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Subspace Identification for Industrial Processes

S.D.M. BORJAS[1], DCEN, Universidade Federal Rural do Semi-árido, UFERSA, Campus Mossoró, Av. Francisco Mota 570, Bairro Costa e Silva, 59625-900 Mossoró, RN, Brasil.

C. GARCIA[2], Escola Politécnica da Universidade de São Paulo - EPUSP, Av. Prof. Luciano Gualberto, trav. 3, nr. 158 - Butantã, São Paulo, SP, Brasil.

Abstract. Subspace identification has been a topic of research along the last years. Methods as MOESP and N4SID are well known and they use the LQ decomposition of certain matrices of input and output data. Based on these methods, it is introduced the MON4SID method, which uses the techniques MOESP and N4SID.

Palavras-chave. Subspace identification, state space models.

1. Introduction

Nowadays, an engineer's work consists more and more of obtaining mathematical models of the studied processes [15]. A major part of the literature referring to system identification deals with how to find polynomial models as Prediction Error Methods (PEM) and Instrumental Variable Methods (IVM). In case of complex systems, the state space model appears as an alternative to PEM and IVM models [22]. For multivariable systems, these methods provide reliable state space models directly from input and output data. As systems of large dimensions are usually found in industry, the application of subspace identification algorithms in this field is very promising [3, 8, 10, 14, 16, 17, 24]. Each subspace identification method is different from the other ones in concept, interpretation and computational implementation. An identification generalization is presented in [18]. In [9] a comparative study among the three most commonly used algorithms, i.e., Canonical Variate Analysis (CVA) [13], Multivariable output-Error State sPace (MOESP) [21] and Numerical algorithms for Subspace State Space System IDentification (N4SID) [19] was made. For further details about these algorithms, the reader can consult [2, 6, 7, 12, 17, 21, 22, 23, 4].

In this work it is shown, in a summarized form, the operation of POMOESP [20] and N4SID [19] methods. Combining these methods, it is obtained the MON4SID algorithm, which estimates the extended observability matrix in the same way that POMOESP method; the state sequence is computed through the oblique projection as is done in N4SID method; from this sequence it is obtained the past and future

[1]santos.borjas@ufersa.edu.br
[2]clgarcia@lac.usp.br

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states and finally a consistent estimate of the system matrices is obtained applying the least squares method.

Other form for integrating these methods can be found in [11], in which the author aimed at reducing the computational complexity, dividing MOESP and N4SID algorithms in modules, separating them in independent parts. Afterwards, he used the computationally fastest modules of each algorithm.

1.1. Subspace identification

Linear time invariant systems models, operating in discrete time, are dealt with in the dynamic subspace identification methods. These systems can be described by models in the innovation form

$$x(k+1) = Ax(k) + Bu(k) + Ke(k) \tag{1.1}$$

$$y(k) = Cx(t) + Du(k) + e(k), \tag{1.2}$$

where the symbols represent the input $u(k) \in \Re^m$, the output $y(k) \in \Re^l$, the state $x(k) \in \Re^n$ and the Kalman filter gain K. $e(k) \in \Re^l$, is zero-mean Gaussian white noise and independent of past input and output data. A, B, C, and D are matrices with appropriate dimensions.

1.2. Subspace identification problem

The subspace identification problem: given $u(k)$ and $y(k)$ a set of input-output measurements aims to, determine the order n of the unknown system, the system matrices (A, B, C, D) up to within a similarity transformation and Kalman filter gain K [19].

1.3. Subspace matrix equation

Performing successive iterations in equation (1.1) and (1.2), one can derive the following matrix equation

$$Y_f = \Gamma_i X_f + H_i^d U_f + H_i^s E_f, \tag{1.3}$$

where subscript f stands for the "future" and p for the "past". The matrices H_i^d and H_i^s are defined as

$$H_i^d = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{i-2}B & CA^{i-3}B & \cdots & D \end{bmatrix}, \tag{1.4}$$

$$H_i^s = \begin{bmatrix} I & 0 & \cdots & 0 \\ CK & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{i-2}K & CA^{i-3}K & \cdots & K \end{bmatrix}. \tag{1.5}$$

