Soils and Rocks
An International Journal of Geotechnical and Geoenvironmental Engineering
www.soilsandrocks.com
ISSN 1980-9743
ISSN-e 2675-5475

Primary consolidation settlement due to ramp loading: Terzaghi (1943) method revisited

Vitor dos Santos Albuquerque[1#], Celso Romanel[1], Raphael F. Carneiro[1]

Technical Note

Keywords
Primary consolidation
Ramp loading
Settlement

Abstract

Terzaghi (1943) developed an empirical method for primary consolidation due to a load applied at constant rate (ramp load) until the end of construction at time t_c. The method considers that the settlement at a time t during construction, can be evaluated admitting the load applied instantaneously at time $t/2$. In this research, two alternative modifications are proposed for this Terzaghi's empirical recommendation. The first one is based on a variable fraction of time t and the second modification keeps Terzaghi's suggestion ($t/2$) but makes reductions in the average degree of consolidation U_v. Computed results for different construction time factors T_v were compared to Olson (1977) analytical solution. The first approach yielded a maximum difference of approximately 2.40% while the second alternative gave results that are practically the same as those calculated by Olson's solution. The validity of these new approaches was also proven by reproducing odometer test results with good agreement.

1. Introduction

One of Terzaghi's most significant contributions to geotechnical engineering was the theory of one-dimensional consolidation (Terzaghi, 1923, 1925), which was also a consequence of another Terzaghi's (1923) fundamental contribution given by the principle of effective stresses in saturated soils of low permeability.

Terzaghi's consolidation theory relies on some simplifying assumptions, among them the hypothesis of loading of infinite extent applied instantaneously. Several methods for estimating the excess of pore water pressure and primary consolidation settlement due to a non-instantaneous ramp loading have been presented in the literature (Terzaghi, 1943; Schiffman, 1958; Olson, 1977; Zhu & Yin, 1998; Conte & Troncone, 2006; Hanna et al., 2013; Carneiro et al., 2021). The two most known approaches are the empirical method proposed by Terzaghi (1943) and the analytical solution developed by Olson (1977).

Terzaghi (1943) empirical method estimates the average degree of consolidation U_v at time factor $T_v \le T_c$ by assuming the loading applied instantly at $T_v/2$, multiplied by the ratio between the load fraction applied at T_v and the total construction load applied at T_c. For the post-construction period ($T_v > T_c$), the average degree of consolidation is calculated considering the total load applied instantly at ($T_v - T_c/2$), according to Equation 1:

$$U_c\left(T_v\right) = \begin{cases} \dfrac{T_v}{T_c} U_v\left(\dfrac{T_v}{2}\right) & T_v \le T_c \\ U_v\left(T_v - \dfrac{T_c}{2}\right) & T_v > T_c \end{cases} \tag{1}$$

Olson (1977) subdivided the ramp load into infinitesimal load increments and applied for each load increment the Terzaghi (1923, 1925) consolidation solution for instantaneous loading. A differential equation was obtained and integrated over time, which permitted the calculation of excess pore water pressures and the average degree of consolidation (Equation 2).

$$U_c\left(T_v\right) = \begin{cases} \dfrac{T_v}{T_c}\left[1 - \dfrac{1}{T_v}\displaystyle\sum_{m=0}^{\infty}\dfrac{2}{M^4}\left(1 - e^{-M^2 T_v}\right)\right] & T_v \le T_c \\ 1 - \dfrac{1}{T_C}\displaystyle\sum_{m=0}^{\infty}\dfrac{2}{M^4}\left(e^{M^2 T_c} - 1\right)e^{-M^2 T_v} & T_v > T_c \end{cases} \tag{2}$$

where $M = \dfrac{\pi}{2}\left(2m + 1\right)$

Terzaghi (1943) empirical method tends to overestimate the average degree of consolidation when compared to Olson (1977) solution, with a difference of about 10% (Hanna et al., 2013). In order to decrease this difference, Hanna et al. (2013) proposed a slight modification in Terzaghi's method so that

[#]Corresponding author. E-mail address: vitorsalbuquerque@aluno.puc-rio.br
[1]Pontifícia Universidade Católica do Rio de Janeiro, Departamento de Engenharia Civil e Ambiental, Rio de Janeiro, RJ, Brasil.

