



## Interpretation of bi-directional tests on piles with the evaluation of stress relief at the pile toe

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Technical Note

### Keywords

Bi-directional test  
Expansive cell  
Equivalent top-down curve  
Elastic shortening  
Stress relief

### Abstract

This paper presents the interpretation of bi-directional load tests performed on three auger piles, in the city of São Paulo, Brazil, using a method based on transfer functions for the shaft and toe. Elastic shortenings of the shaft were directly measured through a displacement indicator at the pile top and two telltales at the upper and bottom plates of the expansive cell. The equivalent top-down load-settlement curves were estimated and compared with two other methods from the literature, one which considers the pile infinitely rigid; and the other, which takes the pile elastic shortening into account. The curves resulted in good agreement considering the pile compressibility. Yet for the infinitely rigid pile, the settlements resulted in up to 75% smaller. Furthermore, the influence of stress relief on the toe behavior due to shaft lifting was investigated. For the cases studied, involving bored and auger piles with the slenderness ratio ( $L_s/r$ ) greater than 20, the percentage of this effect was generally small, up to 5% of the toe load, being negligible for practical uses.

## 1. Introduction

To perform the pile bi-directional load test, one or more expansive cells (or O-cells) are usually installed near the pile toe. They are hydraulically expanded, pushing the shaft upward and the toe downward. Load-displacement curves are obtained for the pile shaft and toe separately.

The resulting force in the shaft corresponds to the load applied by the expansive cell minus the buoyant weight of the pile shaft (Fellenius, 2021). At the pile segment below the cell, taken as a “fictitious” toe, acts the force applied by the cell plus the pore pressure at the cell level.

This paper presents a modification of the method described in Dada & Massad (2018b), based on the model of Coyle & Reese (1966), which can be used to estimate the equivalent top-down load-settlement curve, simulating a conventional static load test.

A practical application of the method is made on continuous flight auger (CFA) piles installed in São Paulo City, Brazil. Displacement measurements were made at the pile top, by a displacement indicator, and at the upper and bottom cell plates, by means of displacement gauges and two telltales.

In addition, the possible influence of the stress relief on the toe behavior, due to the shaft lifting, was evaluated.

## 2. Methods of interpretation

To obtain the equivalent top-down load-settlement curve, a modified version of the method based on the model of Coyle & Reese (1966) will be used. Two other methods will also be applied for comparisons, namely: a) the Elísio-Osterberg’s method (Silva, 1986; Osterberg, 1998), which considers the pile infinitely rigid; and b) the method of Massad (2015), which contemplates pile elastic shortening.

Coyle & Reese (1966) developed a model to predict the load-settlement curve of a pile axially loaded at the top, based on known load transfer functions for the shaft and the toe. The pile is divided into  $n$  elements and the soil is replaced by independent springs that interact with the pile in the centers of each element.

For the bi-directional test, a hyperbolic (Chin, 1970) or an elastoplastic (Cambefort, 1964) relation is fitted to the load-displacement curve, measured at the bottom cell plate, and is used as the load transfer function of the “fictitious toe”. Figure 1 illustrates the use of a hyperbolic relation.

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For the shaft, first, a hyperbola is fitted to the test curve measured at the pile top, as shown in Figure 1. Then, the hyperbola is translated to the center of compression, i.e., to the level at which half of the total shaft elastic shortening occurs. The soil surrounding the shaft pile is assumed to consist of an equivalent layer of homogeneous soil; the subsoil heterogeneity is incorporated by means of the coefficient  $c$  of Leonards & Lovell (1979).

In effect, to obtain the translated hyperbola simulating a pile loaded on top, the upward movement measured at the pile top,  $y'_p$ , should be increased by half of the elastic shortening for top-down loads, as given in Equation 1.

$$y_f = y'_p + \frac{1}{2} \left( \frac{cA_l}{K_r} + \frac{Q'_p}{K_r} \right) \tag{1}$$

where  $y_f$  is the shaft displacement at the center of compression.  $A_l$  and  $Q'_p$  are respectively the shaft and the toe load for the same displacement  $y'_p$ , as shown in Figure 1. For  $K_r$  and  $c$ , see the list of symbols.