The past and future input block-Hankel matrices are defined as

$$U_p = \begin{bmatrix} u_0 & u_1 & \cdots & u_{j-1} \\ u_1 & u_2 & \cdots & u_j \\ \vdots & \vdots & \ddots & \vdots \\ u_{i-1} & u_i & \cdots & u_{i+j-2} \end{bmatrix}, \tag{1.6}$$

where U_p, and $U_f \in \Re^{mixN}$. The output and noise innovation block-Hankel matrices Y_p, e $Y_f \in \Re^{lixN}$ and E_p, and $E_f \in \Re^{mixN}$, respectively, are defined in a similar way to (1.4). The states are defined as

$$X_p \quad = \quad X_0 = \begin{bmatrix} x_0, & \cdots & x_{j-1} \end{bmatrix}, \tag{1.7}$$

$$X_f \quad = \quad X_i = \begin{bmatrix} x_i, & \cdots & x_{i+j-1} \end{bmatrix}. \tag{1.8}$$

The extended observability matrix Γ_i is given by

$$\Gamma_i = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{i-1} \end{bmatrix}. \tag{1.9}$$

1.4. Orthogonal projection

The orthogonal projection of the row space of A_x into the row space of B_x is, [19],

$$A_x/B_x \quad = \quad A_x B_x (B_x B_x^t)^\dagger B_x, \tag{1.10}$$

where $(.)\dagger$ denotes the Moore-Penrose pseudo-inverse of the matrix $(.)$. The projection of the row space of A_x into the orthogonal complement of the row space of B_x is, [19],

$$A_x/B_x^\perp \quad = \quad A_x - A_x/B_x. \tag{1.11}$$

1.5. Oblique projection

The oblique projection of the row space of G along the row space H into the row space of J is, [19],

$$G/_H J \quad = \quad [G/H^\perp].[J/H^\perp]^\dagger.J\,. \tag{1.12}$$

Properties of the orthogonal and oblique projections

$$A_x/A_x^\perp \quad = \quad 0, \tag{1.13}$$

$$A_x/_{A_x} C_x \quad = \quad 0. \tag{1.14}$$

For a proof, see [19].

2. Identification Methods

2.1. MOESP identification Method

Inside the MOESP family, there is the POMOESP method, which solves the problem in section 1.3, by means of an approximation of the extended observability matrix Γ_i. Therefore, it is necessary to eliminate the last two terms in the right hand side of equation (1.3). That is done in two steps:

First, eliminating the term $H_i^d U_f$ in (1.3), performing an orthogonal projection of equation (1.3) into the row space of $U_f^\perp$ yields

$$Y_f/U_f^\perp = \Gamma_i X_f/U_f^\perp + H_i^d U_f/U_f^\perp + H_i^s E_f/U_f^\perp . \tag{2.1}$$

By the property (1.11) it results in $U_f/U_f^\perp = 0$. Equation (2.1) can be simplified to

$$Y_f/U_f^\perp = \Gamma_i X_f/U_f^\perp + H_i^s E_f/U_f^\perp . \tag{2.2}$$

Second, to eliminate the noises in (2.2), it is defined an instrumental variable $Z = [U_p^t Y_p^t]^t$. Multiplication of (2.2) by Z yields

$$Y_f/U_f^\perp Z = \Gamma_i X_f/U_f^\perp Z + H_i^s E_f/U_f^\perp Z . \tag{2.3}$$

As it is assumed that the noise is uncorrelated with input and output past data [20], that means that $E_f/U_f^\perp Z = 0$.Therefore, (2.3) is written as

$$Y_f/U_f^\perp Z = \Gamma_i \hat{X}_f . \tag{2.4}$$

In equation (2.4) $X_f/U_f^\perp Z = \hat{X}_f$ is the estimate of the Kalman filter state. Equation (2.4) indicates that the column space of Γ_i can be calculated by the SVD decomposition of $Y_f/U_f^\perp Z$. For further details, see [19].

2.2. N4SID identification method

N4SID method solves the problem in section 1.3 by means of an approximation of past and future Kalman filter state sequence. From the Theorem 12 in [19],

$$\tilde{X}_i = \Gamma_i^\dagger \Theta_i, \tag{2.5}$$

where $\Theta_i = Y_f/_{U_f} W_p$ which is achieved by performing an oblique projection of equation (1.3), along the row space U_f onto the row space of W_p, that is

$$Y_f/_{U_f} W_p = \Gamma_i X_f/_{U_f} W_p + H_i^d U_f/_{U_f} W_p + H_i^s E_f/_{U_f} W_p . \tag{2.6}$$