Submitted on January 29, 2022; Final Acceptance on November 10, 2023; Discussion open until May 31, 2024.
https://doi.org/10.28927/SR.2024.003522

the ramp load is considered instantaneously applied at time factor $T_f = 2T_v/5$, during the construction period, instead of the generally used time factor fraction $T_f = T_v/2$.

The main objective of this technical note is to revisit Terzaghi (1943) empirical method introducing two simple methodologies to improve primary settlement estimates, comparing their results with Olson (1977) analytical solution and laboratory oedometer tests data.

2. Terzaghi (1943) method revisited

2.1 Methodology 1: new time fractions

The adjustment of time factor fractions was carried out by assuming Olson (1977) analytical solution given by a function f and the new approximated solution by a function g, both belonging to the same vector space. The Euclidean norm that estimates the distance between them should be as close to zero as possible. To calculate this distance, a sequence of equally spaced points ($\Delta T_v = 0.01$) was taken within the interval $0.01 \le T_c \le 2$.

For different construction times T_c, Table 1 shows the adjusted time factor fractions T_f that may be used for $T_v \le T_c$ and $T_v > T_c$ with Terzaghi (1943) method. In the post-construction period, the values of the average degree of consolidation U_v were computed assuming that the total load was instantaneously applied at time factor equal to $(1 - T_f)T_c$. The computed data allowed a representation of time-dependent loading curves for several construction time factors, as shown in Figure 1 for $T_c = 0.5$.

Based on the results listed in Table 1, a correlation (Equation 3) between the time factor fraction T_f and the construction time factor T_c could be obtained with coefficient of determination $R^2 = 0.994$.

$$T_f = 0.0090T_c^2 - 0.0845T_c + 0.4833 \; for \; 0 < T_c \le 2 \quad (3)$$

Considering a sequence of ΔU_v curves separated by an increment of the construction time factor $\Delta T_c = 0.1$ within the interval $0.1 \le T_c \le 2$, the curves in Figure 2 indicate that the constant time factor fraction $T_v/2$ may overestimate the average degree of consolidation up to $\Delta U_v = 9.67\%$ with respect to Olson (1977) analytical solution at $T_c = 2$.

On the other hand, the proposed methodology based on a variable time factor fraction yields a very good agreement with the analytical solution, with a overestimation of approximately $\Delta U_v = 2.14\%$ at $T_c = 2.0$, as can be seen in Figure 2.

2.2 Methodology 2: reduction of the average degree of consolidation

Another methodology to correct Terzaghi (1943) method during the construction period was proposed by Hanna et al. (2013) considering the load applied instantaneously at $T_f = T_v/2$ (that is, keeping Terzaghi's recommendation) but considering a 10% reduction of the overestimated average

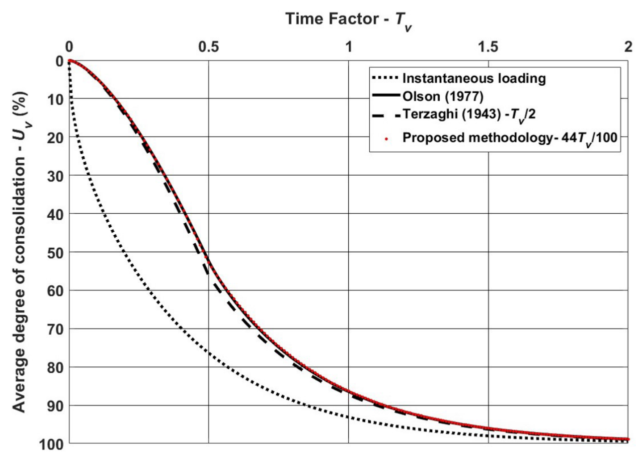

Figure 1. $U_v - T_v$ curves for instantaneous (Terzaghi, 1925) and ramp loads for $T_c = 0.5$.

Table 1. Time factor fractions T_f for clay layers with single drainage ($0.05 \le T_c \le 2$).

