Soils and Rocks
An International Journal of Geotechnical and Geoenvironmental Engineering
www.soilsandrocks.com
ISSN 1980-9743
ISSN-e 2675-5475

Teaching modern soil mechanics

Emanuel Maranha das Neves[1#]

Article

Keywords
Soil mechanics education
Plasticity
Classic soil mechanics
Critical state soil mechanics

Abstract

The important role of the critical state theory in the modern soil mechanics is undeniable. It is true that the number of soil mechanics courses that not cover this subject is progressively decreasing. However, when the critical state theory is introduced, this topic cannot be seen as a simple extension of the classic soil mechanics. On the contrary, it is essential that some significant differences between modern and classic soil mechanics are adequately clarified and understood. This subject is a relevant objective of this paper, besides the large benefits brought by the modern soil mechanics. This discipline, like the mechanics applied to other materials, is fundamentally a preliminary learning to prepare for the professional practice of geotechnical engineering. When the main objective is to teach methods to solve the engineering problems (foundations, excavations, embankments, tunnels, etc.), the matters transmitted to the students are sometimes focused on the geotechnical engineering methods, where, nevertheless, soil mechanics, naturally, has an irreplaceable role. It is true that a design is unique in itself. However, all designs must have in common the same theoretical principles of soil mechanics, regardless of the particularities of the geotechnical design. This cannot be neglected in the modern soil mechanics teaching. Brief ideas concerning where and how soil mechanics has been taught, is also introduced. The fundamentals about plastic design of geotechnical structures are highlighted. The article ends calling attention to the outstanding contribution of the critical state theory for a unified understanding of the soil behavior. Its pedagogic benefits are invaluable.

1. Introduction

After a short comment about the plasticity role in soil mechanics and to the limitations of elasticity concerning the description of the soil mechanical behavior, the link between the perfect-plasticity and the classic soil mechanics is characterized and exemplified. A brief reference to the role of finite element method on limit equilibrium analysis and on displacements evaluation is also presented. The tight relationship between modern plasticity and the modern soil mechanics has been taken into consideration. Failure soil criteria are reexamined, in light of classic and modern soil mechanics. Understanding of soil behavior according to the critical state theory is introduced and its contribution to modern soil mechanics is underlined, with a particular emphasis on the displacements prediction capacity. Finally, some considerations about the plastic design of geotechnical structures are presented.

2. Previous considerations

There are some basic assumptions, applicable to both classic soil mechanics (CSM) and critical state soil mechanics ($CSSM$) that deserve to be mentioned.

2.1 Continuum mechanics

Continuum mechanics, like in other branches of mechanics, is by far one of the underlying sciences on the case of soils. Modern theories that describe mechanical soil behavior considering, explicitly, the particulate nature of soils, do exist. But in nearly all geotechnical engineering applications, theoretical and practical, the soil is idealized as a continuum, i.e., a body that may be subdivided indefinitely without altering its character.

2.2 Effective stresses principle

The principle of effective stresses (Terzaghi, 1936), a fundamental concept for the establishment of soil mechanics itself, is obviously another basic assumption.

2.3 The soil material

It is necessary to make some considerations about this material. Currently, the material object of any mechanics is an archetype of the real material. So, a prerequisite in any design problem involving the real materials is the assumption of certain simplifying material properties to assist mathematical analysis (Chen & Baladi, 1985). In this

[#]Corresponding author. E-mail address: emanuelmaranhadasneves@gmail.com
[1]Universidade de Lisboa, Instituto Superior Técnico, Lisboa, Portugal.

Submitted on May 30, 2023; Final Acceptance on January 18, 2024; Discussion open until August 31, 2024.
https://doi.org/10.28927/SR.2024.006823

case, the soil is considered a homogeneous mechanical mixture of two phases. One represents the structure of solid particles and the other constitutes the fluid water filling up the voids of the aggregate (the soil will always be regarded as saturated). The only forces among the particles are due to the effective stress. In practical terms it can be designated by saturated disturbed soil.

2.4 Disturbed and undisturbed soils

To justify the inevitable but powerful simplification just presented, some brief considerations resulting of the contrast between real and ideal soils must be pointed out. The particles of natural soils exhibit different degrees of structure, which may significantly influence the soil behavior. Ultimately, each natural soil could be considered a different material, but to bypass this troublesome allegation, it is necessary to make use of classifications, frames of reference, etc. To exemplify, the proposal for the use of mechanical characteristics of reconstituted (disturbed) clays as a basic frame of reference for interpreting the corresponding characteristics of natural sedimentary clays can be cited (Burland, 1990). The properties of reconstituted clays are termed intrinsic properties since they are inherent to the soil and independent of the natural state. Otherwise, the properties of natural clay differ from the intrinsic properties due to the influence of soil structure (fabric and bonding). The intrinsic properties provide a frame of reference for assessing the in-situ properties of natural clay and the influence of structure on its in situ properties.

3. Soil mechanics education, elasticity, and plasticity

The emphasis on plasticity issues in a text about the education on soil mechanics needs a justification. The aim of the paper is not to interfere with the course syllabus but remember that the theory of plasticity is an essential part of the education in soil mechanics. As it will be seen, the references to plasticity are also done to get a better understanding of the transition of the classic to the modern soil mechanics.

3.1 Linear elasticity in soil mechanics

Elasticity has sometimes been used successfully in classic soil mechanics to describe the general behavior of soil deformation under short-term working load conditions. Certainly, soil is by no means an elastic material. But it is attracting to assign values to a Young modulus (E) and to a Poisson's ratio (ν) and take profit of a large number of solutions for stresses and displacements due to the application of many types of loads to the surface of an elastic half-space that are available on catalogs of solutions (Poulos & Davis, 1974). Mainly due to the soil dilation phenomena, the elastic parameters largely used in soil mechanics are the bulk modulus (E) and shear modulus, (G) instead of E and ν.

The use of elasticity in the well-known one-dimensional consolidation theory (Terzaghi, 1925) must also be referred.

On the practice field, the elasticity application on the prediction of the foundations settlements due to vertical stress changes, whose reliability was demonstrated by Burland et al. (1977), is used to a great extent even today. This option can be a significant advantage when compared with the time and effort involved in obtaining numerical solutions that employ one of the numerous plasticity soil models available.

Nevertheless, elasticity fails to predict the behavior and strength of a soil-structure interaction problem near ultimate strength condition, because plastic deformation at this load level attains a dominating influence, while elastic deformation becomes of minor importance. This aspect strongly impairs any role of elasticity on the evaluation of structural safety through methods other than the maximum allowable stresses.

3.2 Perfect-plasticity and soil mechanics

The application of plasticity to soil mechanics begun more than 200 years ago, based on the celebrated contribution of Coulomb (1773). Until the 1950-60 decade, a lot of work concerning the rigid-perfectly plastic and the elastic-perfectly plastic models, most of them focused in metals, was accomplished and well understood. This knowledge field can be called classic plasticity.

But, by the time, not only the soil mechanics as a scientific discipline was consolidated, as well as a remarkable activity in the field of plasticity theory was in progress, with particular emphasis on the strain hardening (and strain softening) elastic-plastic models. This scientific work is called modern plasticity.

During the following text it will become clear that *CSM* is tightly associated with classic plasticity and *CSSM* is closely linked to modern plasticity.

3.2.1 The soil mechanics and classic plasticity

It can be admitted that despite the marked difference between metals and soils, the research into soil classic plasticity, notwithstanding its historical application to earth masses by Coulomb (op. cit.), arose as a result of investigations carried out on the mechanical behaviour of metals. For instance, the use, even in the present days, of the bearing capacity formula for continuous footings (Terzaghi, 1943), was inspired on the work about the use of slip lines theory applied to the metal indentation (Prandtl, 1921). Fundamental aspects of the theoretical background of classic plasticity, with reflex in soil mechanics, are the stability postulate and the associated flow rule (Drucker et al., 1952), and, above all, the important theorems of plastic collapse.