It is easy to see that the last two terms of equation (2.6) are null, $U_f/_{U_f} W_p = 0$ by the property of the oblique projection, equation (1.12); $E_f/_{U_f} W_p = 0$ by the assumption that the noise is uncorrelated with input and output past data [19]. Thus, equation (2.6) can be simplified to

$$Y_f/_{U_f} W_p = \Gamma_i \tilde{X}_i, \tag{2.7}$$

where $\tilde{X}_i = X_f/_{U_f} W_p$ and $W_p = [U_p^t Y_p^t]^t$. Then equation (2.7) is written as

$$\Theta = \Gamma_i \tilde{X}_i. \tag{2.8}$$

Equation (2.8) indicates that the column space of Γ_i can be calculated by the SVD decomposition of Θ and Γ_i can be calculated as, [19],

$$\Gamma_i = U_1 S_1^{1/2}. \tag{2.9}$$

Once are known Θ and Γ_i, it is easy to compute $\tilde{X}_i$ (calculated from (1.6)). Now $\tilde{X}_{i+1}$ can be calculated as, [19],

$$\tilde{X}_i = \Gamma_{i-1}^{\dagger} \Theta_{i+1}, \tag{2.10}$$

where $\Theta_{i+1} = Y_f^-/_{U_f^-} W_p^+$ and Γ_{i-1} denotes the matrix Γ_i without the last l rows. For further details, see [19] or [6].

2.3. MON4SID identification method

In this subsection, it is presented the MON4SID method. To solve the problem in section 1.3, it is used the POMOESP method to calculate the extended observability matrix Γ_i and the N4SID method is employed to calculate the matrices (A, B, C, D) through the least squares method.

Γ_i, given in (2.4), can be derived from a simple LQ factorization of a matrix constructed from the block-Hankel matrices U_f, U_p and Y_f , Y_p, in the form

$$\begin{bmatrix} U_f \\ W_p \\ Y_f \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}, \tag{2.11}$$

and the orthogonal projection in the left side of (2.4) can be computed by matrix L_{32} [20]. The SVD of L_{32} can be given as

$$L_{32} = \begin{bmatrix} U_1 & U_2 \end{bmatrix} = \begin{bmatrix} S_n & 0 \\ 0 & S_2 \end{bmatrix} \begin{bmatrix} V_1^t \\ V_2^t \end{bmatrix} = USV^t. \tag{2.12}$$

The order n of the system is equal to the number of non-zero singular values in S. The column space of U_1 approximates that of Γ_i in a consistent way [20], that is

$$\Gamma_i = U_1. \tag{2.13}$$

The system (1.1)-(1.2) can be written as

$$\begin{bmatrix} \tilde{X}_{i+1} \\ Y_{i|i} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \tilde{X}_i \\ U_{i|i} \end{bmatrix} + \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}. \tag{2.14}$$

In equation (2.14), suppose (ideally) that $\tilde{X}_{i+1}$ and $\tilde{X}_i$ are given, then the system matrices (A, B, C, D) could be computed through the least squares method.

Therefore, the problem now is to find the state sequences. The oblique projection Θ_i given in equation (2.8) can be computed from (2.11) by

$$\Theta_i = Y_f/_{U_f} Wp = L_{32}[L_{32}]^{-1} \begin{bmatrix} L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}. \tag{2.15}$$

An estimate of the state sequence X is given by

$$\Gamma_i = \Gamma_i^{\dagger} L_{32}[L_{32}]^{-1} W_p. \tag{2.16}$$

The following matrices are defined: $\tilde{X}_i = X(:, 1 : N-1)$, $\tilde{X}_{i+1} = X(:, 2 : N)$. Thus, the system matrices can be estimated from equation (2.14). The Kalman gain K can be estimated from [19] or [20]

$$K = [G - APC^T][\Lambda_0 - CPC^T]^{-1}, \tag{2.17}$$

where P is the forward state covariance matrix, which can be determined as solution of the forward Riccati equation

$$P = APA^T + [G - APC^T][\Lambda_0 - CPC^T]^{-1}[G - APC^T]^T. \tag{2.18}$$

We have then the following algorithm

Algorithm MON4SID

1) Compute the matrices U_f, U_p and Y_f, Y_p and the LQ factorization given in (2.11).

2) Compute the SVD of the matrix L_{32} from (2.11).

3) Determine the system order by inspection of the singular values in S given in (2.12).

4) Determine Γ_i from equation (2.13) and the state sequence X from (2.16), determine X_{i+1} and X_i. 5) Compute the matrices A, B, C, and D from equation (2.14).

