negative pore-water pressures *in-situ* is for verification of proposed theories of unsaturated soil behavior. The ability to undertake field verification studies would enhance confidence in the application of unsaturated soil theories.

Let us suppose, however, that the physical behavior of unsaturated soils has been adequately verified in the laboratory. Once laboratory verification has been achieved, then the unsaturated soil theories can be assumed to be applicable for use in the field with reasonable confidence. It is fair to say that most unsaturated soil theories have been adequately tested and verified in the laboratory and can therefore be used with confidence in geotechnical engineering practice (Fredlund, 2017).

Let us now further consider the questions related to the permanency of suctions in the field.

3.2 Is it possible to rely on suctions in the field through wet weather conditions?

Moisture infiltration conditions can be divided into two categories; namely, i) the situation where the average ground surface flux is lower than the saturated coefficient of permeability of the soil near ground surface, and ii) the situation where the ground surface moisture flux is maintained at an intensity equal to or greater than the saturated coefficient of permeability of the soil near ground surface (Kasim et al. 1998; Lu and Griffiths, 2004; Srivastava and Yeh, 1991). Let us consider the case of a homogeneous soil deposit where the moisture flux is low but continues over a long-time.

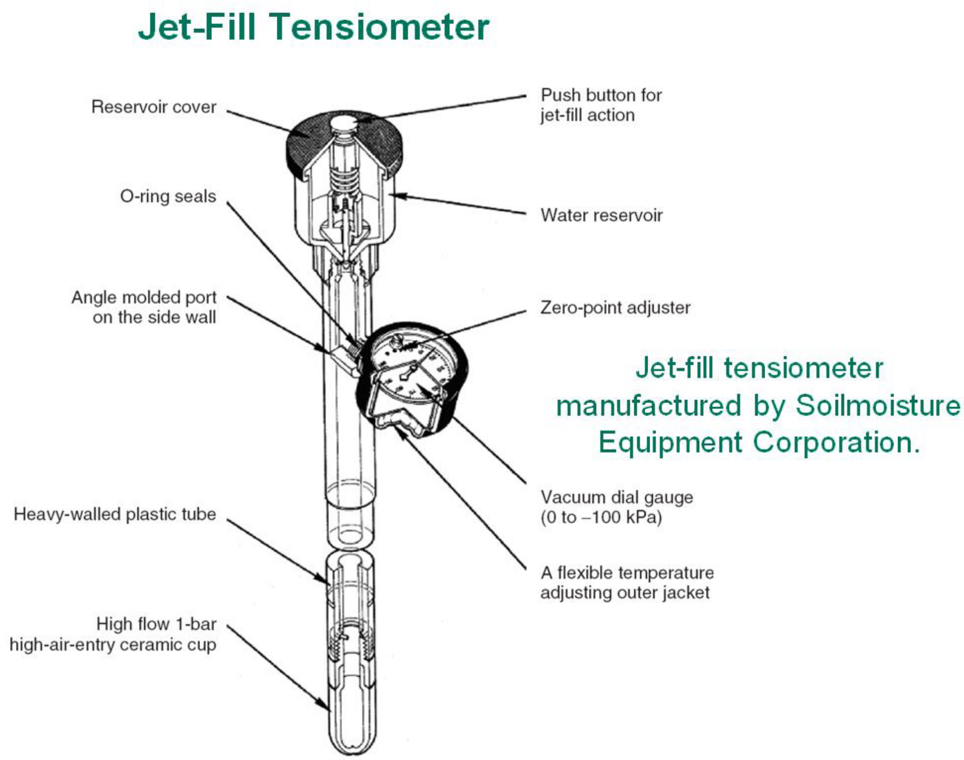

Figure 5. Typical Jet-Filled manufactured by *Soil Moisture Equipment Corporation*, *California.*

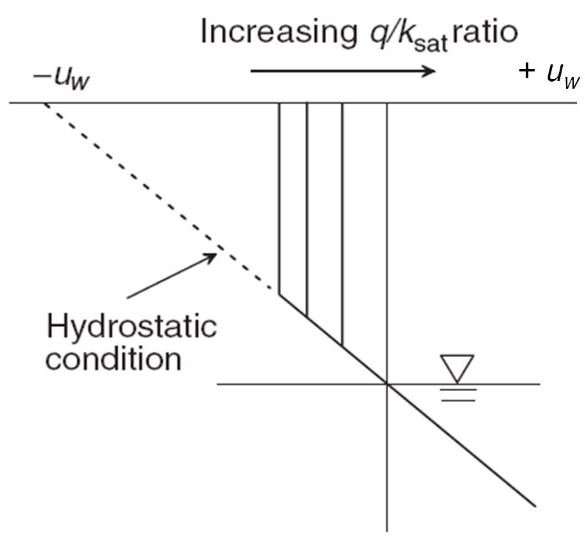

Figure 6. Infiltration into an unsaturated soil under steady state conditions with various ground surface moisture fluxes expressed as a ratio of the saturated coefficient of permeability at ground surface.

Figure 6 presents an approximation of pore-water pressures when the average infiltration rate is less than the saturated coefficient of permeability (Kasim et al., 1998). Under steady state conditions, infiltration occurs under a gradient of 1.0 and can be represented by a vertical line until the hydrostatic line is intercepted. The location of the vertical line is a function of the ratio of the average (ground surface) moisture flux, q, to the saturated coefficient of permeability of the soil, k_{sat}. The soil below ground surface remains at a near constant suction value regardless of how long the rain falls. It could rain constantly at the same rate year after year (i.e., steady state) and our verified unsaturated flow theory indicates that a certain amount of suction would always be maintained in the soil. It should be noted that the pore-water pressure profiles shown in Figure 6 are in reality an indication of the maximum pore-water pressures possible because the condition shown takes time to develop. In other words, it takes time for the equilibrium condition shown to develop.

Steady state rainfall can be considered as an extreme condition in the sense that rainfall does continue forever. In reality, rainfalls stop and the pore-water pressures slowly tends towards hydrostatic conditions.

Let us now consider the second possibility where the moisture infiltration rate exceeds the saturated coefficient of permeability of the near ground surface soil as shown in Figure 7. The pore-water pressure at ground surface reduces to zero and a wetting-front forms. Meanwhile, the remainder of the soil profile maintains negative pore-water pressures. If the excessive rainfall continues over a long period of time, the wetting front slowly moves downward. Fortunately, rainfalls stop and the pore-water pressures tend towards the original hydrostatic condition. It can be observed that a positive pressure hydrostatic condition also commences to form at the ground surface and slowly move downward forming a wetting front.

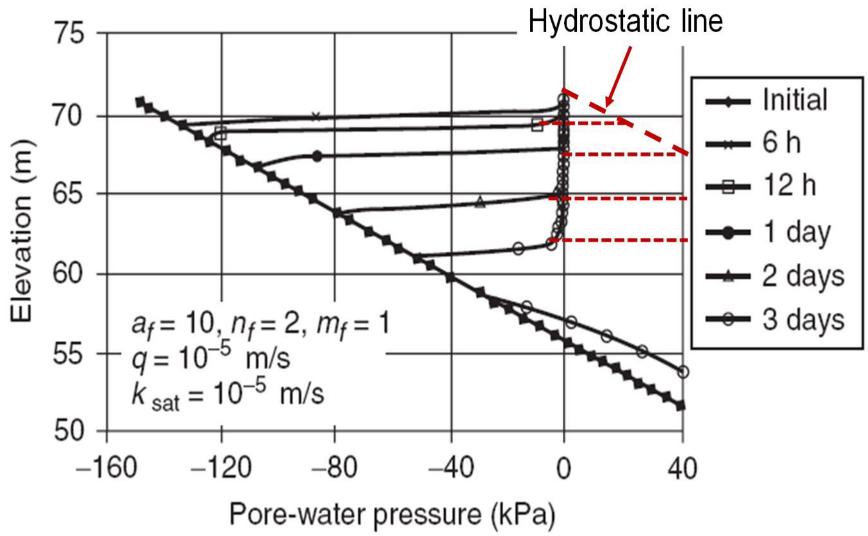

Figure 7. Pore-water pressure profiles under transient rainfall conditions that are in excess of the saturated coefficient of permeability of the soil (modified from Zhang et al., 2004).