3.2.2 Limit analysis

As it is well known, the solution of any limit equilibrium problem can be obtained through the system formed by the equilibrium equations and the equation of the adopted failure criterion. This set of equations is normally known as basic equations. In cases where geometry and actions are simple, it is possible to obtain exact solutions only based on the Mohr circle. As a matter of fact, this circle is a graphic representation of the equilibrium conditions. If the failure condition (represented by two straight lines inclined and tangent to the Mohr circle) is added, a graphical representation of the basic equations is obtained and the necessary conditions to solve limit equilibrium problems exist (when geometry and actions are simple, as was already mentioned).

But the integration of these equations, taking into account the boundary conditions, is, analytically, unmanageable. This difficulty attracted the mathematicians to work hard in this area of the plasticity (in the same way as, some years before, they have done with the integration of the Laplace differential equation applied to seepage problems taking account complex boundary conditions). Once more, exact solutions were obtained only in relatively simple cases.

Therefore, it became common to use approximate methods, as is the case, for instance, of the numerical solutions proposed by Sokolovski (1960). He developed the theory of critical stress equilibrium. But since then no new methods or practical applications worth mentioning have been developed in this area.

Despite the simplicity of the strength expressions at failure, it is quite difficult to obtain exact solutions. So, standard methods used in geotechnical engineering involve simplifications. Two basic approaches exist: the bound methods and the limit equilibrium method.

3.2.3 The theorems of plastic collapse

Making use of these theorems of the perfect plasticity, it is possible, without satisfying all of equilibrium and compatibility conditions, to introduce important simplifications in the stability calculations (Davis & Selvadurai, 2002).

More in detail, to calculate an upper bound it is necessary to satisfy the conditions of compatibility and of the material properties (which govern the work done by the stresses in the soil) but nothing is said about the equilibrium conditions. On the other hand, to calculate a lower bound is mandatory to satisfy the conditions of equilibrium and the material properties (which determine the strength), but nothing is said about displacements or compatibility. This has important consequences on the procedures for safety evaluation of simple geotechnical structures.

3.2.4 Discontinuous equilibrium stress states

Contrary to what happens in elastic media, in the plastic media, the stresses do not impose any strain condition, so it is not necessary to verify the compatibility requirement of the elasticity. So, it is admissible to consider possible discontinuities in the stress fields of plastic equilibrium in order to obtain solutions that comply with the boundary conditions. Such discontinuities are characterized by abrupt changes of direction and on the value of the principal stresses. Then, the lower bound theorem allows the attainment of approximate solutions for a lot of classical problems of plasticity. In such cases it is simpler than any other type of solution, namely those that use numerical methods.

The most well-known methods derived from the theorems of plastic collapse, are those based on the Mohr representation, the numerical method due to Sokolovski (1960), the consideration of discontinuities on the stress field and the slip line theory. Correct solutions to the limiting earth pressure problems with given stresses on the boundaries were given for example by Sokolovski (op. cit.). Despite their very low use, these methods of analysis deserve to be referred to. All solutions that use the slip plane model neglected strain.

3.2.5 The method of the associated fields

In order to predict deformations using perfect plasticity, Serrano (1972) and James et al. (1972), among others, proposed a solution allowing the determination of the stress and strain for each point of the soil mass. It was imperative to equilibrate the resultant stress field, not only with the applied exterior actions but also with the internal stresses. Regarding de strain field, the boundary conditions and the internal kinematic compatibility need to be satisfied. Obviously, it was necessary to postulate a stress-strain law. However, this apparently promising way to obtain displacements of soil structures had no continuity. Since the beginning of years 70, the subject has lost any research and practical interest.

3.2.6 The limit equilibrium method

The limit equilibrium method is the most used to evaluate the stability of geotechnical structures. The method puts together characteristics of both the upper and lower bound theorems. The geometry of the slip surfaces must form a mechanism that will allow the collapse to occur (upper bound) and the overall conditions of equilibrium of forces on blocks within the mechanism must be satisfied (lower bound). The limit equilibrium method leads to correct solutions (there is no formal proof of this allegation) and is one of the reasons for the large use of the method, even in our days. This is also the case with the old wedge analysis methods of Coulomb and the Rankine (1857) method. A large number of computer programs for analysis of geotechnical structures that make use of the limit equilibrium method are available today.

3.2.7 Perfectly plasticity and dilation

One of the more distinctive mechanical properties of soil is dilation. This phenomenon, common to all particulate media, was already known from the 19th Century (Reynolds, 1885). This property is quantified through the angle of dilation, ψ, defined as the relation between the volumetric and deviator strain increment components $(\delta\varepsilon_v \text{ and } \delta\varepsilon_s)$ of the resultant deformation, $\delta\varepsilon$. It is then interesting to see how this important property can be integrated on the continuum mechanics and the elastic perfectly-plastic theories used in the classic soil mechanics.

A condition required to prove the bound theorems is that the material must be perfectly-plastic. This implies an associated flow rule, i.e., the increment of plastic strain must be normal to the yield function.

Figure 1 presents a yield function, which is also the failure envelope, corresponding to a drained loading of a soil.

According to the definition of ψ, at failure, it will be

$$\tan\psi_f = -\frac{\delta\varepsilon_v^p}{\delta\varepsilon_s^p} = \tan\phi_f^{'} \tag{1}$$

As will be seen later on (when dealing with *CSSM*), a soil state at failure is constant, which also means that the volume is invariant. Consequently $\psi_f = 0$. But, according to the normality rule, $\delta\varepsilon_p$ must be normal to the yield function. This condition cannot be satisfied because, as previously proven, $\psi_f = \phi_f^{'}$. This means that the plastic behavior during a drained test cannot be taken as perfectly plastic, at least if the yield and the plastic potential functions are identical. This is important due to the large use of the elastic-perfectly plastic non-associated Mohr-Coulomb models in geotechnical practice, even in our days.

These considerations merely mentioned the incompatibility between the perfectly-plastic model and dilation, which could install doubts about the use of dilation on elastic perfectly-plastic models. This subject will be clarified further below.

Consider a plastic potential function, G (different and possibly more adequate than the yield function,), as can be seen in Equation 2 (Wood, 2004)

$$G(\boldsymbol{\sigma}) = G(p', q) = q - \mathrm{M}^* p' + k = 0 \tag{2}$$

where $\boldsymbol{\sigma}$ is the stress tensor, k an arbitrary value which allows $G(p', q)$ to be defined at the current state stress and M^* is a soil property related to the dilation (see Figure 2). In this context and still in the perfectly plastic framework, the increment of plastic deformation complies with normality rule.

In practice, to use an elastic perfectly-plastic model, the elastic parameters $(E \text{ and } G)$ and the resistance, ϕ', must be determined. If the information regarding the yielding volume variation his needed, the dilation parameter, M^*, or ψ, is also required. All these parameters are constant.

In a drained loading context, if $\psi < 0$, the soil volume diminishes at constant rate. If $\psi > 0$, the soil volume increases at constant rate. This model is employed on the safety analysis of geotechnical structures. When an undrained loading is considered, the ψ value must be zero (Maranha & Maranha das Neves, 2009). In fact, if $\psi \neq 0$, the soil will never fail.

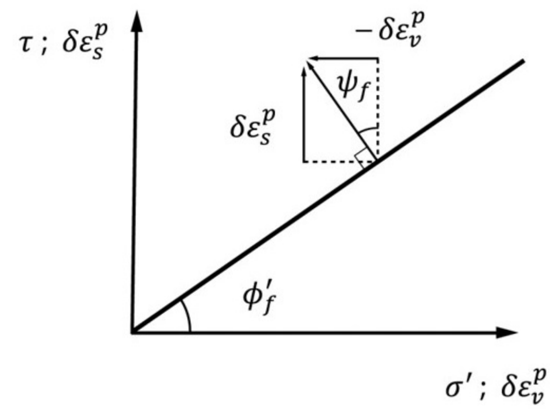

Figure 1. Plastic strain increment in a perfectly-plastic soil model (in the case of a drained loading).