3. Simulation

In this section it is employed the Shell benchmark [5], [25] to evaluate the performance of the MON4SID algorithm and to compare it with other existing identification algorithms (PEM, MOESP and N4SID). The Shell benchmark is a model of a two-input two-output distillation column. The inputs are overhead vapor flow (D) and reboiler duty (Q); the outputs are column pressure (P) and product impurity (X). The model of the process is given as follows

$$P(t) = \frac{-0.6096 + 0.4022q^{-1}}{1 - 1.5298q^{-1} + 0.5740q^{-2}} D(t) + \frac{-0.1055 + -0.0918q^{-1}}{1 - 1.5298q^{-1} + 0.5740q^{-2}} Q(t) + \\ + \frac{Ns}{1 - 1.5945q^{-1} + 0.5945q^{-2}} e_p(t), \tag{3.1}$$

$e_p(t)$ is the white noise which generates the disturbance in the pressure. The parameter Ns is used to set the noise level in the simulation. For this study three values are used: 0.2, 0.5 and 1 and they are called 20 % , 50 % and 100 % noise level simulation. The standard deviation of the noise is 1.231. The model of the impurity (X) is slightly nonlinear. It is given by

$$X(t) = 0.0765\frac{500000}{Q(t-7)-1500} + 0.9235X(t-1) + \\ + \frac{Ns}{1-1.6595q^{-1}+0.6595q^{-2}}e_x(t), \tag{3.2}$$

where $e_x(t)$ is white noise with standard deviation 0.677. The Shell benchmark works around the nominal values $D_{nom} = 20$; $Q_{nom} = 2500$; $P_{nom} = 2800$; $X_{nom} = 500$. It was used a PRBS signal which is persistently exciting of any finite order, obtaining sufficiently informative open-loop experiments. It was collected 1000 samples, 800 samples were used for estimation and the remaining 200 samples were used for validation. The pre-treated signals were of the form $F_x = (F - F_{nom})/(std(F))$ where F can be D, Q, P or X.

4. Model Order Estimation

There is an extensive literature on algorithms to estimate the model order of a linear system in state space. In the subspace algorithms the determination of the model order n is very subtle. Ideally, this information can be determined by the number of singular values different from zero of the orthogonal or the oblique projections of row spaces of the Hankel's block matrix of input and output data. Nevertheless, when the system data contains noise, this number is not easy to calculate. In Figure 1, it is shown the spectrum of SVD values of the process with different values of Ns. Another procedure is to select the value n, which minimizes the estimation errors, a technique used by algorithms such as PEM, which requires a higher computational effort. There is a statistical criterion that can help obtaining the model order: the Akaike Information Criterion (AIC), defined in [1],

$$AIC(n) = \rho.ln[\sigma^2_{error}(n)] + 4P_n, \tag{4.1}$$

where ρ is the number of samples used in the identification, $\sigma^2_{error}(n)$ is the variance of modeling error for a model of order n with P_n parameters. The index $AIC(n)$ normally reaches a minimum for a certain number of parameters in the model. The application of the AIC criterion to the process with Ns = 0.2 is shown in Figure 2, where one can observe that the minimum value occurs for $n = 6$.

5. Model Performance

When there are families of models, one wants to know which of them is the most appropriate for a certain objective, since there is not a model that represents the real system in all its aspects. To evaluate the quality of the model, it is necessary

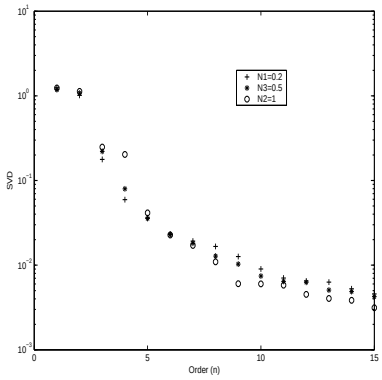

Figure 1: Spectrum of SVD for different values of Ns.

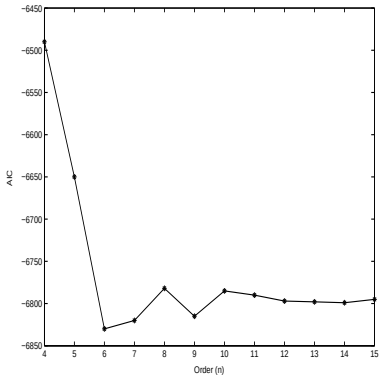

Figure 2: Spectrum of AIC with Ns = 0.2.

a criterion to measure the distance between the model and the real system. Performance indicators very used are mean relative square error (MRSE) and mean related variance (MVAF), which are defined as

$$MRSE(\%) = \frac{1}{l}\sum_{i=1}^{l}\sqrt{\sum_{j=1}^{N}(y-\hat{y})^2 / \sum_{j=1}^{N}(\hat{y})^2}. \quad 100, \tag{5.1}$$

$$MVAF(\%) = \frac{1}{l}\sum_{i=1}^{l}\Big(1 - \mathrm{var}(y-\hat{y})/\mathrm{var}(y)\Big). \quad 100, \tag{5.2}$$

where y is the real output and $\hat{y}$ is the output estimated by the obtained model. The MRSE index is usually used in the literature and the MVAF index is used by the SMI Toolbox. Both performance indexes are used to evaluate the quality of the model obtained by each algorithm, as shown in Table 1 (for $Ns = 0.2$ and order

$n = 6$), Table 2 (for $Ns = 0.5$ and order $n = 6$) and Table 3 (for $Ns = 1$ and order $n = 7$).