T_c	$T_v \le T_c$	$T_v > T_c$	T_c	$T_v \le T_c$	$T_v > T_c$	T_c	$T_v \le T_c$	$T_v > T_c$	T_c	$T_v \le T_c$	$T_v > T_c$
0.05	0.48	0.52	0.55	0.44	0.56	1.05	0.40	0.60	1.55	0.37	0.63
0.10	0.47	0.53	0.60	0.44	0.56	1.10	0.40	0.60	1.60	0.37	0.63
0.15	0.47	0.53	0.65	0.43	0.57	1.15	0.40	0.60	1.65	0.37	0.63
0.20	0.47	0.53	0.70	0.43	0.57	1.20	0.39	0.61	1.70	0.37	0.63
0.25	0.46	0.54	0.75	0.43	0.57	1.25	0.39	0.61	1.75	0.36	0.64
0.30	0.46	0.54	0.80	0.42	0.58	1.30	0.39	0.61	1.80	0.36	0.64
0.35	0.45	0.55	0.85	0.42	0.58	1.35	0.39	0.61	1.85	0.36	0.64
0.40	0.45	0.55	0.90	0.41	0.59	1.40	0.38	0.62	1.90	0.36	0.64
0.45	0.45	0.55	0.95	0.41	0.59	1.45	0.38	0.62	1.95	0.35	0.65
0.50	0.44	0.56	1.00	0.41	0.59	1.50	0.38	0.62	2.00	0.35	0.65

degree of consolidation. Keeping the same methodology proposed by the authors but considering both construction and post-construction periods in the analysis, a second methodology is presented as follows.

During the construction period ($T_v \leq T_c$), the best percentage of U_v reduction for a given construction time factor T_c was again obtained by calculating the Euclidean norm between the functions describing Olson (1977) analytical solution and the new approximate method. The distance between both functions should be as close to zero as possible in order to minimize the error with respect to Olson's results. For the post-construction period ($T_v > T_c$), the time factor T_v was incrementally increased until the average degree of consolidation U_v has practically reached the same value determined during the construction period at $T_v = T_c$. For several construction time factors, Table 2 shows the corresponding coefficients that should multiply the U_v values calculated with Terzaghi (1943) empirical method for $T_v \leq T_c$ and $T_v > T_c$.

Considering the results in Table 2 it was possible to replot the ramp loading curve for $T_c = 1.5$ in Figure 3, which now appears practically superimposed to Olson (1977) solution and gives more accurate predictions than those previously obtained with methodology 1.

3. Experimental validation

The accuracy of the two alternative methodologies was evaluated considering their abilities to predict oedometer test data.

Laboratory ramp loading oedometer tests were conducted by Sivakugan et al. (2014) on artificially mixed kaolinite/sand blend that was mixed in equal proportions with the following characteristics: 0.6 m^2/year (coefficient of consolidation), $q_c = 215.1$ kPa (maximum load at the end of the ramp loading), H_o = 18.241 mm (initial specimen thickness), $\Delta H = 0.272$ mm (total consolidation settlement), ρ_c = 0.22 mm (consolidation settlement at T_c). The ramp

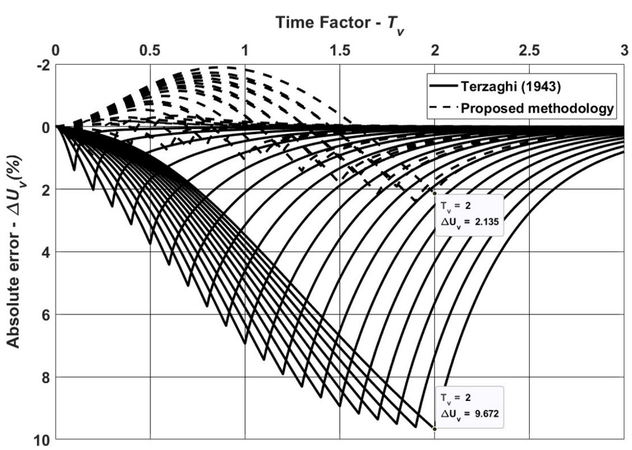

Figure 2. ΔU_v *vs* T_v for $0.1 \leq T_c \leq 2$.