The translated hyperbola  $A_l = f(y_f)$  can be used as the load transfer function of the shaft. It corresponds to the modification of the method originally proposed by Dada & Massad (2018b).

Finally, the equivalent top-down load-settlement curve can be obtained using the model of Coyle & Reese (1966), with the load transfer functions described above.

### 3. Case studies

Six continuous flight auger (CFA) piles, installed in São Paulo, Brazil, were submitted to bi-directional tests. Table 1 presents the general data and some parameters related to the shaft. The typical subsoil profile is shown in Figure 2.

The bi-directional test results for the CFA Pile PCE06 are presented in Figure 1 as an illustration. Note that the displacements were measured at three levels. The difference between the measurements at the cell top and the pile top gives the shaft elastic shortening  $\Delta e$ , which varies with  $A_l$ .

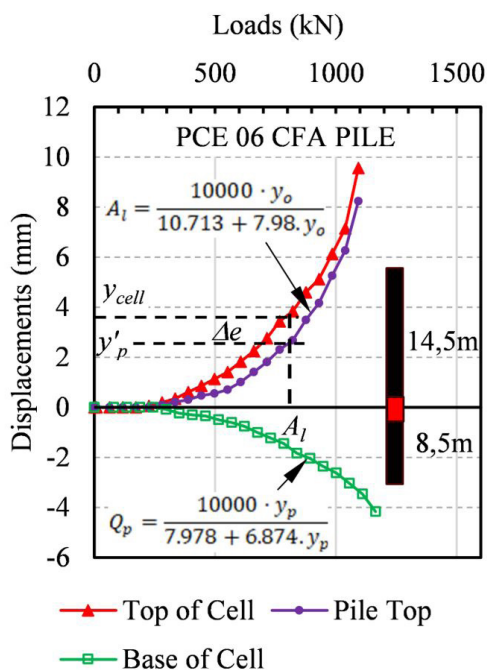


Figure 1. Example of bi-directional test results (see the list of symbols).

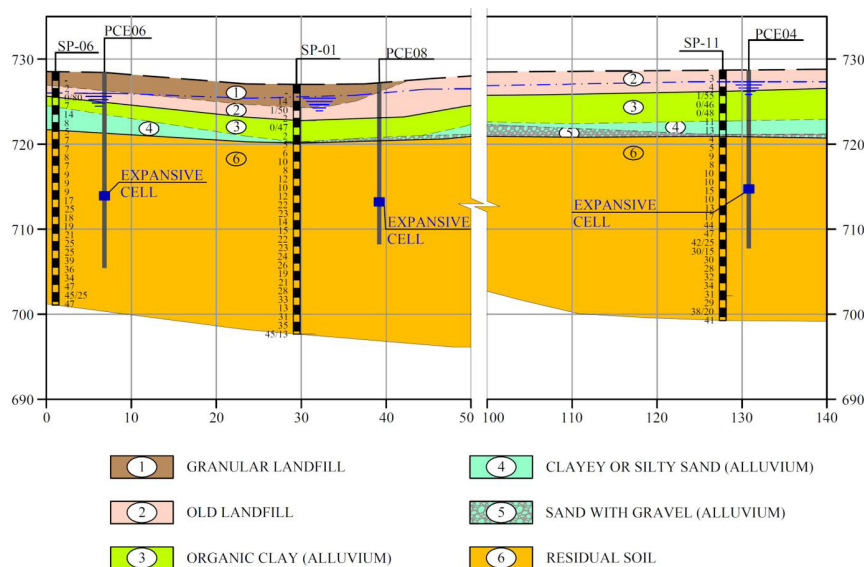


Figure 2. CFA piles: subsoil profile inferred from SPT tests near piles PCE04, PCE06 and PCE08. A similar profile was observed for the entire workplace.