Table 1: Numerical results of algorithm performance for $Ns = 0.2$.

Algorithm	Processing time(s)	MRSE (%)	MVAF (%)
N4SID	0.703	2.7623	99.7376
MOESP	0.312	7.0001	98.4672
MON4SID	0.407	2.4367	99.7670
PEM	3.609	7.5114	98.9544
PEM (n=7)	4.953	2.3579	99.7364

Analyzing the values of Table 1, the MON4SID model is the best one in terms of cross validation. It is verified that the processing time to obtain the model is smaller for MOESP and is much larger for PEM. The MON4SID model was chosen to identify the Shell benchmark process, due to the good balance between its performance and processing time. The PEM model gives better performance with $n = 7$.

Table 2: Numerical results of algorithm performance for $Ns = 0.5$.

Algorithm	Processing time(s)	MRSE (%)	MVAF (%)
N4SID	0.672	3.5803	99.3617
MOESP	0.297	5.3490	98.3046
MON4SID	0.400	2.9910	99.4758
PEM	0.3594	3.5728	99.3316

Table 3: Numerical results of algorithm performance for $Ns = 1$.

Algorithm	Processing time(s)	MRSE (%)	MVAF (%)
N4SID	0.703	3.7106	99.1107
MOESP	0.396	4.3724	98.4669
MON4SID	0.407	3.1619	99.3167
PEM	4.453	3.4401	99.1423

Figure 3 shows the outputs generated by the process and by the identified model for $Ns = 0.2$. It can be observed that, for a given operating range, the identified model reproduces very well the main dynamic characteristics of the Shell benchmark process. Zero initial conditions were considered.

Analyzing the values in Tables 2 and 3, the MON4SID model is the best one in terms of cross validation. It is verified that the processing time to obtain the model is smaller for MOESP and is much larger for PEM.

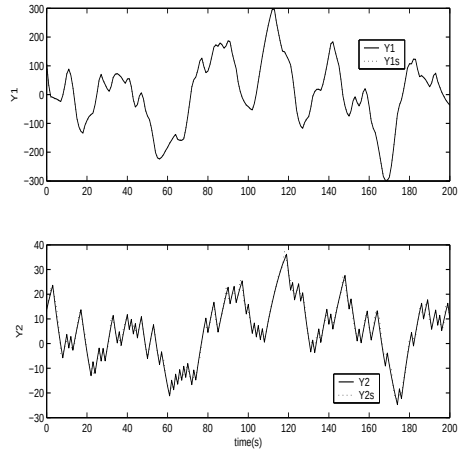

Figure 3: Comparison of process response (broken line) versus MON4SID model response (continuous line).

6. Conclusions

In this work, it was presented the MON4SID algorithm, which uses LQ factorization in the same way as the MOESP method, which is used to compute the oblique and orthogonal projections; given these projections it is computed the state sequence and the extended observability matrix, respectively. The past and future state sequences are computed from the state sequences which have only one initial state, it does not happen in the N4SID method, since for each oblique projection (Θ_i and Θ_{i+1}) different state sequences ($\tilde{X}_{i+1}$ and $\tilde{X}_i$) are computed, generating a problem of bias in the estimates. To evaluate this algorithm, three identification algorithms (MOESP, N4SID, PEM) were applied to the Shell benchmark process, to identify a MIMO model in discrete time state space and their results were compared. It is possible to observe that a linear model can provide a good description of a non-linear system within a certain operating range. The Akaike criterion provided the model order. The performance comparison was made according to cross validation for each algorithm, employing two different performance criteria. For this specific case, the MON4SID model was chosen, due to its performance and processing time. PEM was the slowest model out of the four tested ones. The obtained model is observable, controllable and asymptotically stable within a certain range of operation, and it can be applied in control and monitoring applications.

Resumo. Identificação por subespaços tem sido um tema de pesquisa ao longo dos últimos anos. Métodos como MOESP e N4SID são bem conhecidos eles usam a decomposição LQ de certas matrizes de dados de entrada e saída. Com base nestes métodos, é apresentado o método MON4SID, que usa a técnica dos métodos MOESP e N4SID.

Palavras-chave. Identificação por subespaços, método MOESP, método N4SID, processos industriais.

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