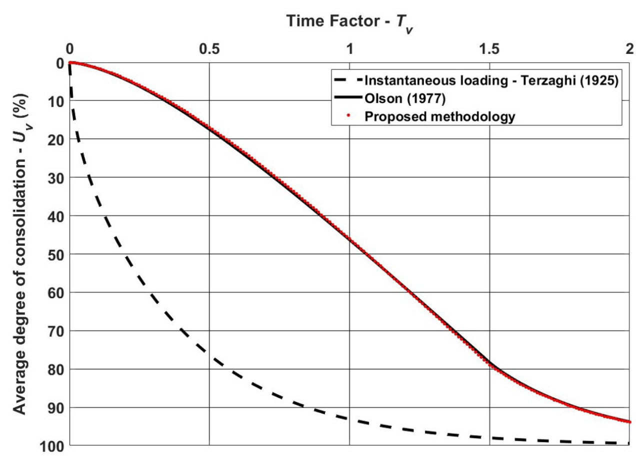

Figure 3. $U_v - T_v$ curves for instantaneous and ramp loads for $T_c = 1.5$.

Table 2. Multiplying coefficients to reduce *Uv* calculated by Terzaghi (1943) method for $0.05 \leq T_c \leq 2$.

T_c	$T_v \leq T_c$	$T_v > T_c$	T_c	$T_v \leq T_c$	$T_v > T_c$	T_c	$T_v \leq T_c$	$T_v > T_c$	T_c	$T_v \leq T_c$	$T_v > T_c$
0.05	0.9477	0.5510	0.55	0.9356	0.5646	1.05	0.9164	0.5992	1.55	0.9040	0.6396
0.10	0.9450	0.5535	0.60	0.9337	0.5673	1.10	0.9148	0.6032	1.60	0.9032	0.6437
0.15	0.9438	0.5547	0.65	0.9317	0.5703	1.15	0.9132	0.6073	1.65	0.9025	0.6476
0.20	0.9434	0.5550	0.70	0.9297	0.5734	1.20	0.9118	0.6113	1.70	0.9019	0.6515
0.25	0.9430	0.5554	0.75	0.9276	0.5768	1.25	0.9104	0.6154	1.75	0.9013	0.6554
0.30	0.9424	0.5561	0.80	0.9256	0.5803	1.30	0.9092	0.6193	1.80	0.9007	0.6594
0.35	0.9415	0.5571	0.85	0.9237	0.5839	1.35	0.9080	0.6234	1.85	0.9003	0.6632
0.40	0.9404	0.5584	0.90	0.9217	0.5877	1.40	0.9069	0.6275	1.90	0.8999	0.6670
0.45	0.9390	0.5601	0.95	0.9199	0.5914	1.45	0.9058	0.6316	1.95	0.8995	0.6708
0.50	0.9374	0.5622	1.00	0.9181	0.5953	1.50	0.9049	0.6356	2.00	0.8992	0.6745

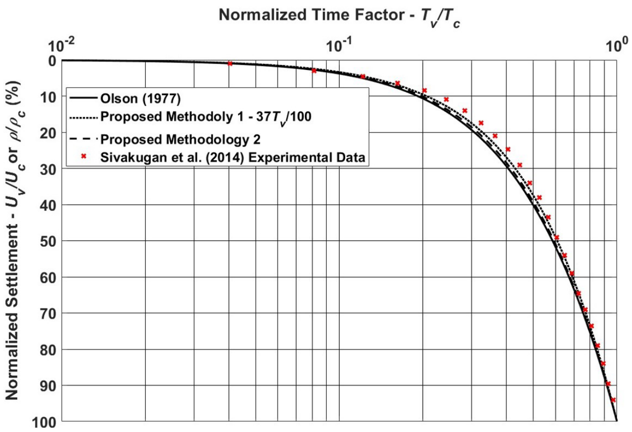

Figure 4. Experimental and theoretical normalized settlement ramp loads curves for T_c = 1.6, considering different methodologies.

loading was applied by filling a bucket on the loading arm with scoops of sand over a period of 1-2 h. Figure 4 presents the experimental results for T_c = 1.60 with the predictions obtained by Olson (1977) analytical solution and the two methodologies herein presented. In the settlement-time curves the time factor T_v was normalized with respect the construction time factor T_c and the consolidation settlement ρ at time t was normalized with respect to the settlement ρ_c at the end of the ramping load t_c, which is equivalent to the ratio U_v/U_c where U_c is the average degree of consolidation at time factor T_c.

As can be seen in Figure 4, the ramp loading laboratory tests clearly demonstrate that the normalized settlement-time plots fall within a narrow band, matching the theoretical predictions. The maximum observed difference in this case was about 5% between $0.2 < T_v/T_c < 0.50$.