Negative pore-water pressures throughout the unsaturated soils profile can only disappear under high moisture fluxes conditions over long periods of time. For a slope to fail, the rainfall must be excessive and remain excessive. Under these conditions, the wetting front has time to move downward throughout the unsaturated soil profile. Consequently, it is **not** easy to wipe-out negative pore-water pressures (or suctions) by subjecting a slope to rainfall. There are two distinct questions that should be given consideration by the geotechnical engineer, namely, i.) How long can suctions be maintained? and ii.) What would be an adequate suction value to use for design purposes?

Failure of a natural slope will most likely occur when the slope is subjected to moisture flux conditions that are larger and longer than has ever occurred in the past. The "trigger" that allows a soil mass to become unstable lies in a knowledge of the state of stress in the pore-water phase above the groundwater table. Numerical seepage modeling can provide geotechnical engineers with value-added information regarding the conditions under which a slope might become unstable.

An understanding of moisture flow into an unsaturated soil assists the geotechnical engineer in finding engineered solutions for slope stability concerns. Even so, geotechnical engineers tend to be reluctant to rely upon *in-situ* suction for the stability of a slope. It might be of assistance to rephrase the question that should be addressed. *"Is it possible to significantly increase the stability of a slope by reducing the near-ground-surface coefficient of permeability of the soil by one, two or more orders of magnitude?"* Or, if the near-ground-surface permeability can be reduced, would the stability of the slope be increased by one, two or three orders of magnitude in terms of elapsed time to failure?

Figure 8. Picture of the Po Shan Landslide in Hong Kong in 1972.

The decrease in the coefficient of permeability of the near ground surface material is essentially what was done in Hong Kong following the disastrous slope failure that passed through the mid-levels of Hong Kong Island in 1972 killing 78 persons (Figure 8). The maintenance of slopes in Hong Kong has proven to be extremely effective by covering sloping surfaces with a mixture of "chunam" (i.e., a paste of decomposed granite, flyash and cement). The plastering of chunam over the surface of cut-slopes has been found to reduce moisture infiltration by approximately 90%; thereby largely solving the landslide problem. Figure 9 shows a typical cut-slope in Hong Kong that has been covered with chunam, guiding the surface rainfall water into a drainage system that guides water into the harbour.

3.3 Quantifying ground surface moisture flux conditions

All unsaturated soil deposits have a ground surface that is exposed to atmospheric weather conditions. The quantification of moisture flux boundary conditions (i.e., Neumann boundary conditions), has not been a part of historical soil mechanics which was largely restricted to zero moisture flux boundary conditions or a "hydraulic head" boundary condition. The lack of a soil mechanics methodology for quantifying actual net moisture flux boundary conditions has provided an excuse for not analyzing moisture flux boundary condition problems into unsaturated soils.

The moisture flux at ground surface is the net value obtained from the summation of downward moisture flux (i.e., rainfall and snowfall), upward moisture movement (i.e., actual evaporation and evapotranspiration), and surface runoff (Figure 10). The "bad" news is each of these components needs to be independently quantified with respect to time (i.e., often over one or more years). The "good" news is that related disciplines (e.g., surface hydrology and agriculture) have proposed and verified methodologies for the quantification of the components of moisture flux. In addition, weather stations have been installed and programmed to collect moisture flux data all around the world. The data collected in most weather stations (i.e., temperature, wind speed, relative humidity and net solar radiation) provide the necessary input for the calculation of potential evaporation through use of the Penman, Wilson-Penman, or some other method (e.g., Thornthwaite, Monteith, etc.).

Figure 9. Use of chunam in Hong Kong to maintain the stability of slopes.

Independent methodologies have been proposed and tested for the calculation of water runoff as well as "actual evaporation". The calculation of net moisture flux constitutes a new "tool" that is available to the geotechnical engineer. The calculation of net moisture flux lends itself well to database technologies (e.g., EXCEL). The primary engineered application making use of net moisture flux calculations has involved the design of soil cover systems (i.e., store-and-release covers).

4. Debunking myths related to measurement of unsaturated soil properties

I have attempted to debunk several misconceptions or myths that are often used as excuses for not accepting and applying unsaturated soil mechanics in engineering practice. Let us consider the following verbal exchange in response to an inquiry about an unsaturated soil mechanics problem.

Let us assume that a potential client phones my office and says to me, *"I have a problem that involves unsaturated soils"*. Before listening to my client's explanation of the problem

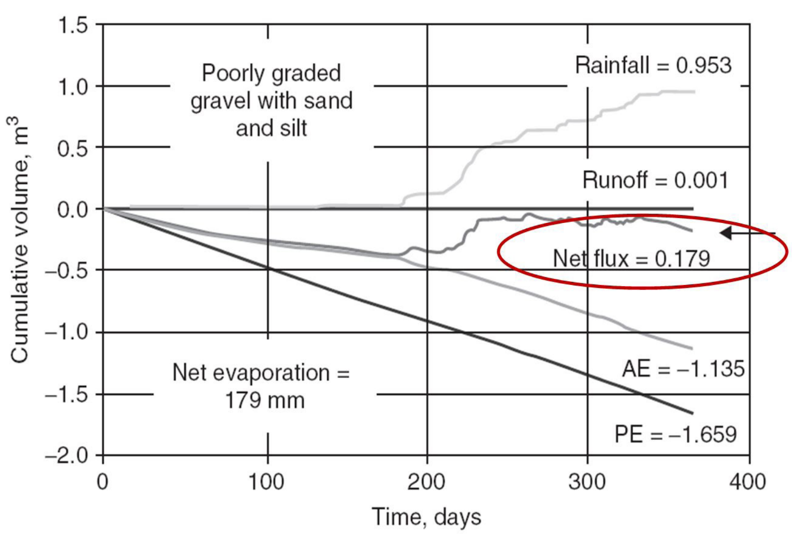

Figure 10. Net moisture infiltration at the ground surface calculated from weather station data (Fredlund et al., 2012).

I respond as follows, "*We need to measure the soil-water characteristic curve for the soil*". The client might rightfully say, *"But I haven't told you what's the problem and you are already saying you need to do some laboratory testing"*.

My response may have sounded arrogant and extreme; however, there is a point to be made and it can be stated as follows. There is one piece of unsaturated soil property information that stands out as being of paramount importance when addressing virtually any problem involving unsaturated soils. An understanding of the soil-water characteristic curve, SWCC, (along with a simple shrinkage curve test), provides the unsaturated soils information that is required for the geotechnical engineer to extend saturated soil properties into the unsaturated soils range.

4.1 Dispelling myths related to measuring unsaturated soil properties

The cost of measuring the unsaturated permeability function for a soil can be orders of magnitude more costly than measuring the saturated soil properties for the same soil (Fredlund & Rahardjo, 1993). Staying within the historic paradigm for saturated soil mechanics simply provides a cost-based excuse for not becoming involved in unsaturated soil mechanics. In other words, using the old paradigm that involves obtaining undisturbed soil sampling and direct laboratory testing for the physical soil properties of interest invokes costs that are prohibitive. As a result, it is necessary for geotechnical engineers to ask some serious questions regarding the need to consider using a new soil property assessment paradigm that would cost less while still providing sufficient accuracy and reliability for engineering practice.