**Table 1.** Bi-directional tests on 6 CFA piles - general data (adapted from Dada et al., 2019; Dada, 2019).

Pile	$D$	$L_s$	$L_{toe}$	Pile shaft parameters		
	(m)	(m)	(m)	$K_r$ (kN/mm)	$c_{eq}$	$c'_{eq} = I - c_{eq}$
PCE03	0.5	14	7	393	0.74	0.26
PCE04	0.5	14	7	393	0.7	0.3
PCE05	0.5	14.7	7.3	374	0.74	0.26
PCE06	0.5	14.5	8.5	379	0.53	0.47
PCE07	0.5	16	7	344	0.72	0.28
PCE08	0.4	14	5	251	0.72	0.28

See list of symbols.

For each pile, Massad's (2015) coefficients  $c'$  were estimated with Equation 2.

$$\Delta e = \frac{c' A_l}{K_r} \quad (2)$$

The average values ( $c'_{eq}$ ) are indicated in Table 1.

To simulate the download conventional test by the Method of Massad (2015),  $y_p$  related to  $A_l$  is settled equals to the toe movement  $y'_p$ , associated with  $Q'_p$ , as indicated in Figure 1 for CFA Pile PCE06. A pair  $y_o - P_o$  of the equivalent curve is determined by Equations 3 and 4.

$$y_o = y'_p + \Delta e \frac{c}{c'} + \frac{Q'_p}{K_r} \quad (3)$$

$$P_o = A_l + Q'_p \quad (4)$$

As far as the method based on Coyle & Reese's model is concerned, the application of Equation 1 to the results given in Figure 1 leads to the translated hyperbola of Equation 5, which was used as the load transfer function of the shaft for CFA Pile PCE06. For its toe, the hyperbolic transfer function is shown in Figure 1.

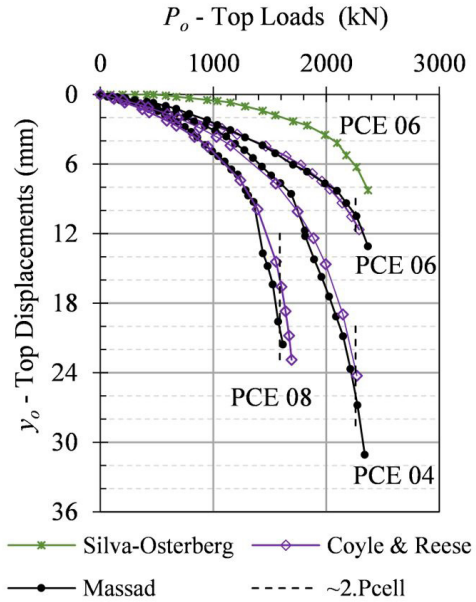
$$A_l = \frac{10000 y_f}{23.348 + 6.995 y_f} \quad (5)$$

The equivalent top-down curves, given by these two methods, are shown in Figure 3 for three CFA Piles of Table 1, revealing good convergence when compared to each other.

The application of the Elísio-Osterberg method (Silva, 1986; Osterberg, 1998), which assumes the pile as infinitely rigid, resulted in settlements up to 75% smaller, as shown in Figure 3 for the PCE06.

#### 4. Evaluation of stress relief

Next, the influence of stress relief on the toe behavior due to shaft lifting during the bi-directional test (up-top loads) was evaluated.



**Figure 3.** Equivalent top-down curves - CFA piles: PCE04, PCE06 and PCE08.

#### 4.1 Loading at the pile top (top-down loads)

For loads applied at the pile top, Martins (1945) and Geddes (1966) developed elastic solutions to obtain the load increase at the pile toe, due to shaft load ( $\Delta Q_{p,f}$ ), by integrating Mindlin's (1936) influence factors. Vargas (1978) adopted Martins's (1945) solutions, which assumed uniform skin friction ( $f$ ) and Poisson's ratio  $\nu = 0.5$  for the soil. Poulos & Davis (1974) suggested the use of Geddes' (1966) solutions, which in turn considered a linear variation of  $f$  and  $\nu = 0.3$ .