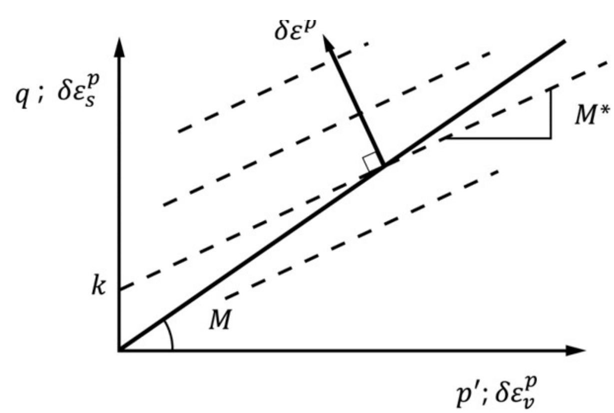

Figure 2. Plastic potential functions of an elastic perfectly plastic material, different of the yield function, and that observes the Mohr-Coulomb criterion.

4. The role of the finite element method on limit equilibrium analysis and on displacements evaluation

Practical results of the research on this area had occurred from 1970, and we cannot ignore their role on the prediction of the deformational behavior of geotechnical structures (Duncan, 1996).

4.1 Static stability and deformation analysis in geotechnical structures

The finite element method has been developed and adapted to these applications. Improvements on this approach followed the increasing availability of computers and related software. The finite element method was confirmed as the most widely used method of analysis of deformations on geotechnical design. Geotechnical engineers had long been aware of the limitations of the linear elastic analysis of stresses and strains in earth masses, and it was immediately apparent that the ability to consider nonlinear stress-strain behavior gave the finite element method great potential for use in geotechnical problems.

The finite element method allowed the calculation of deformations of soil structures before failure, considering non-linear behavior of the soil (with de elastic-perfect plastic model, before failure, only elastic strains, reversible, were obtained). This was a remarkable aspect as it allowed the analysis of the serviceability of a soil structure.

4.2 The incremental analysis and the new breath of elasticity concerning the serviceability limit states of geotechnical structures

The use of incremental analysis involved the simulation of the overall problem as a series of events, and to interpret each event as a simple linear problem. Nonlinear and stress-strain dependent behavior is modeled by changing the stiffness values assigned to each element during each increment of the analysis. Different stress-strain relationships were used, namely the hypoelastic approaches (Naylor et al., 1986; Maranha das Neves & Veiga Pinto, 1988).

Examining more in detail the impact of the use of finite element and finite differences methods in the geotechnical area is out of the scope of this paper. But the growing and useful influence of these numerical techniques on the applications to geotechnical engineering cannot be denied.

5. The classic soil mechanics

5.1 Generalities

As already pointed out, classic soil mechanics and plasticity are tightly connected. It must be highlighted the role of the plane and its omnipresence on the theory of the classic soil mechanics. Consequently, any role of the intermediate principal stress is omitted. But perfectly-plasticity may be profitably used since as it permits to take advantage of the powerful bound theorems. Nevertheless, its practical use is restricted to safety evaluations of geotechnical structures.

Today, excluding the use of the actual version of the Coulomb method, the Rankine method, the limit equilibrium method, as well as the use of the Mohr – Coulomb perfectly-plastic model (with or without dilation) on numerical safety evaluation of geotechnical structures, it must be recognized that the use of the classic plasticity in the soil engineering practice, is, in a certain way, modest. The main debility is its impossibility to predict the deformations of a geotechnical system under working loads, i.e., the evaluation of the potential serviceability limit states. Indeed, classical geotechnical education concentrates its attention on the determination of shear strength and on the failure of geotechnical structures, i.e., the evaluation of ultimate limit states (Wood, 2012).

5.2 Fundamental aspects of the classic soil mechanics not shared with the modern soil mechanics

There are other fundamental aspects that must be cited, though they are not shared with the modern soil mechanics. In particular, the use of the concept of true cohesion, the maintenance of the classical ideas of Terzaghi centred on an approach to the strength and stress-strain relationships as practically independent entities. Another feature is to think about clays and sands as soils that need to be dealt with in separate, and finally, the necessity of an *ad hoc* explanation of the concepts of drained and undrained behavior, as well as the concept of the undrained strength (Alonso, 2005). These topics will be appreciated later on when dealing with $CSSM$.

Another interesting point refers to the role of mineralogy and colloidal chemistry on the mechanical behavior of clays. More precisely, the role of inter-particle forces when they have dimensions lower than $1\mu m$. This was a subject widely and deeply developed in the soil mechanics textbooks, mainly in the decades 70-80. This is the case, for instance, of the well-known and appreciated books of Scott (1963) or Lambe & Whitman (1979). But in more recent publications, mainly those who include the critical states theory, the theme of the particle's mineralogy and inter-particle forces resulting of the surface chemistry is completely ignored. See for instance, Schofield & Wroth (1968), Bolton (1979), Wood (1990) and Atkinson (1993).

It is true that in geotechnical civil engineering and in some situations, these inter-particle forces may have a significant role, for instance those related with the occurrence of piping, scour, self-filtering etc. But nothing that could justify the relevance assigned to this type of forces in soil mechanics (Maranha das Neves, 2007).

6. The critical state soil mechanics and the modern plasticity

The emergence of the modern soil mechanics $(CSSM)$ is also a consequence of more recent developments in the plasticity theory. The results of these significant advances are called modern plasticity. In short, the coming in site of the $CSSM$ is also due to the modern plasticity, a fact that must be underlined by those who teach soil mechanics.

6.1 Work hardening plasticity

One of the major advances is the application of the elastic-plastic work hardening (and work softening) theory to soil and is due to Drucker et al. (1957). The innovation is based in the idea that the usual soil consolidation curve is a case of work hardening stress-strain relationship, as well as the successive yield envelopes, as those marked 1 and 2 in Figure 3. Another innovation concerns the isotropic normality consolidated condition, such as point A in Figure 3:

an increase in the mean effective stress causes yield so that the yield envelope must pass through point A'. The yield surface changes according a hardening law, usually based on the accumulated volumetric plastic work (Wroth, 1973).

6.2 The emergence of the critical state soil mechanics

In the last part of this paper, besides drawing your attention to the unique facets of the *CSSM*, the main aim is to make clear the differences between modern and classic soil mechanics. According to a highly impressive generic appreciation it can be said that the *CSM* is based in critical stresses while the *CSSM* is based in critical states.

The family of soil models developed at Cambridge University (UK) resulted not only from the introduction of work hardening plasticity into soil mechanics, as well as from the important innovation that has been the concept of critical state, conceived by Roscoe et al. (1958).

A soil is said to be in a critical state when exhibits shear strains with invariance of q, p' and v. According to this concept, the critical state line (CSL) is the locus of the end condition (failure) of all shear paths, considering that soil remains homogeneous during those trajectories.

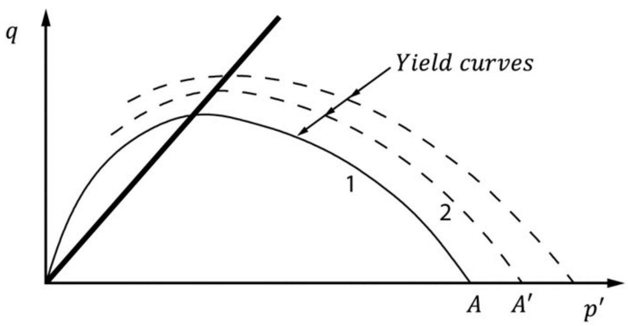

Figure 3. Possible yield surfaces produced by normal consolidation.

The failure criterion, according to the critical states, is defined not only in a stress plane q e p' (as in the *CSM*), but also considers the specific volume v. (Figure 4).

The failure criterion is defined by the Equations 3 and 4. The Equation 3, in the (q, p') plane, is

$$q = \mathrm{M}p' \tag{3}$$

and Equation 4, which represents the critical state line, *CSL,* in a (q, p', v) space, will be

$$v = \Gamma - \lambda \ln p' \tag{4}$$

where Γ is the v value at the CSL, for $p' = 1kPa$.

The Equation 5, not represented in Figure 4, is the unload-reload line in the $(v, \ln p')$ plane

$$v = \mathrm{N} - \kappa \ln p' \tag{5}$$

where κ is the slope of that line and N is the v value at the *NCL* for $p' = 1kPa$.