Figure 11 attempts to show the similar basic soil property requirements between saturated soil mechanics theories and unsaturated soil mechanics theories. Moving from below the water table (i.e., saturated soil conditions) to above the water table (i.e., unsaturated soil conditions) changes soil mechanics from having soil properties that can be viewed in terms of a series of soil constants to soil properties that are functions of negative pore-water pressures (or soil suction). The change from "soil constants" to "soil property functions" occur at the top of the capillary zone (or at the air-entry value) of the soil.

Above the capillary zone, the soil moves away from saturation and the soil properties can change quite rapidly. The transition in the designation of the soil properties transforms soil mechanics analyses from having linear constitutive relations to having nonlinear constitutive relations. The nonlinearity gives rise to a new challenge when analyzing practical engineering problems. However, on the positive side, research studies have verified that it is possible to "estimate" all unsaturated soil property functions through use of a modestly priced laboratory measurement of the soil-water characteristic curve, SWCC along with the shrinkage curve. In each case, the soil property is anchored to the saturated soil properties with a change in soil properties as the air-entry value of the soil is exceeded.

Figure 11 shows the three basic application areas for soil mechanics (i.e., water flow, shear strength, and volume change). The figure shows the constitutive equation form of the laws governing the behavior of saturated and unsaturated soils. The difference between the treatment of saturated and unsaturated soils lies in the form of the mathematical equations required to describe unsaturated soil constitutive relations.

4.2 Emergence of a new paradigm for unsaturated soil property functions

Colleagues in soil physics and other agriculture-related disciplines have long recognized that a new approach was

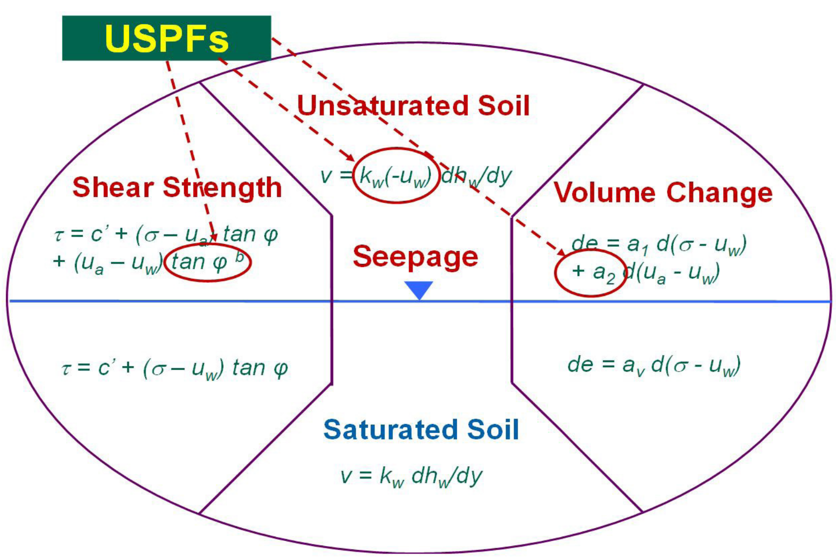

Figure 11. Three fundamental unsaturated soil property functions required in saturated/unsaturated soil mechanics.

required for soil property evaluation when dealing with unsaturated soils (van Genuchten, 1980). Initial interest was primarily limited to the evaluation of the unsaturated hydraulic conductivity properties of the soil (Fredlund et al., 1994). The new approach that evolved required the laboratory measurement of "*something other*" than the direct unsaturated soil property functions. That "*something other*" was the soil-water characteristic curve, SWCC, or the relationship between soil suction and the amount of water in the soil (Klute, 1965, 1986) as depicted in Figure 12.

The assumption was made that water flow through a soil was related to the amount of water in the voids. As a result, it was reasoned that the SWCC could be used to calculate a permeability function that would extend from saturated soil conditions to essentially the dry state. However, it was obvious that this approach had its challenges, mainly because there was not a unique relationship between the amount of water in the soil and the stress path adhered to during the test. Rather, there was a family of SWCC curves including scanning curves.

The primary curves were referred to as the *Initial Drying* curve (from 100% saturation), the *Main Drying* curve, and the *Main Wetting* curve. Soil samples were generally tested under zero total stress conditions and the *Main Drying* curve and the *Main Wetting* curves formed bounding or limiting conditions. The laboratory characterization of all branches of the family of SWCC was time consuming and costly.

The *Main Drying* curve was the easiest laboratory relationship to measure and with time became the main SWCC quantified. It became most common practice to control matric suctions using the axis-translation technique in the low suction range (i.e., up to 1500 kPa) and measure total suction in the high suction range (greater than 1500 kPa) using hygrometric means as shown by the raw data in Figure 13. The data was fitted using a regression analysis based on one of the numerous sigmoidal type equations that have been proposed for SWCCs. It is important that the equation selected be applicable over the entire range of suctions from a fraction of 1 kPa to 1,000,000 kPa (e.g., Fredlund & Xing, 1994).

There are two points on the drying SWCC that are of primary interest to geotechnical engineers; namely, i) the true air-entry value of the soil, and ii) the rate of desaturation of the soil with respect to soil suction. Both variables require the determination of the degree of saturation with respect to changes in soil suction, (*S*-SWCC). However, gravimetric water content SWCC was the easiest to measure in the laboratory. Changes in gravimetric water content as soil suction was changed could possibly signify changes in degree of saturation or might also reflect changes in overall volume. It is important to be able to separate out the effects of volume change from the effects of desaturation of the soil. The separation of volume change and desaturation can be accomplished through measurement of the shrinkage curve for the soil, (SC) (Fredlund & Zhang, 2013). By using the drying shrinkage curve, the same stress path is followed during both the SWCC test and the shrinkage curve measurement for the soil (see Figure 14).

4.3 Combining the *w*-SWCC and the shrinkage curve, SC, laboratory results

Combining the results of the gravimetric water content SWCC and the shrinkage curve allows the calculation of all volume-mass relationships versus soil suction; that is, the void ratio characteristic curve, *e*-CC, the volumetric water content characteristic curve, θ-SWCC, and the degree of saturation characteristic curve, *S*-SWCC. The point of greatest importance along the *S*-SWCC is the true air-entry value of the soil, (i.e., the top of the capillary zone) where unsaturated soil properties come into effect (Shown in Figure 15). It is noted that the true air-entry value for a soil is often more than one order of magnitude greater than the "apparent" air-entry value observed on the gravimetric water content SWCC.

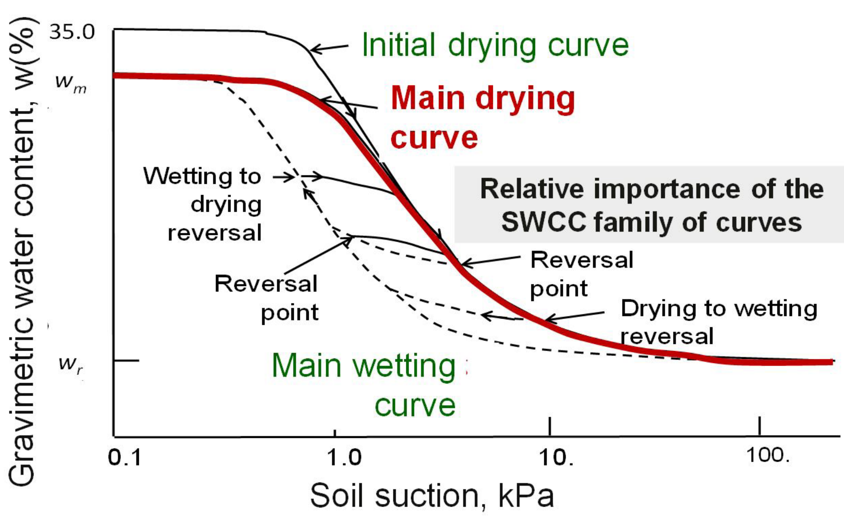

Figure 12. Basics of the soil-water characteristic curve, SWCC, family of curves (from Klute, 1965, 1986).