Vargas (1978) proposed the following equation, rewritten for this paper:

$$\frac{\Delta Q_{p,f}}{Q_p} = K_{zz} \pi \alpha \left( \frac{1}{L_s/r} \right)^2 \quad (6)$$

where  $\alpha = A_l/Q_p$ ;  $L_s$  is the pile shaft length;  $r$  is the pile radius and, therefore,  $L_s/r$  is the slenderness ratio.

The term  $K_{zz}$  is an influence factor at a depth  $1.05 \cdot L_s$ , proposed by Vargas (1978), and is equal to 4.73 or 6.70, according to Martins (1945) or Geddes (1966) solutions, respectively. Vargas concluded that the  $\Delta Q_{p,f}$  is usually small and may be disregarded. Randolph & Wroth (1978) made a similar statement: the stress changes at the pile toe would be uncoupled from the shaft load, adding the condition  $L_s/r \geq 20$ , that is, slenderness ratio not less than 20.

#### 4.2 Bi-directional tests (up-top loads)

Analogous analyses were made for the bi-directional tests performed on the CFA piles plus 3 others, as indicated in

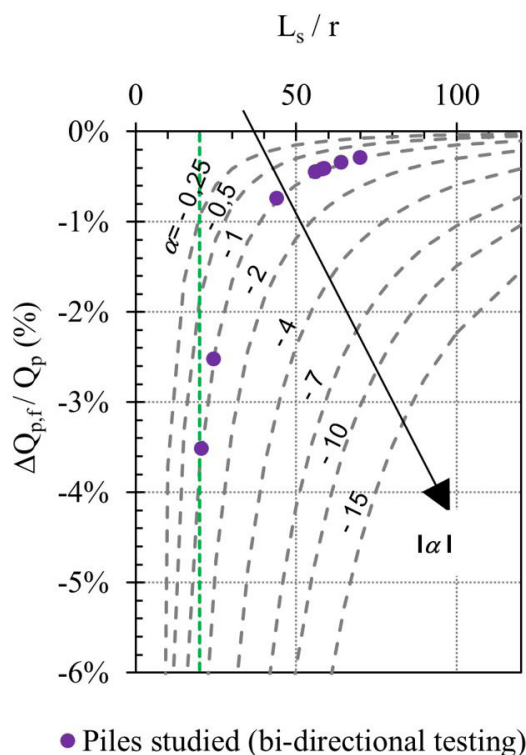
Table 2, together with some parameters (see list of symbols). Note that the shaft loads take a negative signal in the elastic analysis since they are upward loads.

The load relief ratios were estimated using Equation 6. Figure 4 presents the results for Martins (1945) solutions,

**Table 2.** Case studies -  $L_s/r$  ratio, maximum loads reached in bi-directional tests and  $\alpha$  parameter (adapted from Dada, 2019).

Pile type	$L_s/r$	$A_{l,max}^{(1)}$ (kN)	$Q_{p,max}^{(1)}$ (kN)	$\alpha^{(3)}$	Data source
Root (E-B3)	44	-1218	1264	-1	Dada & Massad (2018a)
Omega (PC-02)	24	-931 <sup>(2)</sup>	931 <sup>(2)</sup>	-1	Fellenius (2014)
Omega (PC-07)	21	-761 <sup>(2)</sup>	761 <sup>(2)</sup>	-1	Fellenius (2014)
CFA (PCE03)	56	-1095	1163	-0.9	Dada et al., (2019)
CFA (PCE04)	56	-1095	1164	-0.9	
CFA (PCE05)	59	-1187	1260	-0.9	
CFA (PCE06)	58	-1093	1165	-0.9	
CFA (PCE07)	64	-1087	1165	-0.9	
CFA (PCE08)	70	-772	816	-0.9	

<sup>(1)</sup>Sign convention: upward load negative; downward load positive; <sup>(2)</sup>Data as presented by Fellenius (2014); <sup>(3)</sup> $\alpha \cong 1$ , because the expansive cell applies almost the same load to shaft and toe, due to the above-mentioned corrections.  $A_{l,max}$  and  $Q_{p,max}$  are compression loads.