In this model, the elastic and plastic behavior is entirely specified by only four basic soil constants: $\lambda, \kappa, \mathrm{M}$ or ϕ' and Γ.

The *CSL* is the critical state locus. This line links together successive yield locus by connecting the points as C in the Figure 4.

It was first proven that the use of the Coulomb's failure envelope as a yield locus (Drucker et al., 1957) was mistaken. In reality this envelope is the locus of separate failure points (see Figure 5b).

The first model to make use of the work hardening plasticity in the context of the critical state theory was presented under the name of Cam-clay. The different limit surfaces in a (q, p', v) space, in addition to the *CSL* and to the *NCL*, are presented in Figure 6. The Cam-clay model diffusion in multiple variants, many concerning the use in practice, occurred in a short time. The use of numerical

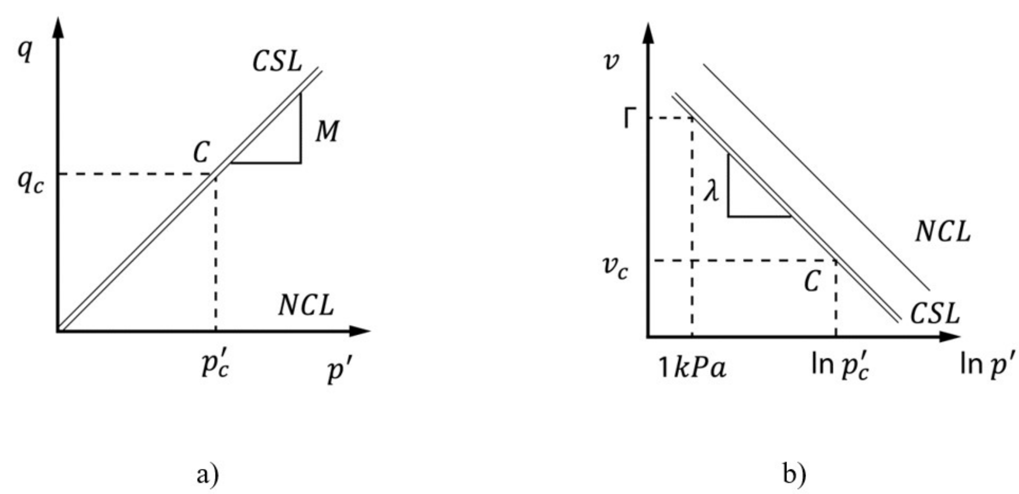

Figure 4. Critical state line (*CSL*) and normal compression line (*NCL*). (a) in the (q, p') plane; (b) in the $(v, \ln p')$ plane.

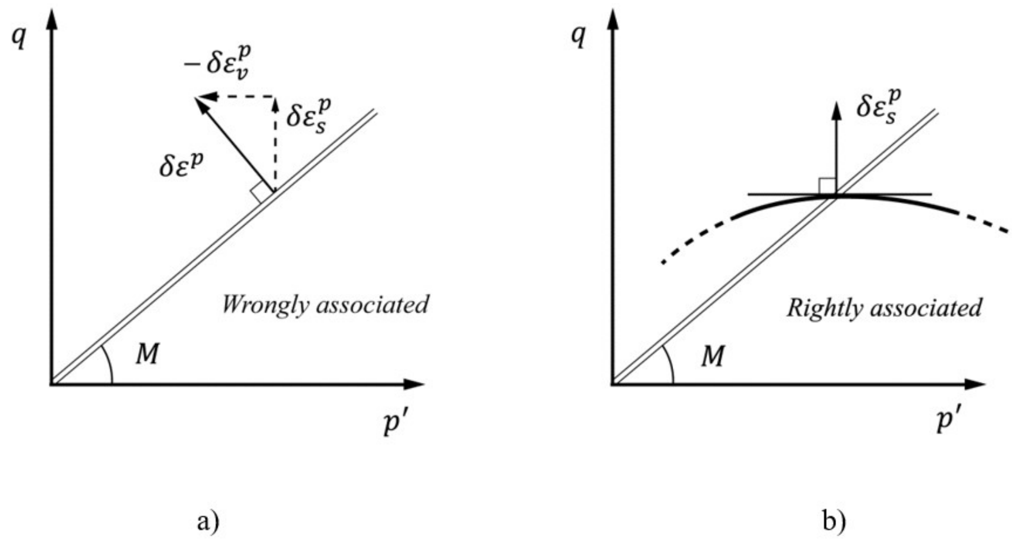

Figure 5. Associated flow for a soil at critical state. (a) wrongly associated; (b) rightly associated.

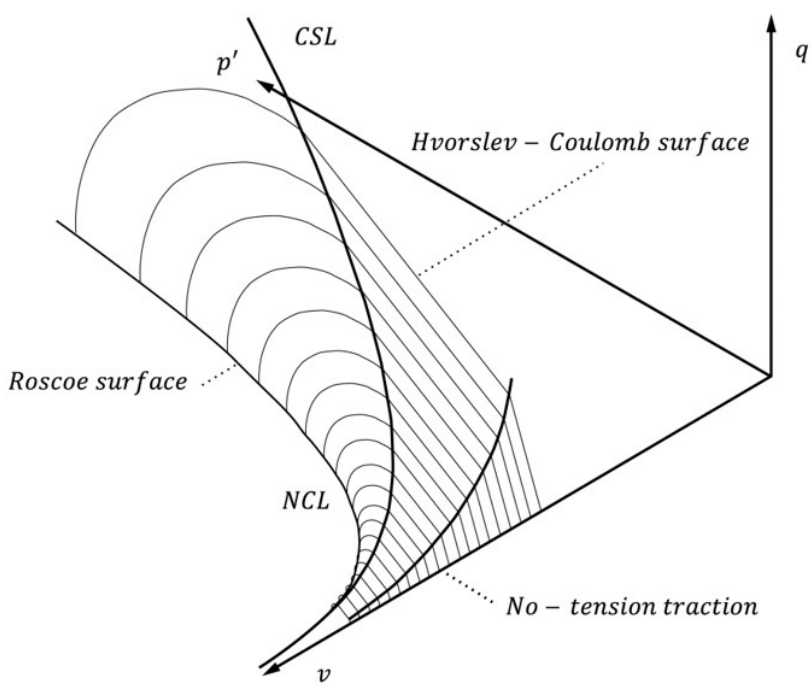

Figure 6. Limiting states surfaces in a (q, p', v) space, according to the Cam-clay model: Roscoe surface, Hvorslev-Coulomb surface and no-tension traction surface (adapted from Wood, 1990).

methods in modern soil mechanics began in the decade 70 (see for instance Naylor & Zienkiewicz, 1972; Naylor, 1975).

After a reexamination of the work of Hvorslev (1937), Roscoe et al. (1958) recognized the huge importance of the incorporation of voids ratio as an essential parameter on critical state theory, with natural reflexes on soil failure criteria. Thirty years later, Schofield & Wroth (1968), in a tribute to the author, named the limit states surface that connects *CSL* and the no-traction failure surface, as Hvorslev-Coulomb surface (see Figure 6).

This surface has the outstanding role of being a frontier between two contrasting disturbed soil behaviours. One, based on the Cam-clay model for isotropic soft soil placed on the *wet* side of the *CSL* during plastic yielding and flow, where the material is considered homogeneous, contractile and exhibits stable yielding (Schofield, 2006). The other disturbed soil behavior (localized on the *dry side*), is characterized by slip planes, indicating dilation and an unstable yielding.

6.3 The failure criteria reexamined

According to the CSM, adhesion, friction and cohesion are the strengths with which soil resists cracks or slip plane failure. It makes use of the failure criterion of Coulomb, which, on the failure plane, is represented by the Equation 6,

$$\left|\tau\right| \le c' + \sigma' \tan\phi' \tag{6}$$

and, graphically, in Figure 7.

To have an idea of the deep marks of Coulomb's failure criterion in soil mechanics, it could be said that, for engineers, the classic soil mechanics is the wide set of design calculations and studies which are based on the Coulomb equation.