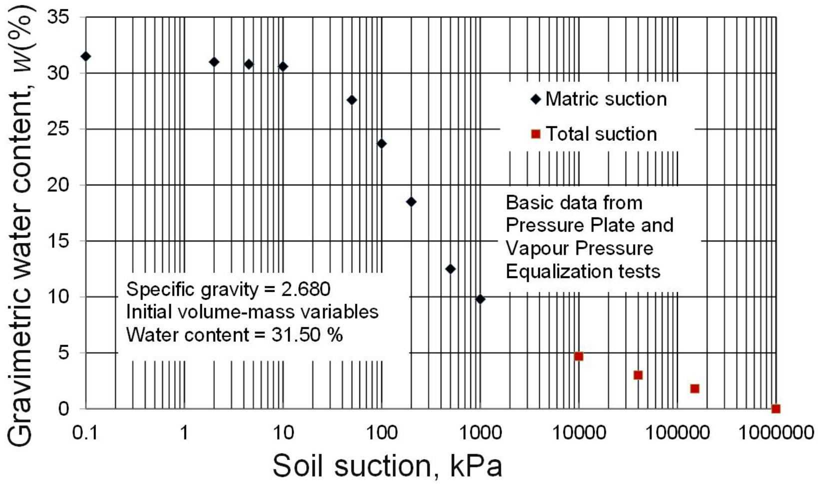

Figure 13. Gravimetric soil-water characteristic curve measurements over the entire range of soil suctions (Fredlund, 2019a).

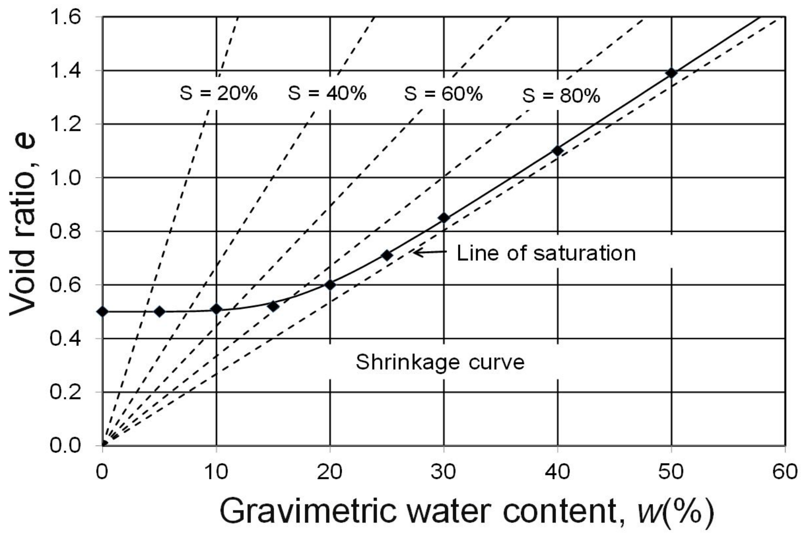

Figure 14. Typical shrinkage curve relating changes in volume to changes in water content (Fredlund, 2019b).

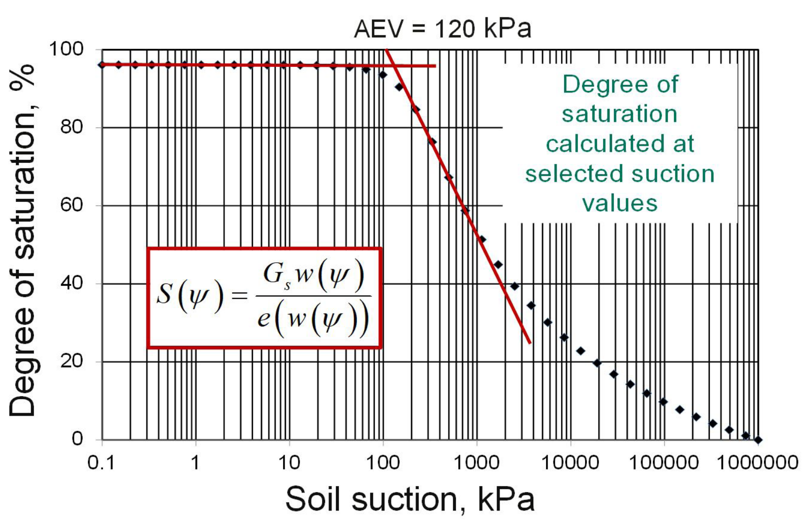

Figure 15. Degree of saturation soil-water characteristic curve, *S*-SWCC, calculated over the entire soil suction range using the *w*-SWCC and the Shrinkage Curve (Fredlund, 2019b).

4.4 Calculation of unsaturated soil property functions

The appropriate volume-mass SWCCs can now be used along with one of the empirical "estimation" procedures proposed in the research literature for calculating the unsaturated soil property functions. An important unsaturated soil property function is the coefficient of permeability function and an example permeability function is shown in Figure 16. The coefficient of permeability is shown to decrease significantly once the air-entry value of the soil is exceeded. Note that the unsaturated soil permeability function can start from a dimensionless value of 1.0 corresponding to saturated soil conditions. The permeability function can then be scaled downward such that under saturated soil conditions the coefficient of permeability corresponds to the actual saturated coefficient of permeability.

The effect of hysteresis in the permeability function can be related back to the difference between the drying and wetting *S*-SWCC. In other words, the difference between the drying and wetting *S*-SWCCs can be used to provide an estimation of the difference between the drying and wetting permeability functions (Pham et al., 2003).

Figure 17 shows a typical water storage function for an unsaturated soil. The formulation for transient seepage through an unsaturated soil is usually formulated such that the water storage function can be designated as the (arithmetic) slope of the volumetric water content SWCC (θ-SWCC). Once again, there is a drying and a wetting water storage function with the maximum water storage value corresponding to the inflection point along the θ-SWCC.

It is also possible to calculate other unsaturated soil property functions. For example, a shear strength function or a volume change function may need to be estimated. The assumption is being made that the soil behaves as a stable-structured soil. In each case, the unsaturated soil constitutive relationship can be calculated from an understanding of the

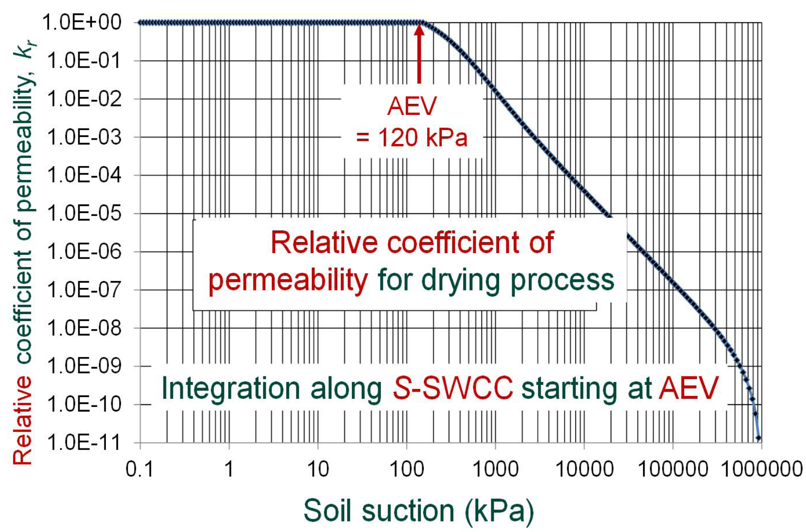

Figure 16. Coefficient of permeability unsaturated soil property function corresponding to the drying *S*-SWCC (Fredlund & Xing, 1994).