**Figure 4.** Load relief ratio ( $\Delta Q_{p,f}/Q_p$ ), estimated with Martins's (1945) solution. The studied piles, subjected to bi-directional tests, are indicated with circles.

highlighting the ratio  $L_s/r = 20$  and  $\alpha = 1$ . The concept of "fictitious toe" was considered in the analysis.

From Figure 4, when  $L_s/r = 20$ , the ratio  $\Delta Q_{p,f}/Q_p$  assumes a value of 3.7%. For Geddes's (1966) solution, this ratio is 5.2% (Dada, 2019). About 75% of the studied piles had  $L_s/r \geq 40$ ; hence, in these cases, the load relief percentages resulted in a maximum of 1%.

## 5. Conclusions

The method for the interpretation of bi-directional test results presented herein, based on the model of Coyle & Reese (1966), lead to equivalent top-down curves with good agreement with the method of Massad (2015), which considers pile elastic shortening. The application of the Elísio-Osterberg Method (Silva, 1986; Osterberg, 1998), which assumes the pile as infinitely rigid, resulted in settlements up to 75% smaller, as was the case of CFA Pile PCE06.

Finally, load reliefs at the pile toe, due to shaft lifting, were estimated for the CFA piles plus 3 others from the literature. The load relief ratios ( $\Delta Q_{p,f}/Q_p$ ) resulted in less than 1% for 75% of the piles, and up to 5% for all of them. These values are not significant for practical purposes and could be neglected.

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## Declaration of interest

The authors declare that there are no known conflicts of interest associated with this publication.

## Authors' contributions

Thais Lucovicz Dada: conceptualization, data curation, methodology, writing – original draft, writing – review and editing. Façal Massad: conceptualization, data curation, methodology, supervision, validation, writing – review and editing.

## List of symbols

$A_l$	Total lateral (shaft) load.
$A_{l,max}$	Maximum lateral (shaft) load reached in the bi-directional test.
$c$	Leonards & Lovell (1979) coefficient.
$c_{eq}$	Value of $c$ related to the average of elastic shortening measurements.
$c'$	Correlate of $c$ for bi-directional tests (Massad, 2015).
$c'_{eq}$	Value of $c'$ related to the average of elastic shortening measurements.

$D$	Pile diameter.
$F$	Unit skin friction.
$K_r$	Pile stiffness, as a structural piece.
$K_{zz}$	Influence factor of the shaft load at the pile toe.
$L_s$	Pile shaft length embedded in soil up to the toe (real or “fictitious”) level.
$L_{toe}$	Length of pile “fictitious toe”.
$n$	Number of pile subdivision elements for iterative calculation.
$P_{cell}$	Load applied by the expansive cell.
$P_o$	Axial load at the pile head.
$Q_p$	Total toe load (real toe or “fictitious” toe).
$Q'_p$	Total toe load of the bi-directional test (“fictitious toe”), related to $y'_p$ .
$Q_{p,max}$	Maximum toe load reached in the bi-directional test (“fictitious toe”).
$r$	Pile radius.
$y_o$	Displacement of the pile at the head (pile top).
$y_f$	Displacement at the center of compression of the pile shaft.
$y_{cell}$	Upward displacement at the expansive cell upper plate.
$y'_p$	Upward displacement of the pile head (bi-directional test) = downward displacement of the pile toe (downward test).
$\alpha$	Ratio of $A_l$ to $Q_p$ .
$\Delta e$	Pile elastic shortening.
$\Delta Q_{p,f}$	Load increase or decrease at the pile toe (real toe or “fictitious toe”).
$\nu$	Poisson’s ratio of subsoil.

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