Equation 6 shows that c' has a constant value, so, independent of σ'. This parameter, also called "true cohesion" according Terzaghi, would result of the closeness of the mineral particles that interferes in the balance between the attractive forces of Van der Waals and the repelling forces originated in the double layer. As was already stated, this is not a plausible reason in the context of the $CSSM$. Once more, it must be underlined that this failure criterion is only defined in function of the state stress.

In a clear contrast, the $CSSM$ treats soil as a paste continuum. It explains how the strength of an unbonded aggregate of strong and stiff soil grains, depends on effective pressure σ' and on specific volume v. This puts into question the Coulomb resisting slip plane and the Terzaghi concepts of "true cohesion",σ', and of "true friction angle", ϕ'.

According the $CSSM$, the failure criterion, due to Taylor (1962), is radically different. It is based on energy concepts and equates the dilation input for the soil shear strength characterization. It uses the term interlocking to describe this important phenomenon. The Taylor's proposal is described by the Equation 7,

$$q\delta\varepsilon_s + p'\delta\varepsilon_v = \mathrm{M}p'\left|\delta\varepsilon_s\right| \tag{7}$$

According to this criterion, the applied energy is divided between the part stored (left side of Equation 7) and the part dissipated (right side). The dissipated energy depends on a frictional constant, or ϕ', as a fundamental parameter in the theory.

It is important to point out that the volumetric change cannot produce work dissipation. This was intuited by Roscoe et al. (1958), but Thurairajah (1961) showed experimentally that the work absorbed internally is independent of the dilation rate. The occurrence of increments of volumetric strains in Equation 7 indicates that dilation has been taken into account to allow modeling of deformations.

6.3.1 Interlocking versus cohesion

It is evident that the definition of the shear strength in each of the criteria is a fundamental feature of the failure theory. Figure 8 shows the differences between both formulations and helps on the choice of which of them is the more appropriate (Schofield, 2006).

Taking into account these considerations, the peak strength that must be considered is the one suggested by Taylor (i. e., the sum of interlocking and the ultimate critical state drained friction) rather the peak strength recommended by the Coulomb criterion, always supported by Terzaghi (sum of the "true cohesion" and of the "true friction"). The first criterion is a fundamental concept of the $CSSM$ and the second one is inseparable of the CSM.

7. The over-consolidated soil behavior before failure

The interpretation of NC (or lightly OC) and OC soil behavior, is quite different, depending on which of the theories of CSM or $CSSM$, is based on. As was shown before, dilation is null at the critical state, so, the important role of interlocking has to do with the over-consolidated soil behavior before failure, particularly the peak stress states.

7.1 The interpretation according to the classic soil mechanics

As can be seen in Figure 9, the peak stresses are represented through the concept of effective cohesion, c'.

But the failure criterion, Equation 6, is only applicable to the situations where

$$\sigma' \le \sigma'_C \tag{8}$$

where c'_C is the pre-consolidation pressure. Some important limitations of this criterion are referred to in the next paragraph.

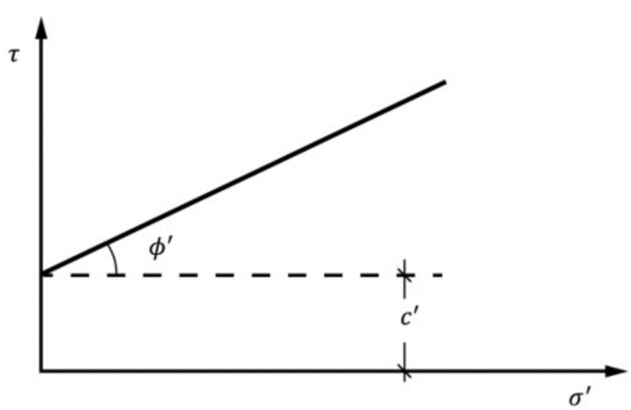

Figure 7. The Coulomb's failure criterion, Equation 6.

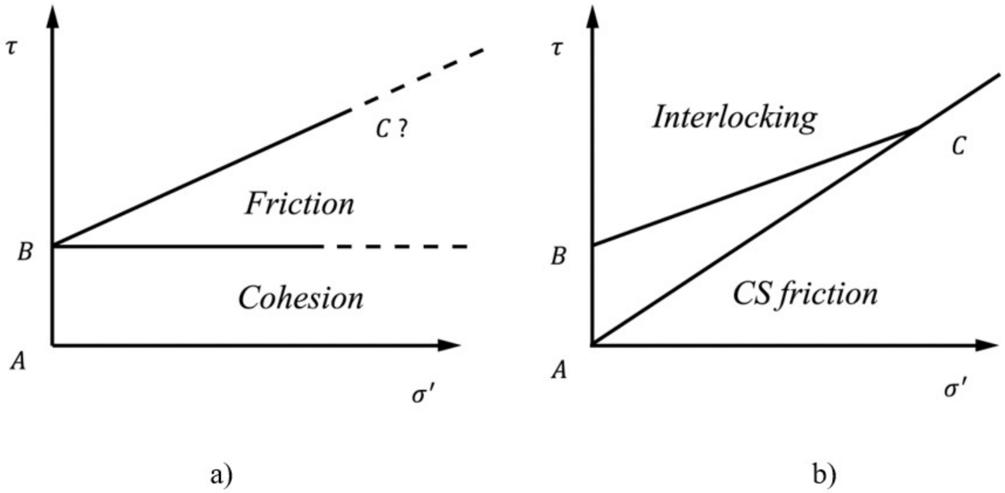

Figure 8. Alternative models for the peak strength of remolded, reconsolidated fine-grain soil: (a) according Coulomb; (b) according Taylor.

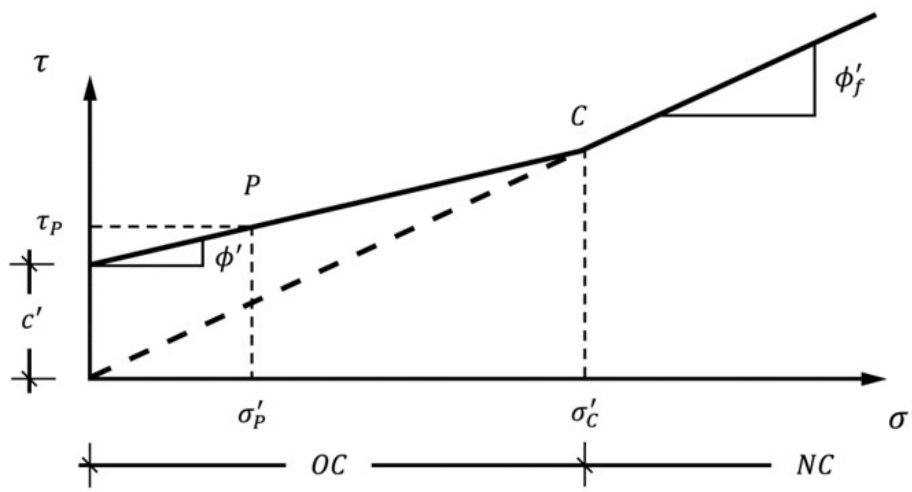

Figure 9. Peak strengths in *OC* soils, based on *CSM* theory. Interpretation in a (τ, σ') plane.

7.1.1 The true cohesion does exist in disturbed soil?

Even before the critical state theory appearance, the experimental work of Hvorslev (1937), reexamined by Schofield & Wroth (1968), showed that cohesion increases exponentially when the specific volume diminishes. This means that the Coulomb's equation, where c' is independent of the normal effective stress, is not verified. The same can be said about Terzaghi's opinion, that not only supported that invariance of σ', but also reinforced this concept by entitling it of true cohesion.

Besides, now in light of the critical state theory, is only obtained for those c' values two to three times lower than the pre-consolidation pressure (σ_C', see Figure 9). This fact was not previously considered. The failure criterion also omits any information about the inexistence of c' at failure (critical state). And, if there were any "true cohesion" on the *dry side* of the CS line, it would also be seen on the *wet side*. Finally, c', for $\sigma' = 0$, was never measured, fact that cannot be ignored. All these reasons confirm that there is no "true cohesion" at all in re-consolidated disturbed soil (Schofield, 2001).