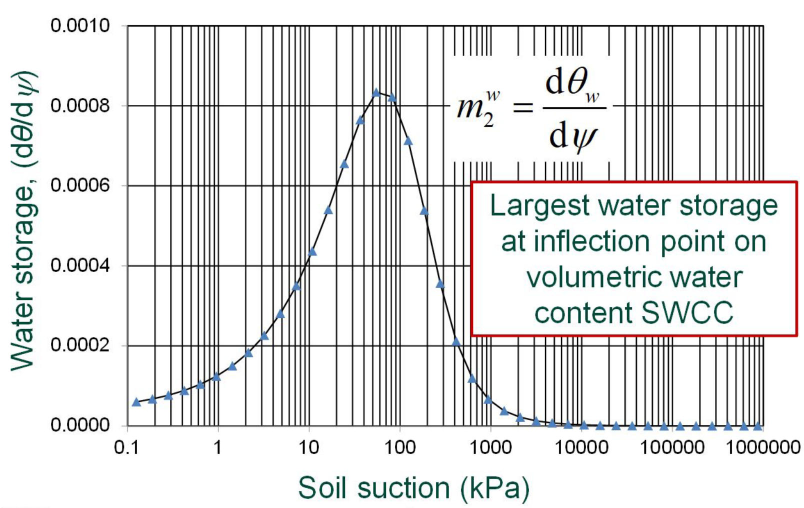

Figure 17. Typical water storage function for an unsaturated soil (Fredlund, 2019b).

SWCC and SC. All unsaturated soil property functions are empirical and based on a variety of assumptions. In some cases, the thought processes and proposed application protocols have originated in other disciplines and simply need to be confirmed for geotechnical engineering applications.

4.5 Calculation of unsaturated soil property functions

Practical geotechnical engineering protocols have been distilled from research studies into unsaturated soil mechanics over the past several decades. In all cases, estimations for unsaturated soil property functions are based on a careful evaluation of the volume-mass properties and the response of the soil to changes in soil suction over a large range of suction values. There are assumptions associated with each empirical procedure that have been proposed for the calculation of unsaturated soil property functions. The assessment of each unsaturated soil property function has been approximated but the functions have proven to be extremely useful in providing geotechnical engineers with information on what will likely happen in response to a series of "*What if?"* questions that might be asked.

5. Debunking computational challenges

The evolution of a science basis for unsaturated soil mechanics did not start at the origination of the science for saturated soils. It appears that geotechnical engineers lacked understanding as to how best to apply unsaturated soil mechanics in engineering practice. In hindsight it is observed that the basic science principles for unsaturated soil mechanics are similar to the principles accepted for saturated soil mechanics. A significant challenge that needed to be addressed was the quantification of unsaturated soil property functions. The determination of unsaturated soil property functions has largely been resolved through use of "estimation" procedures based on laboratory measured soil-water characteristic curves, SWCCs, in conjunction with a shrinkage curve, SC. The last major question that needed to be addressed was, *"Can unsaturated soil behavior be modeled using nonlinear partial differential equation solvers?"*

There are two disciplines that have played an important role in making it possible to solve highly nonlinear partial differential equations of the type associated with unsaturated soil behavior. Mathematicians have developed a variety of mesh generation and mesh refinement and optimization techniques to provide robust numerical modeling techniques for nonlinear formulations. The computing industry has also increased its computing capability at a rapid pace that has been coincident with the emergence of unsaturated soil mechanics theories.

Unsaturated soil property functions have the form of nonlinear mathematical functions and as a result unsaturated soil mechanics requires significant computational capabilities. Nonlinearity arises because the soil properties are related to an (unknown) variable that was part of the solution. In seepage problems, for example, the coefficient of permeability of the soil depends on the (negative) pore-water pressure and the pore-water pressure head is a component of the hydraulic head driving flow. Consequently, it is necessary to assume a value for the coefficient of permeability and then solve the seepage problem. Then a check must be made to ascertain whether the assumed value for the coefficient of permeability was correct. The nonlinear nature of the problem leads to a "trial and error" iterative type of solution. Fortunately, computers lend themselves well to this type of a challenge.

5.1 Resistance to change in soil mechanics

Even though solutions for unsaturated soil mechanics problems have been extensively promoted at research conferences, still there appears to have been some resistance to change within the geotechnical engineering community. It is now possible to view soil continua as having an unsaturated soil zone and a saturated zone with a smooth seamless transition between the zones. Unfortunately, it often seemed easier to revert to crude past protocols rather than embrace the benefits of new, more rigorous procedures applicable for unsaturated soil mechanics. There are, however, considerable value-added benefits to be gained through simultaneously modeling of the saturated and unsaturated soil zones of a continua. The development of recent commercial software packages has made it considerably easier to accommodate saturated/unsaturated soil behavior modeling.

The complexities of the mathematical relations associated with unsaturated soil mechanics (e.g., nonlinear partial differential equations) can be largely obscured from the geotechnical engineer through use of recent software developments. Usage of special purpose software packages require: i) a delineation of the ground surface and the underlying soil strata, ii) the entry to the unsaturated soil properties, and iii) the designation of appropriate boundary conditions.

5.2 Example solution of a saturated/unsaturated seepage problem

Steady state and unsteady state (or transient) solutions to a saturated/unsaturated soils problem are used to illustrate the role of numerical modeling solutions. Steady state problems require the input of a coefficient of permeability function. Unsteady state problems require that a water storage function also be input as a second soil property. It is noteworthy that there are similar transmission type properties and storage type properties required when considering virtually any continuum mechanics field problem (e.g., heat flow, air flow, chemical movement, etc.).

5.3 Modeling a steady state problem involving saturated/unsaturated soil zones

Figure 18 shows a typical coefficient of permeability function for an unsaturated soil. A permeability function must be used to determine the coefficient of permeability once the pore-water pressure becomes negative. However, the soil can be assumed to have a constant coefficient of permeability (i.e., k_{sat}) when the pore-water pressure remains below the air-entry value for the soil (i.e., the capillary zone).

It is possible to hard code into computer software one of several possible mathematical equations for the permeability function. However, it has been found to be just as efficient to input a table of values (e.g., 20 to 40 points) that cover the entire range of possible soil suction values that might be encountered when solving the seepage problem. Interpolation along the soil suction versus permeability should linearize the function because of the logarithmic nature of the suction scale. Different permeability calculation models (or calculation procedures) might be used in various software packages; however, a table of points along the soil property function seems to be quite acceptable to numerical modellers.

Figure 19 shows the hydraulic head along with the zero-pressure line (i.e., piezometric line) under steady state seepage through a dam with a low permeability clay soil core. Subdivision of the geometry into mesh of finite elements may be either part of the numerical solver or it might be controlled by the modeller. It can be observed that the equipotential lines (or lines of hydraulic head) are seamless through the positive and negative pore-water pressure zones. Numerous numerical software codes in the marketplace simultaneously solve for hydraulic heads in the positive and negative pore-water pressure ranges (e.g., SV/Flow, Seep/W, etc.). Numerical modellers can quickly check and observe the response of the model to a series of "*What if –?*" queries.