4. Conclusions

This technical note presented two methodologies that adapt Terzaghi (1943) empirical method to calculate primary consolidation settlement due to a ramp loading. The first methodology is based on a variable time factor fraction T_f dependent on the construction time factor T_c, while the second methodology keeps Terzaghi (1943) original time factor fraction $T_f = T_v/2$ for the construction period but corrects the settlement overestimation through a multiplying coefficient in order to reduce the average degree of consolidation U_v.

Both methodologies showed good agreement with experimental oedometer test data and the theoretical solution presented by Olson (1977). For the engineering practice, the main advantage of the described methodologies is to recommend the use of the well-known Terzaghi (1943) empirical method but applying either of the two corrections: a) new time factor fractions easily determined through a correlation with the construction time factor; b) keeping the time factor fraction suggested by Terzaghi ($T_f = T_v/2$) but using a multiplying coefficient to reduce the average degree of consolidation U_v= 2. The second one yields predictions that are practically the same as those calculated by Olson (1977) method.

Acknowledgements

The authors thank the Brazilian funding agency CAPES and the Pontifícia Universidade Católica do Rio de Janeiro (PUC-Rio) for their support.

Declaration of interest

The authors declare no conflict of interest.

Authors' contributions

Vitor dos Santos Albuquerque: conceptualization, data curation, formal analysis, investigation, methodology, validation, writing – original draft. Celso Romanel: validation, writing – review and editing the final manuscript. Raphael F. Carneiro: validation, writing – review and editing the final manuscript.

Data availability

All data produced or examined in the course of the current study are included in this Technical Note.

List of symbols

c_v	coefficient of consolidation
m	count parameter
q_c	maximum vertical load at the end of the ramp loading
t_c	duration of the ramping load
H_o	initial specimen thickness
M	normalized count parameter
T_c	construction time factor in terms of layer thickness
T_f	time factor fraction
T_v	time factor in terms of layer thickness
U_c	average degree of consolidation at T_c
U_v	average degree of consolidation at T_v
ΔH	total consolidation settlement.
ΔT_v	time factor increment
ΔU_v	absolute error
ρ	consolidation settlement at time t
ρ_c	consolidation settlement at time t_c

References

Carneiro, R., Gerscovich, D., & Danziger, B. (2021). A simple approach to predict settlement due to constant rate loading in clays. *Soil and Rocks*, 44(2), 1107-1116. http://dx.doi.org/10.28927/SR.2021.057120.

Conte, E., & Troncone, A. (2006). One-dimensional consolidation under general time-dependent loading. *Canadian Geotechnical Journal*, 43(11), 1107-1116. http://dx.doi.org/10.1139/t06-064.

Hanna, D., Sivakugan, N., & Lovisa, J. (2013). Simple approach to consolidation due to constant rate loading in clays. *International Journal of Geomechanics*, 13(2), 193-196. http://dx.doi.org/10.1061/(ASCE)GM.1943-5622.0000195.

Olson, R. (1977). Consolidation under time dependent loading. *Journal of the Geotechnical Engineering Division*, 103(1), 55-60. http://dx.doi.org/10.1061/AJGEB6.0000369.

Schiffman, R.L. (1958). Consolidation of soil under time dependent loading and varying permeability. In *Proceedings of the 37th Annual Meeting of the Highway Research Board* (pp. 584-615), Washington DC.

Sivakugan, N., Lovisa, L., Ameratunga, J., & Das, B.M. (2014). Consolidation settlement due to ramp loading. *International Journal of Geotechnical Engineering*, 8(2), 191-196. http://dx.doi.org/10.1179/1939787913Y.0000000017.

Terzaghi, K. (1923). Die berechnung der durchlassigkeitsziffer des tones aus dem verlauf der hydro-dynamischen spannungserscheinungen. *Sitzungberichte Akademie der Wissenschaften*, 132, 125-138.

Terzaghi, K. (1925). *Erdbaumechanik auf bodenphysikalischer grundlage*. Vienna: Franz Deuticke.

Terzaghi, K. (1943). *Theoretical soil mechanics*. New York: Wiley.

Zhu, G., & Yin, J. (1998). Consolidation of soil under depth-dependent ramp load. *Canadian Geotechnical Journal*, 35(2), 344-350. http://dx.doi.org/10.1139/t97-092.