7.2 The interpretation according to the critical state soil mechanics

In Figure 10 is represented, in the planes (τ, σ') and (v, σ'), the failure criterion interpretation, based on the concept of Taylor and on the critical state theory. Obviously, c' doesn't exist.

The state W, located between the NCL and the CSL, when submitted to a shear stress (at σ' constant), displays a decrease of v during its path to W_C (critical state). The W states are NC or lightly OC and, as can be seen, pass by any peak shear stress located on the right of the CSL, (*wet soil*) they have a contractile behavior during the path $\overline{WW_C}$.

The D state, located on the left side of the CSL, (*dry soil*) are OC when submitted to a shear stress. It displays an increase of v during its path to D_c (critical state). The D states are OC and the shear stress evolves to a peak, D', before reaching the critical state, D_c, showing a positive dilation behavior.

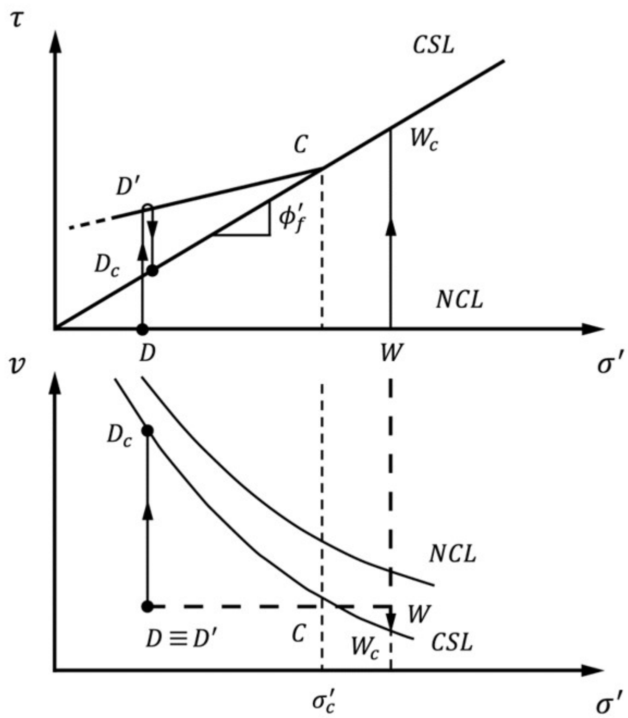

Figure 10. Peak strengths in OC soils, based on $CSSM$ theory. Interpretation in a (τ, σ', v) space.

During the shear path $\overline{DD'}$, the normal stress σ' is constant and has elastic behavior, the soil volume also remains constant. The change of volume occurs only during the path, $\overline{D'D_c}$ with a maximum at D' and zero at D_c.

Summing up, unlike the proposals of the CSM, soils only exhibit positive dilation for OC degrees higher than a certain threshold. This dilation behavior is responsible for the peak stresses that occur before the critical state is attained (the failure). Figure 11 intends to make evident the contribution of the dilation (interlocking) to the peak strength as well as its transient character.

The mentioned weaknesses from the CSM, are due to the use of a failure criterion defined only in a stress space. This inconvenient can be overcome by adding a parameter associated to the volumetric deformation as is the case of the Taylor's failure criterion, Equation 7. Furthermore, it will also allow the characterization of the states before the failure.

Dilation is a concept inseparable of the $CSSM$. As the volume is invariant at the critical state, its performance is mainly related with the behavior of OC soils before failure and specifically with the peak stress states.

This doesn't mean that CSM ignores the dilation phenomenon, but it must be considered in an *ad hoc* way, i. e., it is needed a previous indication of a drained or undrained condition. According to the $CSSM$ theory, the state localization in the (q, p', v) space is enough to quantify its volumetric behavior (or water pressure).

7.3 The OC soil behavior before failure according to the classic and critical state theories

On the application of the deviator stress to an OC soil, elastic and plastic volume changes occur simultaneously.

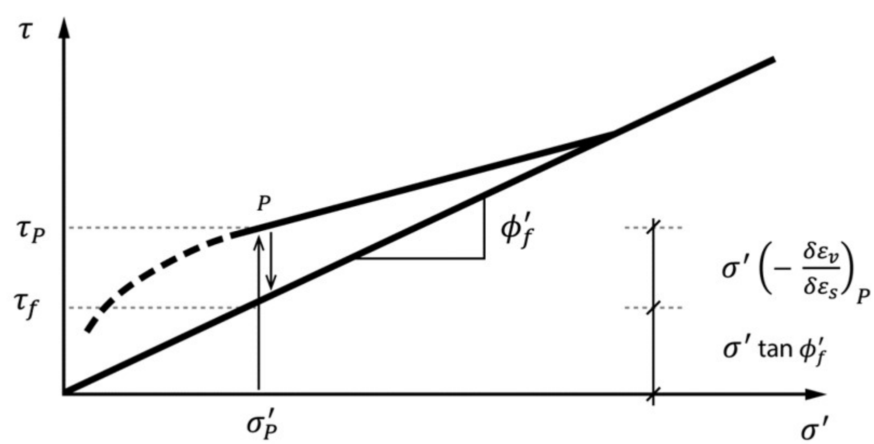

Figure 11. The peak shear strength, τ_P, has two different origins: friction, $\sigma' tan\phi'_f$ and interlocking, $\sigma'\left(-\frac{\delta\varepsilon_v}{\delta\varepsilon_s}\right)_P$.

When being tested, shortly after reaching peak deviator stress, the material deforms on planes or within thin zones, but the correspondent plastic expansions are hardly observed in the test boundary measurements (Parry & Amerasinghe, 1973).

7.3.1 According to the *CSM*

The *NC* soils don't have peak strengths and failure stresses are analyzed on the considered failure plane. Strains before or at failure cannot be obtained.

The *OC* soils exhibit peak strengths (peak values of c' or ϕ', or both) that can fall down to values corresponding to the failure envelope concerning the *NC* soil. Those last failure strength values, due to additional shear strains, can even fall to residual values. The slip plane model, mandatory in *CSM*, doesn't allow the prediction of all the successive forms of a specimen. A body can be divided into separate blocks moving apart from each other, or bulge and flow (Schofield, 2006). The analysis of equilibrium situations is always referred to a plane that, in this case, is a slip surface associated to the peak strength.

7.3.2 According to the *CSSM*

But following the *CSSM*, the *NC* soils are located on the *wet side* of the *CSL*. The material is considered homogeneous, and the soil state can be known before failure. In the state path to the *CSL*, the soil exhibits negative dilation, meaning that the soil plastic behavior is stable. Boundary displacements, originated by the integrated effects through the aggregate, can be observed and measured.

In the case of *OC* soils, the homogeneous character of the soil of the wet zone disappears once the peak strength is attained. Thenceforth, the soil exhibits positive dilation, which diminishes till zero at the *CSL*. In this phase soil plastic yielding behavior is unstable.

Note that a distinction has been made between the peak stress criterion in which the soil body is still considered to be a homogeneous continuum, and the Hvorslev-Coulomb equation for limiting equilibrium between two separate parts of an only just ruptured body. The Hvorslev-Coulomb surface specifies stress components only on the failure plane.

8. The high over-consolidated soils behavior

A soil can exhibit high *OC* behavior when, in an over-compacted state (low specific volume) and a low effective pressure, fails for very high values of the stress obliquity. In reality, under these conditions, the stress obliquity (q/p'), designated by η, can attain values near 3 (in the case of active equilibrium) or near $1,5$ (in the case of passive equilibrium). The soil behavior resulting of the conditions just described is characterized in the following paragraphs.

During the evolution of η from the critical state $(\eta = \mathrm{M})$ to $\eta \approx 3$ (or $\eta \approx 1,5$ for passive equilibrium), the soil begins to develop parallel slip planes, allowing the use of the limit equilibrium design methods, for instance. Note that at these states the global soil mass hydraulic conductivity is not affected.