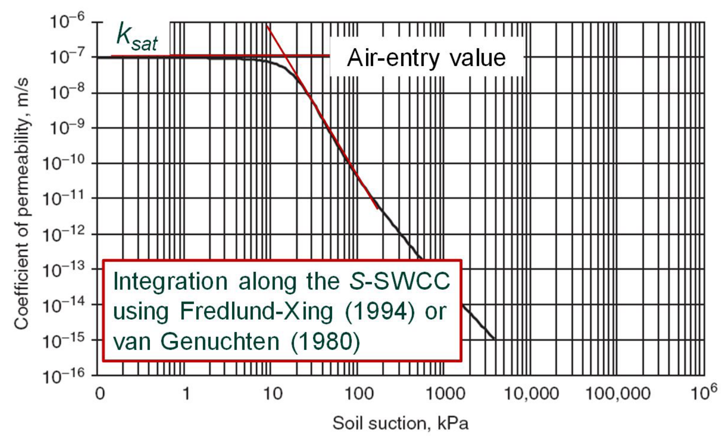

Figure 18. Permeability function for analyzing steady state and unsteady state seepage through an unsaturated soil (Fredlund et al., 2012).

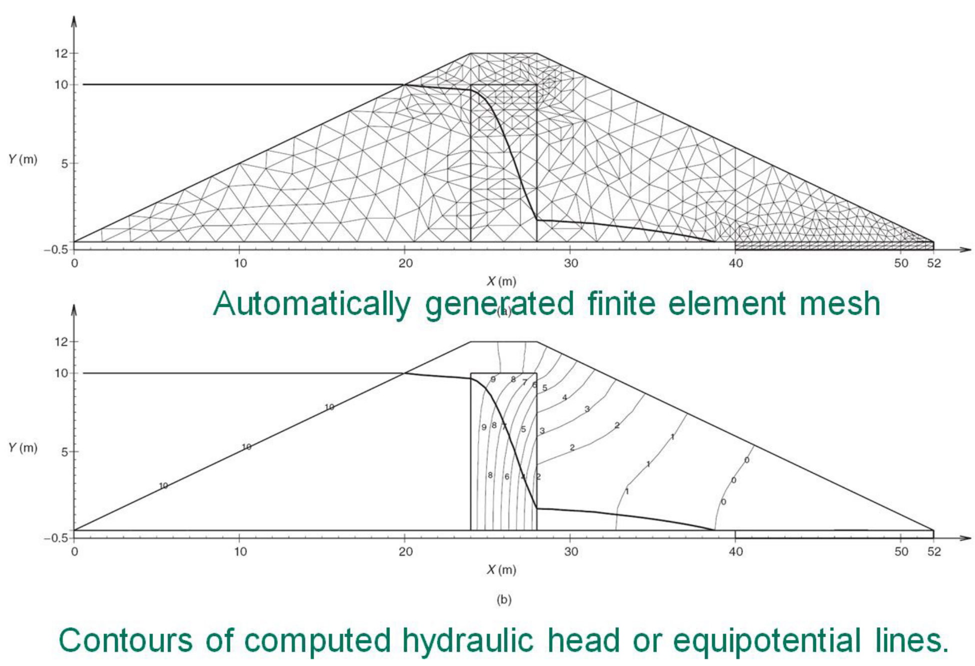

Figure 19. Contours of hydraulic head and the piezometric line under steady state seepage conditions (Fredlund et al., 2012).

5.4 Modeling an unsteady state problem involving saturated/unsaturated soil zones

Transient (or unsteady state) seepage infers that the modeling process commences at an initial point in time and then a process (i.e., seepage in this case) is followed for a period of elapsed time. Figure 20 shows an example of the water storage function required when performing a transient seepage analysis that involves an unsaturated soil. The water storage function is calculated from the slope of the volumetric water content soil-water characteristic curve. The water storage can be input in a tabular format with interpolation performed on a logarithmic scale.

The example problem being considered assumes that water is instantly placed in the reservoir and the process of seepage into the compacted homogeneous earthfill dam is modeled with respect to time. The results at three elapsed times are shown: i) Figure 21 after 25 days, ii) Figure 22 after 60 days, and iii) Figure 23 after 1500 days (Fredlund et al., 2012). The solution reverts to what is essentially a steady state solution after 1500 days.

Figure 21 shows high head gradients as water enters the upstream face of the dam. The line of zero pressure (phreatic line) takes time to develop. Flow goes across the phreatic line as the hydraulic head contours spread out and move towards the downstream portion of the dam.

The phreatic line and the hydraulic head contours take on the form of a steady state solution after an elapsed time of 1500 days.

5.5 Need for the teaching and demonstration of unsaturated soil mechanics at universities

New technologies are not immediately brought into engineering practice as soon as they are proven to be superior and correct. Rather, there commonly appears to be resistance to changing the way things have been done. It is necessary to go through a series of steps to bring about changes in engineering protocols. Usually there is a need to thoroughly understand the new procedures and be assured of the safety or lack of risk associated with putting new protocols into practice. This paper has largely focused on

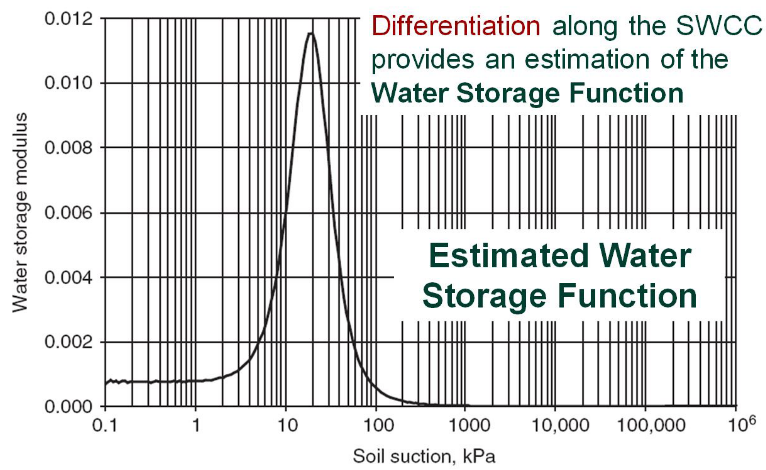

Figure 20. Example of the water storage function required when analyzing transient seepage analysis on an unsaturated soil (Fredlund et al., 2012).

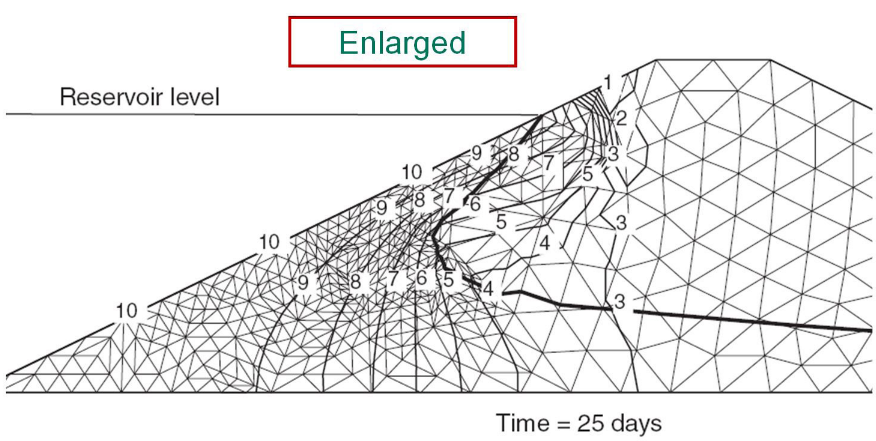

Figure 21. Contours of hydraulic head and the piezometric line under unsteady state seepage for 25 days (Fredlund et al., 2012).

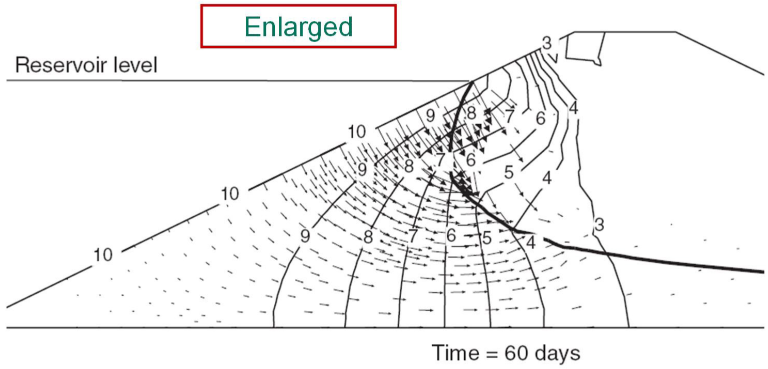

Figure 22. Contours of hydraulic head and the piezometric line under unsteady state seepage for 60 days (Fredlund et al., 2012).

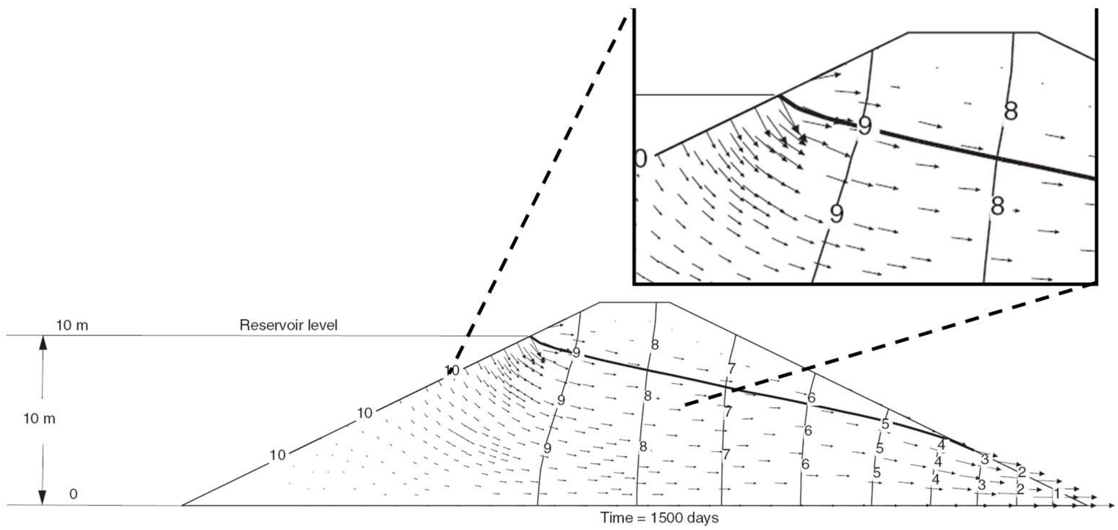

Figure 23. Contours of hydraulic head and the piezometric line under unsteady state seepage after 1500 days (Fredlund et al., 2012).

dispelling myths, misconceptions and unjustified challenges that have interfered with the implementation of unsaturated soil mechanics. An attempt has been made to dispel each of the common myths.

The teaching of unsaturated soil mechanics theories, along with examples and case histories, plays another important role in gaining acceptance of unsaturated soil mechanics. To assist in moving forward with implementation, a series of six one-hour lectures have been recorded on *Webinar* for the International Society of Soil Mechanics and Geotechnical Engineering, ISSMGE. The topics presented are shown in Figure 24. The *Webinar* lectures are presently in the library of ISSMGE and can be accessed free of charge. The lecture material has been synthesized largely based on the book *Unsaturated Soil Mechanics in Engineering Practice*, (Fredlund et al., 2012).

There is an ongoing need for the synthesis of other information related to unsaturated soil mechanics. One example is the need for standard (or generally accepted) testing procedures to be described in detail and adopted by regulatory agencies around the world. Several software companies appear to have been out-in-front with the development of both two-dimensional and three-dimensional software codes that simultaneously model both the saturated and unsaturated soil zones.

5.6 Debunking unsaturated soils misconceptions

The teaching of soil mechanics at universities has mainly focused on the behavior of saturated soils. Not surprisingly, geotechnical engineers have inherited some preconceived inhibitions about the application of unsaturated soil mechanics. An attempt is made herein to debunk some misconceptions related to the acceptance and use of unsaturated soil mechanics. Misconceptions discussed can be listed as follows:

1) It is a commonly expressed viewpoint that theories related to unsaturated soils are too difficult for practicing geotechnical engineers to comprehend. However, the fundamental principles underlying saturated soil mechanics can be shown to also apply for unsaturated soil mechanics (i.e., both are based on a phenomenological continuum mechanics approach);

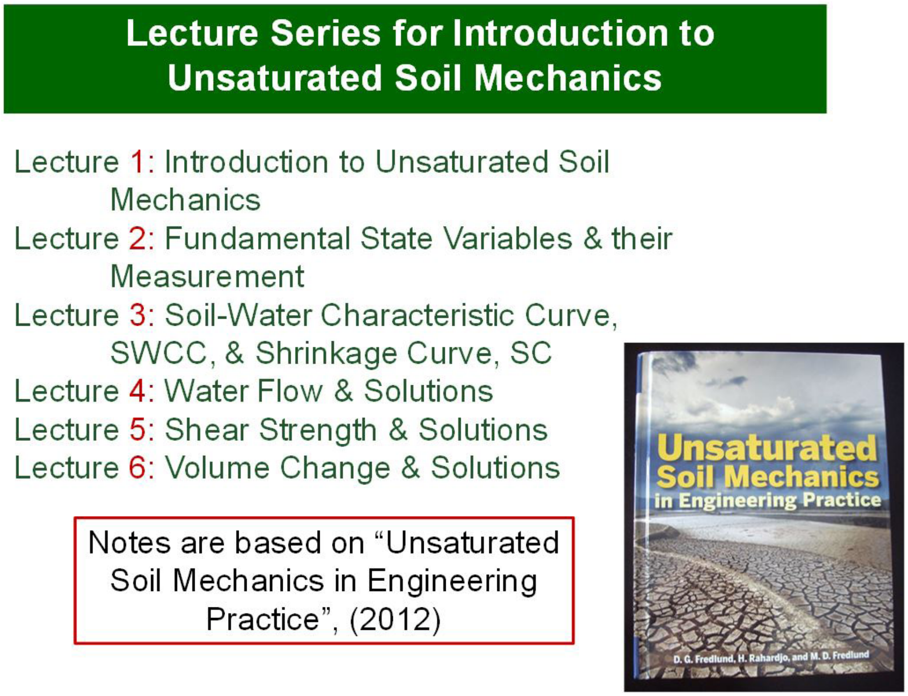

Figure 24. Lecture Notes on Unsaturated Soil Mechanics prepared for the International Society for Soil Mechanics and Geotechnical Engineering in 2019.

2) It is a misconception to conclude that it is necessary to wait in applying unsaturated soil mechanics principles and analyses until a better *in-situ* soil suction measurement device is discovered. Rather, the laboratory verification of unsaturated soil mechanics' theories provides an adequate framework for applying unsaturated soil mechanics theories that function similarly in the field as in the laboratory;
3) It is a misconception to conclude that soil suctions readily disappear when rain falls on the ground surface. Only under conditions of very heavy rainfall over a long, long period of time can a slope become unstable. *In-situ* suctions do not quickly disappear following intense precipitation conditions. It has been shown that it is not easy to "wipe out" negative pore-water pressures either under low intensity or high intensity rainfall conditions. In other words, the dissipation of negative pore-water pressures is a relatively slow process;
4) While it is difficult and costly to measure unsaturated soil property functions in the laboratory, there has been an increase in methodologies proposed for the estimation of the soil property functions through the measurement of the soil-water characteristic curve, SWCC, (and the shrinkage curve, SC) for a soil. These estimation methodologies have been shown to produce adequate results for most unsaturated soils problems. The SWCC and SC are not too costly and difficult to perform in geotechnical testing laboratories;
5) It is not right to argue that the analyses of practical geotechnical engineering problems are too complex for available numerical modeling solutions. Ground surface boundary conditions can also be accommodated. Moisture fluxes are related to highly variable weather conditions; however, practical methodologies have been provided for geotechnical engineers with assistance from colleagues in surface hydrology. The quantification of net moisture flux at the ground surface has been extensively applied in the design of earth cover systems.