But for $\eta \gg \mathrm{M}$, (near 3), the soil mass can even exhibit hydraulic fracture, piping, as well as fluidized rubble, phenomena that can happen, for instance, in embankment dams.

When dealing with natural undisturbed ground, which in reality is a soft rock with an aggregate structure (bonding and fabric), flow debris can occur and, at a first sight, this cannot be associated to soil embankments. However, the high *OC* disturbed soils, not only can fail along slip surfaces and exhibit tension cracks, but also break up into blocks of rubble. In this situation, if subjected to a high hydraulic gradient, it will flow as debris in a catastrophic failure.

9. Critical states soil mechanics education: where and how

Before some considerations about this matter it is necessary to make clear that the title of this paper may mislead the reader. In fact, it is not intended to discuss about pedagogy in soil mechanics in its different branches. It's a topic that deserves certainly great interest and there is a lot of information about this subject (Burland, 1989, 2008; Atkinson, 2008; Herl & Gesellmann, 2008, among many others). Nevertheless, some brief words about the soil mechanics education are pertinent.

9.1 The role of modern soil mechanics

There are recently well established theories in the field of soil mechanics that cannot be left to be taught. In many courses, predominantly undergraduate, these subjects are not yet contemplated in the syllabus of the soil mechanics discipline. Or they are simply added, but without explaining the contradictions that such information can originate. It's mandatory to take into account that some basic concepts that characterize modern soil mechanics, contradict in absolute those considered fundaments in classical soil mechanics. How to deal with this situation? The answer was already approached on this paper.

9.2 Critical states soil mechanics education in undergraduate and graduate courses

Another question that can be pointed out is the acceptable differences between syllabus of undergraduate and graduate courses regarding the critical states soil mechanics.

According to Burland (2008), the geotechnical education matters consist into three distinct but interlinked aspects that can be summarized in the following titles: a) the ground profile; b) the observed behavior and c) the use of appropriate models. All these three aspects can be influenced by a fourth one: the judgment based on empiricism and experience, or rather, "well-winnowed" experience (Burland, 1989).

The boundaries between these four aspects often became not clear and one or more of them are frequently neglected.

Regarding the three aspects previously mentioned, the undergraduate courses should mainly focus on the a) and b), while in the graduate courses, a particular attention should be given to c), where the critical states theory is included.

9.3 Differences between what is taught at the universities and what is used in the geotechnical practice

As early as 1983, Atkinson claimed that *CSSM* terminology became the "lingua franca" of soil mechanics. Nevertheless this rapid spreading wasn't followed by an equivalent expansion of the critical states concepts among the geotechnical professionals. The practical application of the critical states theory occurred mostly through the use of some commercial calculation programs embodying those theoretical concepts. Many of these programs employ very attractive models as they are not excessively complex and need only a reduced number of parameters. And, above all, they complain about the ability to determine deformations.

Nevertheless, many users are not yet familiar with the critical states theory taught in soil mechanics courses, making it difficult a correct interpretation of the program results. According Randolph (2005), the lack of comprehension is not due to the complexity of the concepts or the algebra, but rather about the understanding of the underlying message and the gap between the knowledge that many experienced engineers, academics and practitioners, actually have and the misleading language and teaching that permeate much of soil mechanics education.

To "know when one doesn't know", an extremely useful ability normally recognized to the engineers to have (May, 2008), may not be verified for the type of calculation programs that include critical states assumptions. This is one more reason to adopt a particular care when teaching with this kind of software in university courses.

10. Some brief considerations about plastic design of geotechnical structures

The plastic design methods involve the assessment to the strength of structures and are based on the assumption that the used materials have good ductile properties and can tolerate a certain permanent deformation.

These materials allow internal redistribution of structural forces, and if loads are slowly increased, their collapse values are predictable. The small imperfections of fabrication and construction of hyperstatic structures, which alter so markedly the elastic distribution of internal forces, have no effect on ultimate carrying capacity (Heyman, 1996).

As was already largely commented, the soil mechanics and plasticity were always deeply interconnected. But in modern soil mechanics, which incorporates the critical state concept, this link is even more evident. It wasn't by chance that the plastic design denomination, coming from the structural engineering, was also installed in geotechnical design.

Design of geotechnical structures should be based on plastic theory and on approximate methods of analysis by upper and lower bounds. As any other structure it cannot answer disproportionately to a small load increment. Or be at risk of progressive failure if it is not able to dissipate the required energy through the potential failure mechanisms. The *CSSM* concepts can be the guide to satisfy the plastic geotechnical design principles.

The most relevant aspect is the nature of the plastic flow that *NC* or lightly *OC* soils exhibit before failure. Being contractile, the associated volumetric deformations avoid a progressive collapse. This means a desirable stable structural behaviour.

The behaviour of disturbed soil will depend on the effective pressure on the aggregate of soil grains and the specific volume of the aggregate. If it is not over-compacted, it behaves as a ductile plastic material at the critical state effective pressure. But if over-compacted and lightly stressed, it exhibits the brittle behavior already described in 8.

The engineers should bring structural materials into a tough (avoiding fracture) and ductile state as far as possible. A plastic analysis on a critical state basis will emphasize the benefits of ductility in geotechnical structures (Schofield, 2005).

A debate concerning zoned embankment dams took place on the decades 1960-80, concerning the benefits (or not) of ductility in the behavior of these structures. Instead of heavy compaction and a water content lower than the optimum - that strengthens and hardens the soil - the construction using light compaction and water content at the optimum or slightly higher – which favours the soil ductility – were recommended (Maranha das Neves, 1991). The actual plastic design theory came to bring the scientific basis to justify the previous recommendations, mainly based on experience and structural observation.

11. Conclusion

It is unacceptable nowadays that the theory of critical states is completely excluded from soil mechanics syllabus of civil engineering courses. Sometimes some principles of the *CSSM* are attached, trying what can be considered as a simple refreshment of the *CSM*. But one cannot simply add to the *CSM* a few brief notes about the modern soil mechanics. On the contrary, the importance of the introduced topic and the contradictions that arise in relation between a certain fundamental aspects of the classic programme needs a profound clarification. Finally, as this paper also contemplates the soil mechanics education, the unified understanding of the soil behaviour must be considered one of the outstanding consequences of the critical state soil mechanics launching.

Acknowledgements

To David Maranha and André Maranha, who drew up the figures and revised the text, my grateful acknowledgements.

List of symbols

e	void ratio
p'	octahedral effective stress
q	deviator stress
CSL	critical state line
CSM	classic soil mechanics
$CSSM$	critical state soil mechanics
E	Young modulus
F	yield function
G	plastic potential function; stiffness modulus
K	bulk modulus
M	frictional constant
M^*	dilation parameter
N	v value at the normal compression line for $p' = 1kPa$
NCL	normal consolidation line
NC	normally consolidated
OC	overconsolidated
$\delta\varepsilon$	strain increment
ε_s^p	plastic shear strain
ε_v^p	plastic volumetric strain
η	stress obliquity
κ	gradient of unload-reload compression curve on $(v, \ln p')$ plane; swelling index, (C_s)
λ	NCL gradient; compression index,
ν	Poisson ratio
v	specific volume,
$\boldsymbol{\sigma}$	stress tensor
σ'	normal effective stress
$\sigma_C^{'}$	pre-consolidation stress
τ	shear stress
ψ	dilation parameter
Γ	$v$value at the critical state for $p' = 1kPa$

References

Alonso, E.E. (2005). *Las catástrofes y el progreso de la geotecnia: discurso inaugural, abertura del año académico.* España: Real Academia de Ingeniería.

Atkinson, J.H. (1993). *An introduction to the mechanics of soils and foundations*. London: McGraw-Hill.

Atkinson, J. (2008), *What should geotechnical engineers be able to do and how should they acquire these skills*? In I. Manoliu (Ed.), *Education and training in geo-engineering sciences* (pp. 3-8). London: Taylor & Francis Group.

Bolton, M. (1979). *A guide to soil mechanics*. London: Mcmillan Education.

Burland, J.B. (2008), Personal reflections on the teaching soil mechanics. In I. Manoliu (Ed.), *Education and training in geo-engineering sciences* (pp. 35-48). London: Taylor & Francis Group.