It is difficult to bring about change in geotechnical engineering protocols, even when there are benefits to be gained in applying improved technologies. It is important to move past present-day inhibitions when the most significant roadblock simply involves the will of the geotechnical engineer.

Declaration of interest

The author declares that there are no conflicting interests that could inappropriately bias their work.

List of symbols

AEV	air-entry value
e-CC	void ratio characteristic curve
SC	shrinkage curve
SWCC	soil-water characteristic curve

θ-SWCC volumetric water content characteristic curve
S-SWCC degree of saturation characteristic curve
w-SWCC gravimetric water content characteristic curve
USPFs unsaturated soil property functions

References

Blight, G.E. (2013). *Unsaturated soil mechanics in geotechnical practice.* Boca Raton: South Africa CRC Press, Taylor & Francis Group.

Fredlund, D.G., & Houston, S.L. (2009). Protocols for the assessment of unsaturated soil properties in geotechnical engineering practice. *Canadian Geotechnical Journal*, 46(6), 694-707. http://dx.doi.org/10.1139/T09-010.

Fredlund, D.G., & Rahardjo, H. (1993). Soil Mechanics for Unsaturated Soil*;* John Wiley & Sons: New York, NY, USA.

Fredlund, D.G., Rahardjo, H., & Fredlund, M.D. (2012). *Unsaturated soil mechanics in engineering practice*. New York: John Wiley & Sons.

Fredlund, D.G. (2015). Relationship between the laboratory soil-water characteristic curves and field stress state. In *Proceedings of the Asia-Pacific Conference on Unsaturated Soils*, Quilin.

Fredlund, D.G. (2017). Role of the soil-water characteristic curve in unsaturated soil mechanics. In *Proceedings of the 19th International Conference on Soil Mechanics and Geotechnical Engineering* (The Blight Lecture, (Honours lecture), pp. 57-80), Seoul.

Fredlund, D.G. (2019a). Determination of unsaturated soil property functions for engineering practice. In *Proceedings of the African Regional Conference on Soil Mechanics and Geotechnical Engineering* (The Jerry Jennings Memorial Lecture), Cape Town.

Fredlund, D.G. (2019b). State of practice for use of the soil-water characteristic curve (SWCC) in geotechnical engineering. *Canadian Geotechnical Journal*, 56(8), 1059-1069. http://dx.doi.org/10.1139/cgj-2018-0434.

Fredlund, D.G., & Morgenstern, N.R. (1977). Stress state variables for unsaturated soils. *Journal of the Geotechnical Engineering Division*, 103(GT5), 447-466.

Fredlund, D.G., & Xing, A. (1994). Equations for the soil-water characteristic curve. *Canadian Geotechnical Journal*, 31(3), 521-532. http://dx.doi.org/10.1139/t94-061.

Fredlund, D.G., Xing, A., & Huang, S.Y. (1994). Predicting the permeability function for unsaturated soils using the soil-water characteristic curve. *Canadian Geotechnical Journal*, 31(4), 533-546. http://dx.doi.org/10.1139/t94-062.

Fredlund, D.G., & Zhang, F. (2013). Combination of shrinkage curve and soil-water characteristic curves for soils that undergo volume change as soil suction is increased. In *Proceedings of the 18th International Conference on Soil Mechanics and Geotechnical Engineering*, Paris, France.

Houston, S.V. (2019). It is time to use unsaturated soil mechanics in routine geotechnical engineering practice. *Journal of Geotechnical and Geoenvironmental Engineering*, 145(5), 1-10. http://dx.doi.org/10.1061/(ASCE)GT.1943-5606.0002044.

Kasim, F., Fredlund, D.G., & Gan, J.M.-M. (1998). The effects of steady state rainfall on the matric suction conditions in a slope. In *Proceedings of the 2nd International Conference on Unsaturated Soils, UNSAT-98* (Vol. 1, pp. 78-83), Beijing, China.

Klute, A. (1965). Laboratory measurement of hydraulic conductivity of unsaturated soil. In C.A. Black, D.D. Evans, J.L. White, L.E. Ensminger & F.E. Clark (Eds.), *Methods of soil analysis* (Monograph, No. 9, Part 1, pp. 253-261). Madison, WI: American Society of Agronomy.

Klute, A. (1986). Water retention: laboratory methods. In A. Klute (Ed.), *Methods of soil analysis, part 1 – physical and mineralogical methods* (pp. 635-662). Madison, WI: American Society of Agronomy.

Lu, N., & Griffiths, D.V. (2004). Profiles of steady state suction stress in unsaturated soils. *Journal of Geotechnical and Geoenvironmental Engineering*, 130(10), 1063-1076. http://dx.doi.org/10.1061/(ASCE)1090-0241(2004)130:10(1063).

Morgenstern, N.R. (2012). Foreword. In D.G. Fredlund, H. Rahardjo & M.D. Fredlund (Eds.), *Unsaturated soil mechanics in engineering practice* (pp. xiii). New York: John Wiley & Sons.

Pham, H.Q., Fredlund, D.G., & Barbour, S.L. (2003). A practical hysteresis model for the soil-water characteristic curve for soils with negligible volume change. *Geotechnique*, 53(2), 293-298. http://dx.doi.org/10.1680/geot.2003.53.2.293.

Rahardjo, H., Kim, Y., & Satyanaga, A. (2019). Role of unsaturated soil mechanics in geotechnical engineering. *International Journal of Geo-Engineering*, 10(1), 8. http://dx.doi.org/10.1186/s40703-019-0104-8.

Srivastava, R., & Yeh, T.-C.J. (1991). Analytical solutions for one-dimensional, transient infiltration toward the water table in homogeneous and layered soils. *Water Resources Research*, 27(5), 753-762. http://dx.doi.org/10.1029/90WR02772.

Taylor, D.W. (1948). *Fundamentals of soil mechanics*. New York: John Wiley & Sons.

Tarantino, A., & El Mountassir, G. (2013). Making unsaturated soil mechanics accessible for engineers: preliminary hydraulic-mechanical characterisation and stability assessment. *Engineering Geology*, 165, 89-104. http://dx.doi.org/10.1016/j.enggeo.2013.05.025.

Terzaghi, K. (1943). *Theoretical soil mechanics*. New York: John Wiley & Sons.

Terzaghi, K., & Peck, R.B. (1967). Soil *mechanics in engineering practice*. New York: John Wiley & Sons.

Vanapalli, S. K., & Mohamed, F. M. O. (2006). Bearing capacity of model footings in unsaturated soils. In T. Schanz (Ed.), *Experimental unsaturated soil mechanics* (pp. 483-493). Berlin: Springer. https://doi.org/10.1007/3-540-69873-6_48.

van Genuchten, M.T. (1980). A closed-form solution for predicting the hydraulic conductivity of unsaturated soils. *Journal of Soil Science of America*, 44(5), 892-898. http://dx.doi.org/10.2136/sssaj1980.03615995004400050002x.

Zhang, L.L., Fredlund, D.G., Zhang, L.M., & Tang, W.H. (2004). Numerical study of soil conditions under which matric suction can be maintained. *Canadian Geotechnical Journal*, 41(4), 569-582. http://dx.doi.org/10.1139/t04-006.