Burland, J.B. (1990). On the compressibility and shear strength of natural clays. *Geotechnique*, 40(3), 329-378. http://dx.doi.org/10.1680/geot.1990.40.3.329.

Burland, J.B. (1989), The teaching of soil mechanics: a personal view, Nash Lecture. In *Proceedings of the 9th European Conference of Soil Mechanics and Foundation Engineering* (Vol. 3, pp. 1427-1447), Dublin.

Burland, J.B., Broms, B.B., & Mello, V.F.B. (1977). Behavior of foundations and structures. In *Proceedings of the 9th International Conference on Soil Mechanics and Foundation Engineering* (Vol. 2, pp. 495-546). Tokio: Japanese Society of Soil Mechanics and Foundation Engineering.

Chen, W.F., & Baladi, G.Y. (1985). *Soil plasticity: theory and implementation.* Amsterdam: Elsevier.

Coulomb, C.A. (1773). Essai sur une application des règles de maximis & minimis à quelques problèmes de statique, relatifs à l'architecture. *Mémoires de Mathématique de l'Académie Royale des Sciences*, 7, 343-382.

Davis, R.O., & Selvadurai, A.P.S. (2002). *Plasticity and geomechanics*. Cambridge: Cambridge University Press.

Drucker, D.C., Gibson, R.E., & Henkel, D.J. (1957). Soil mechanics and work-hardening theories of plasticity. *Transactions of the American Society of Civil Engineers*, 122(1), 338-346. http://dx.doi.org/10.1061/TACEAT.0007430.

Drucker, D.C., Greenberg, J.H., & Praguer, H. (1952). Extended limit design theorems for continuous media. *Quarterly of Applied Mathematics*, 9(4), 381-389. http://dx.doi.org/10.1090/qam/45573.

Duncan, J.M. (1996). State of the art: limit equilibrium and finite-element analysis of slopes. *Journal of Geotechnical Engineering*, 122(7), 577-596.

Herl, I., & Gesellmann, S. (2008), Demonstration experiments in geotechnical education. In I. Manoliu (Ed.), *Education and training in geo-engineering sciences* (pp. 379-382). London: Taylor & Francis Group.

Heyman, J. (1996). *Elements of the theory of structures*. Cambridge: Cambridge University Press.

Hvorslev, M.J. (1937). Über die Festigkeitseigenshaften Gestörter Bindiger Böden. Køpenhavn.

James, R.J., Smith, I.A.A., & Bransby, P.L. (1972). The prediction of stresses and deformations in a sand mass adjacent to a retaining wall. In *Proceedings of the 5th European Conference on Soil Mechanics and Foundation Engineering* (Vol. 1, pp. 39-46), Madrid.

Lambe, T.W., & Whitman, R.V. (1979). *Soil mechanics: SI version*. New York: John Wiley & Sons.

May, I.M. (2008), Let's together. In I. Manoliu (Ed.), *Education and training in geo-engineering sciences* (pp. 73-77). London: Taylor & Francis Group.

Maranha das Neves, E. (1991). Static behavior of earth-rockfill dams. In E. Maranha das Neves (Ed.), *Advances* *in rockfill structures* (NATO ASI Series, No. 200, pp. 375-447). Dordrecht: Springer.

Maranha das Neves, E. (2007). Resistência dos solos: dilatância versus coesão efetiva. *Geotecnia*, 109, 5-23.

Maranha das Neves, E., & Veiga Pinto, A. (1988). Modelling colapse on rockfill dams. *Computers and Geotechnics*, 6(2), 131-153. http://dx.doi.org/10.1016/0266-352X(88)90077-8.

Maranha, J., & Maranha das Neves, E (2009). The experimental determination of the angle of dilatancy in soils. In *Proceedings of the 17th ICSMGE* (Vol. 1, pp. 147-154), Alexandria, Egypt.

Naylor, D.J. (1975). *Non-linear finite element models for soils* [Doctoral thesis]. University College of Swansea.

Naylor, D.J., & Zienkiewicz, O.C. (1972). Settlement analysis of a strip footing using a critical state model in conjunction with finite elements. In *Proceedings of the Symposium Interaction of Structure and Foundations*, Birmingham.

Naylor, D.J., Maranha das Neves, E., Mattar, D., & Veiga Pinto, A.A. (1986). Prediction of construction performance of Beliche dam. *Geotechnique*, 36(3), 359-376. http://dx.doi.org/10.1680/geot.1986.36.3.359.

Parry, R.H.G., & Amerasinghe, S.F. (1973). Components of deformation in clays. In *Proceedings of the Symposium on Plasticity and Soil Mechanics* (pp. 108-126), Cambridge.

Poulos, H.G., & Davis, E.H. (1974). *Elastic solutions for soil and rock mechanics*. New York: John Wiley & Sons.

Prandtl, L. (1921). On the penetrating strengths (hardness) of plastic construction materials and the strength of cutting edges. *Zeitschrift für angewandte Mathematik und Physik*, 1(1), 15-20.

Randolph, M.F. (2005), Foreword. In A. Schofield (Ed.), *Disturbed soil properties and geotechnical design* (pp. vi-ix). London: Thomas Telford Publishing.

Rankine, W.J.M. (1857). On the stability of loose earth. *Philosophical Transactions of the Royal Society of London*, 147, 9-27. http://dx.doi.org/10.1098/rstl.1857.0003.

Reynolds, O. (1885). On the dilatancy of media composed of rigid particles in contact: with experimental illustrations. *The London, Edinburg and Dublin Philosophical Magazine and Journal of Science*, 20(127), 469-481.

Roscoe, K.H., Schofield, A.N., & Wroth, C.P. (1958). On the yielding of soils. *Geotechnique*, 8(1), 22-53. http://dx.doi.org/10.1680/geot.1958.8.1.22.

Schofield, A.N. (2001). Re-appraisal of Terzaghi's soil mechanics. In *Proceedings of the 15th ICSMGE* (Special Lecture, pp. 2473-2480), Istambul.

Schofield, A.N. (2005). *Disturbed soil properties and geotechnical design*. London: Thomas Telford Publishing.

Schofield, A.N. (2006). Interlocking, and peak and design strengths. *Geotechnique*, 56(5), 357-358.

Schofield, A., & Wroth, P. (1968). *Critical state soil mechanics*. Berkshire: McGraw-Hill.

Scott, R.F. (1963). *Principles of soil mechanics*. Reading: Addison-Wesley Publishing Company.

Serrano, A.A. (1972). The method of associated fields of stress and velocity and its application to earth pressure problems. In *Proceedings of the 5th European Conference on Soil Mechanics and Foundation Engineering* (Vol. 1, pp. 77-84), Madrid.

Sokolovski, V.V. (1960). *Statics of soil media*. London: Butterworths Scientific Publications.

Taylor, D.W. (1962). *Fundamentals of soil mechanics*. New York: John Wiley & Sons.

Terzaghi, K. (1925). *Erdbaumechanik auf bodenphysicalisher grundlaje*. Leipzig: Deuticke.

Terzaghi, K. (1936). The shearing resistance of saturated soils. In *Proceedings of the 1st International Conference on Soil Mechanics and Foundations Engineering* (Vol. 1, pp 54-56), Cambridge, Massachusetts.

Terzaghi, K. (1943). *Theoretical soil mechanics*. New York: John Wiley & Sons.

Thurairajah, A.H. (1961). *Some properties of kaolin and of sand* [Doctoral thesis]. University of Cambridge.

Wood, D.M. (1990). *Soil behavior and critical state soil mechanics*. Cambridge: Cambridge University Press.

Wood, D.M. (2004). *Geotechnical modeling*. London: Spon Press.

Wood, D.M. (2012). Soils: virtues and vices: spatial awareness. In *Proceedings of the 7th National Conference of the Portuguese Geotechnical Society* (Invited Lecture). Lisbon: LNEC.

Wroth, C.P. (1973). A brief review of the applications of plasticity to soil mechanics. In *Proceedings of the Symposium on Plasticity and Soil Mechanics* (pp. 1-11), Cambridge.